

New Integer Factorizations

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Abstract. New factorizations of Fibonacci numbers, Lucas numbers, and numbers of the form $2^n \pm 1$ are presented together with the strategy (a combination of known factorization methods) used to obtain them.

1. Introduction. This paper presents new complete factorizations of 174 large integers of interest. The factorizations have been obtained by the author over the past four years using a combination of known methods. Our factorization strategy, which has been implemented on a micro-programmable computer, makes extensive use of J. M. Pollard's factorization methods (Pollard [8] and Pollard [9]). The strategy also includes the well-known method of trial division, the continued fraction method (Morrison and Brillhart [6]), and the primality tests of Brillhart, Lehmer and Selfridge [3].

Section 2 contains a summary of our factorization strategy. Since we use known methods, no details of these methods are given. Section 3 contains the actual results and describes their form. We have attacked the cofactors (i.e. the factors remaining after the known factors are removed) of Fibonacci numbers, Lucas numbers, and numbers of the form $2^n \pm 1$, whose factorizations were not previously known.

The paper is a condensed version of Naur [7], which, in addition to the material in this paper, contains summaries of the factorization methods used and full factorization tables.

It should be noted that most of the factorizations presented here have been independently discovered by J. Brillhart, D. H. Lehmer, J. L. Selfridge, B. Tuckerman, S. S. Wagstaff, or their colleagues, possibly using a strategy similar to the one described here (Lehmer [5], Pollard [10], and Wagstaff [12]).

2. Strategy. The factorization strategy combines known methods and may be summarized as follows:

Step 1. Trial divide to 10^6 . This step hardly needs further comment (for an efficient implementation of trial division, see Wunderlich and Selfridge [18]).

Step 2. Use Pollard's two methods simultaneously. The $P - 1$ method is able to discover a prime factor p , if the factors of $p - 1$ are small. We use the first stage of the method as described at the end of Pollard [8]. For previous uses of the $P - 1$ method, see Williams and Judd [15] and [16], Williams and Seah [17], and Williams [13], the latter giving an account of the related $P + 1$ method.

Received August 11, 1982; revised February 23, 1983.
1980 *Mathematics Subject Classification.* Primary 10-04, 10A25, 10A40.

The rho method (or Monte Carlo method, see Pollard [9]) is usually able to find a prime factor p in about $O(p^{1/2})$ steps. Improvements of the rho method (see Brent [1] and Brent and Pollard [2]) have only been recently published and have not been used.

Both the $P - 1$ method and the rho method can be broken into smaller steps, and a step from each method is executed alternately.

Step 3. Having executed a suitable number of steps of Pollard's methods, the continued fraction method (Morrison and Brillhart [6]) is invoked, if the number has 53 or fewer digits.

Step 4. Whenever a factorization is discovered in Steps 2 or 3, both factors are tested for primality using the tests from Brillhart, Lehmer and Selfridge [3] (the number itself is also tested after Step 1). Since these tests require a certain number of factors of $N \pm 1$ to be known before the primality of N can be verified, these auxiliary factorizations are attempted simultaneously using our strategy recursively. When a sufficient number of factors is found, the factoring is stopped and the primality tests are carried out.

The above strategy has been implemented on the micro-programmable computer Mathilda, developed at the Aarhus University (Schriver and Kornerup [11]).

TABLE 1. *Fibonacci numbers*

n	Factorization of U_n
191.	4870723671313 * 757810806256989128439975793
221.	233 * 1597 : 203572412497 * 90657498718024645326392940193
233.	139801 * 25047390419633 * 631484089583693149557829547141
239.	10037 * 62141 * 2228536579597318057 * 28546908862296149233369
251.	582416774750273 * 21937080329465122026187124199656961913
253.	89 * 28657 : 4322114369 * 2201228236641589 * 1378497303338047612061
257.	5653 * 32971978671645905645521 * 1230026721719313471360714649
259.	13 * 73 * 149 * 2221 : 1553 * 404656773793 * 3041266742295771985148799223649
265.	5 * 953 * 55945741 : 15901 * 2741218753681 * 926918599457468125920827581
267.	2 * 1069 * 1665088321800481 : 122887425153289 * 64455877349703042877309
269.	5381 * 2517975182669813 * 32170944747810641 * 169360439829648789853
289.	1597 : 577 * 1733 * 98837 * 101232653 * 106205194357 * 658078658277725444483848541
291.	2 * 193 * 389 * 3084989 * 361040209 : 76674415738994499773 * 227993117754975870677
303.	2 * 743519377 * 770857978613 : 8550224389674481 * 96049657917279874851369421
305.	5 * 4513 * 555003497 : 2441 * 6101 * 20415253966247698801 * 647277670717998240933861
319.	89 * 514229 : 1913 * 578029 * 1435522969 * 1535414556003613 * 18626243184683463348283529
333.	2 * 17 * 73 * 149 * 2221 * 145900030513721 : 12653 * 124134848933957 * 930507731557590226767593761
355.	5 * 6673 * 46165371073 : 4261 * 75309701 * 309273161 * 9207609261398081 * 49279722643391864192801
361.	37 * 113 : 6567762529 * 1196762644057 * 3150927827816930878141597 * 12020126510714734783009241
367.	733 * 17969789 * 75991753 * 5648966761 * 43397676601 * 114150315493 * 797357235624701499134444201
369.	2 * 17 * 2789 * 59369 * 68541957733949701 : 8117 * 199261 * 84738793193 * 9382599520669 * 117838518633351469
377.	233 * 514229 : 104264251753 * 361575655741 * 608146585345567981670893199985449202015060094237

TABLE 2. *Lucas numbers*

n	Factorization of V_n
166.	$3 : 6464041 * 245329617161 * 10341247759646081$
197.	$31498587119111339 * 4701907222895068350249889$
211.	$33128448586319 * 3768695026320506495615952689771$
221.	$521 * 3571 : 2337127044022973021 * 3531495042124863863141$
227.	$39499 * 5098421 * 4311537234701 * 317351386961794678797301$
230.	$3 * 41 * 4969 * 275449 : 3116523496881881 * 2224700455311857347241$
242.	$3 * 43 * 307 : 200872171147 * 3564873012035809 * 13253086025993542387$
244.	$7 : 487 * 52471477541626010209 * 5500902230146438151405489047$
252.	$2 * 7^2 * 23 * 167 * 14503 * 103681 * 65740583 : 503 * 4322424761927$ $* 571385160581761$
253.	$139 * 199 * 461 : 13343097459037867049 * 439589715274978576995097049$
256.	$34303 * 73327699969 * 125960894984050328038716298487435392001$
257.	$2107028233569599 * 125090447782502159 * 1945042261468790758531$
258.	$2 * 3^2 * 313195711516578281 : 7772507 * 73254041816089 * 258422401920467$
269.	$13451 * 49098524855733491 * 290341026883813109 * 860882346042166879$
272.	$2207 : 4470047 * 7378607647 * 42848407775681 * 224189164930816106106049$
274.	$3 : 547 * 27947 * 86409516719752275209 * 461963939612677343458490143601$
277.	$1109 * 5923369 * 1003666289 * 322458613167451 * 3647646099535497480264359$
281.	$20567460049 * 46415343154434259 * 55678135331080359350346681814561$
286.	$3 * 43 * 307 * 90481 : 5147 * 2441129996120243$ $* 13092861035652370656608696909281$
287.	$29 * 370248451 : 256579 * 319973431 * 101731310703289$ $* 10635841025639256246541$
288.	$2 * 1087 * 4481 * 11862575248703 : 270143 * 25033626656641$ $* 1974737795746080149567$
292.	$7 : 839207 * 121355783 * 2864461601 * 4953066392881$ $* 1045794092558661358680161$
296.	$47 : 15400289 * 19088449 * 77894162661647 * 89311781152481$ $* 754276330346432303$
299.	$139 * 461 * 521 : 599 * 2233531 * 1194215681621 * 143236388738249$ $* 40197222522537856361$
301.	$29 * 6709 * 144481 : 39488879317091 * 1050474234583201$ $* 689529693448123842995171$
302.	$3 : 70963651961 * 95305716283 * 64119657493918388500959028976916724219027$
304.	$2207 : 607 * 1823 * 20063 * 91807 * 1156984541407 * 12441241017224321$ $* 52601970578546783$
308.	$7^2 * 263 * 881 * 967 * 14503 : 7872253927 * 9623520524969002343$ $* 1935298980672778761041$
309.	$2^2 * 619 * 1031 * 5257480026438961 : 1270029990781$ $* 2216051880587916003268636813231$
310.	$3 * 41 * 3020733700601 : 180501911066713425499001$ $* 911316263659755894779625401$
320.	$127 * 186812208641 : 62379555831803099867272961$ $* 5079180256659675431743744001$
322.	$3 * 281 * 4969 * 275449 : 643 * 770867 * 25154641 * 163674763583689$ $* 8357802723902097130683089$
331.	$526291 * 54184296181 * 4386848568249611$ $* 11957954590103942275063852978039182929$
334.	$3 : 821641 * 7162963 * 50187047747 * 14167898020159929481$ $* 504752765667203736366779801$
338.	$3 * 90481 : 2027 * 141283 * 404112157123 * 478061565712797524641$ $* 2892106995173496522201467$
339.	$2^2 * 412670427844921037470771 : 44607276283528829839$ $* 954423225346040964978868549$
340.	$7 * 2161 * 23230657239121 : 5441 * 897601 * 17276792316211992881$ $* 38349368324041346444974961$
348.	$2 * 7 * 23 * 299281 * 834428410879506721 : 56058952425321966662183$ $* 1185031046372137517381447$
350.	$3 * 41 * 281 * 401 * 570601 * 12317523121 : 2801 * 28001$ $* 248773766357061401 * 7358192362316341243805801$
374.	$3 * 43 * 67 * 307 * 63443 : 2243 * 49369 * 3827019260681$ $* 1586361987756363049 * 12812807672672518125975550387$

TABLE 2. (Continued)

n Factorization of V_n

- 391. 139 * 461 * 3571 : 10949 * 2476673936041
* 834484880498372128537891307445941852925566419249911616229
- 393. 2^2 * 1049 * 414988698461 * 5477332620091 : 32845130922638389
* 43274370280887890687749341750584741809
- 399. 2^2 * 29 * 211 * 229 * 9349 * 95419 * 10694421739 * 2152958650459 : 28729
* 519499 * 561434197549 * 252171167457207277136719
- 400. 2207 * 23725145626561 : 107420801 * 177736001 * 51793685214662401
* 7601587101128729489773008667804801
- 402. 2 * 3^2 * 6163 * 201912469249 * 2705622682163 : 1609 * 2915186157721
* 3625049985433518620724629754945150956569
- 410. 3 * 41^2 * 163 * 800483 * 350207569 : 628774904181521
* 143860188296781167161 * 2322429099336692919718294260481
- 411. 2^2 * 541721291 * 78982487870939058281 : 242491 * 104446810724929
* 18070186267894449189347092077457768249
- 441. 2^2 * 19 * 29 * 211 * 1009 * 31249 * 65269 * 620929 * 8844991 * 599786069
: 35281 * 80642113181244469 * 16247350756640617732192770750349
- 444. 2 * 7 * 23 * 10661921 * 114087288048701953998401 : 887 * 2663 * 17761
* 21423730326721 * 1753583251175771127559228156664349601
- 450. 2 * 3^3 * 41 * 107 * 401 * 601 * 2521 * 570601 * 10783342081
* 87129547172401 : 1801 * 186374563189054810201
* 427694148584338087778220001
- 457. 143392891 * 48175086409 * 864351271995241
* 53865562038701008975397146407705442118820462326130285905669299
- 474. 2 * 3^2 * 21803 * 5924683 * 14629892449 * 184715524801 : 947 * 10279163
* 8411395441 * 5922309413062354009 * 376943442492584130991581889
- 478. 3 : 632890270126128456721 * 414612475582425401119754697066276232016-
4758443697460091641243037362105014072881
- 483. 2^2 * 29 * 139 * 211 * 461 * 691 * 1289 * 1485571 * 1917511
* 965840862268529759 : 9661 * 236235695207989
* 9952648158500556841649035737455844289
- 498. 2 * 3^2 * 6464041 * 245329617161 * 10341247759646081 : 20276569
* 93750172283 * 212216314620580244514251999177476639338737695720283

3. Results. The factorization results are given in Tables 1 through 6. All factorizations listed are complete, i.e. all factors are prime. The algebraic factors (if any) are separated from the primitive factors by a colon (:). We have not given any details of the actual methods used to obtain the various factorizations.

The author has investigated the Fibonacci numbers U_n , defined by $U_0 = 0$, $U_1 = 1$, $U_{n+2} = U_{n+1} + U_n$, for odd n , $1 \leq n \leq 399$, and the Lucas numbers V_n , defined by $V_0 = 2$, $V_1 = 1$, $V_{n+2} = V_{n+1} + V_n$, for all n , $0 \leq n \leq 500$. Tables 1 and 2 contain factorizations of U_n and V_n , respectively, which have not been listed as complete in Jarden [4], Morrison and Brillhart [6], Williams and Judd [15] and [16], Williams and Holte [14], or Williams [13].

The numbers $2^n - 1$ have been investigated for odd n , $1 \leq n \leq 299$, and Table 3 contains factorizations of numbers of this form which have not been listed as complete in Brillhart, Lehmer and Selfridge [3] or Williams [13]. The numbers $2^n + 1$ have been investigated for $0 \leq n \leq 300$, $2 \leq n \leq 598$, if $n = 4k + 2$. Table 4 contains factorizations of $2^n + 1$ with $n \neq 4k + 2$, which have not been listed as

TABLE 3. $2^n - 1$, n odd

n Factorization of $2^n - 1$

169. 8191 : 4057 * 6740339310641 * 3340762283952395329506327023033
 173. 730753 * 1505447 * 70084436712553223 * 155285743288572277679887
 185. 31 * 223 * 616318177 : 1587855697992791 * 7248808599285760001152755641
 191. 383 * 7068569257 * 39940132241 * 332584516519201 * 87274497124602996457
 193. 13821503 * 61654440233248340616559 * 14732265321145317331353282383
 199. 164504919713 * 4884164093883941177660049098586324302977543600799
 205. 31 * 13367 * 164511353 : 2940521 * 70171342151
 * 3655725065508797181674078959681
 213. 7 * 228479 * 48544121 * 212885833 : 66457 * 2849881972114740679
 * 4205268574191396793
 215. 31 * 431 * 9719 * 2099863 : 1721 * 731516431 * 514851898711
 * 297927289744047764444862191
 217. 127 * 2147483647 : 5209 * 62497 * 6268703933840364033151
 * 378428804431424484082633
 219. 7 * 439 * 2298041 * 9361973132609 : 3943 * 671165898617413417
 * 4815314615204347717321
 223. 18287 * 196687 * 1466449 * 2916841 * 1469495262398780123809
 * 596242599987116128415063
 235. 31 * 2351 * 4513 * 13264529 : 2391314881 * 72296287361
 * 73202300395158005845473537146974751
 237. 7 * 2687 * 202029703 * 1113491139767 : 1423 * 49297 * 23728823512345609279
 * 31357373417090093431
 243. 7 * 73 * 2593 * 71119 * 262657 * 97685839 : 487 * 16753783618801
 * 192971705688577 * 3712990163251158343
 247. 8191 * 524287 : 15809 * 6459570124697 * 402004106269663
 * 1282816117617265060453496956212169
 267. 7 * 618970019642690137449562111 : 78903841 * 28753302853087
 * 24124332437713924084267316537353
 271. 15242475217
 * 248927757868131890277330541567820045256364273970773286542188386932989391
 273. $7^2 * 79 * 127 * 337 * 911 * 8191 * 121369 * 112901153 * 23140471537$
 : 108749551 * 4093204977277417 * 86977595801949844993
 279. 7 * 73 * 2147483647 * 658812288653553079 : 16183 * 34039 * 1437967
 * 833732508401263 * 2034439836951867299888617
 283. 9623 * 68492481833
 * 23579543011798993222850893929565870383844167873851502677311057483194673
 285. 7 * 31 * 151 * 191 * 32377 * 524287 * 1212847 * 420778751 * 30327152671
 : 1491477035689218775711 * 25349242986637720573561
 287. 127 * 13367 * 164511353 : 17137716527
 * 51954390877748655744256192963206220919272895548843817842228913
 295. 31 * 179951 * 3203431780337 : 4721 * 132751 * 5794391 * 128818831
 * 3812358161 * 452824604065751 * 4410975230650827973711

complete in Brillhart, Lehmer and Selfridge [3] or Williams [13]. For $n = 4k + 2$,

$$2^n + 1 = (2^{2k+1} - 2^{k+1} + 1)(2^{2k+1} + 2^{k+1} + 1),$$

which has been investigated for $1 \leq 2k + 1 \leq 299$. Tables 5 and 6 contain factorizations of these factors which have not been listed as complete in Brillhart, Lehmer and Selfridge [3] or Williams [13].

TABLE 4. $2^n + 1$

n Factorization of 2^n+1

151. 3 : 18717738334417 * 50834050824100779677306460621499
 152. 257 : 27361 * 69394460463940481 * 11699557817717358904481
 157. 3 : 15073 * 2350291 * 17751783757817897 * 96833299198971305921
 163. 3 : 11281292593 * 1023398150341859 * 337570547050390415041769
 169. 3 * 2731 : 4929910764223610387 * 18526238646011086732742614043
 172. 17 : 3855260977 * 64082150767423457 * 1425343275103126327372769
 179. 3 : 58745093521 * 4347868190665879373495950562775707707143803
 184. 257 : 43717618369 * 549675408461419937 * 3970299567472902879791777
 188. 17 : 1198107457 * 23592342593 * 4501946625921233 * 181352306852476069537
 193. 3 : 6563 * 35679139 * 1871670769 * 7455099975844049
 * 1280761337388845898643
 197. 3 : 197002597249 * 1348959352853811313 * 251951573867253012259144010843
 208. 65537 : 928513 * 18558466369 * 23877647873 * 21316654212673
 * 715668470267111297
 209. 3 * 683 * 174763 : 419 * 3410623284654639440707
 * 1607792018780394024095514317003
 211. 3 : 4643 * 9878177 * 5344743097 * 199061567251
 * 22481127512575175864234185190299
 219. 3^2 * 1753 * 1795918038741070627 : 9070197542196643
 * 3278244690156222434135906137
 221. 3 * 2731 * 43691 : 443 * 4714692062809
 * 4507513575406446515845401458366741487526913
 223. 3 : 219256122131 * 2049349592095043950407650450918171260318303154708405513
 227. 3 : 297371 * 3454631579714210387
 * 69982170658265444713117545258712031103399659
 235. 3 * 11 * 283 * 165768537521 : 328006342451 * 461797907949997211
 * 235457374510092115086834691
 243. 3^6 * 19 * 163 * 87211 * 135433 * 272010961 : 1459 * 139483
 * 10429407431911334611 * 918125051602568899753
 245. 3 * 11 * 43 * 281 * 86171 * 4363953127297 : 491 * 15162868758218274451
 * 50647282035796125885000330641
 249. 3^2 * 499 * 1163 * 2657 * 155377 * 13455809771 : 9202419446683
 * 3388098290567587377052016525627948593
 253. 3 * 683 * 2796203 : 4049 * 85009 * 31797547 * 81776791273
 * 2822551529460330847604262086149015242689
 260. 17 * 61681 * 858001 * 308761441 : 42641 * 5746001 * 2400573761
 * 65427463921 * 173308343918874810521923841
 261. 3^3 * 19 * 59 * 3033169 * 96076791871613611 : 523 * 6929826139
 * 3453412901832690553 * 33563856450515702761
 264. 97 * 257 * 673 * 229153 * 119782433 * 43872038849 : 16875081675650881
 * 86945388997210442828259494992321
 273. 3^2 * 43 * 2731 * 5419 * 224771 * 1210483 * 22366891 * 25829691707 : 547
 * 105310750819 * 292653113147157205779127526827
 277. 3 : 25792643401363
 * 3138280009399679017344631051542622769205877134953845128202334345822857
 279. 3^3 * 19 * 529510939 * 715827883 * 2903110321 : 26227 * 119232435043
 * 85384915399027 * 6444365376140611199022187
 288. 641 * 6700417 * 18446744069414584321 : 3457 * 816769
 * 1562985901350085709953 * 1422346738975853644793916289

TABLE 5. $2^n - 2^{(n+1)/2} + 1$, n odd

n	Factorization of $2^n - 2^{(n+1)/2} + 1$
149.	5 : 12961064789 * 11011808951971745915313242336927641
157.	5 : 2790467761 * 5941035366826969 * 2203942033439148343973
169.	53 * 157 : 677 * 615946323850313 * 215656329382891550920192462661
173.	5 : 13625405957 * 175739665310505752968877740350313227534889
179.	5 : 31815461 * 416115013830990336221 * 11575709336636595278866333
181.	5 : 9413 * 178925762979037 * 3830538323149121 * 95016376135553173181
187.	5 * 397 * 26317 : 26509131221 * 35155077044989397 * 4029292065629191839853
219.	5 * 293 * 9929 * 649301712182209 : 877 * 1013533 * 704710824913 * 142406868765525436670617
225.	13 * 37 * 41 * 61 * 101 * 1201 * 8101 * 29247661 * 1182468601 : 413150254353901 * 3192261504216112476901
235.	5^2 * 3761 * 7484047069 : 941 * 894434441 * 3357909154141 * 38425816980821 * 722501809616926841
243.	5 * 109 * 246241 * 106979941 * 168410989 : 3333950193493 * 1753477469677913202190537606674204157
245.	5^2 * 29 * 197 * 47392381 * 19707683773 : 306178659371201 * 1372226516822701 * 1008787906424294727221
249.	13 * 997 * 46202197673 * 209957719973 : 136453 * 218166829 * 41732461753 * 5791487405427228378717709
253.	5 * 397 * 1013 * 1657 : 6994042018866541 * 621109541542884571802304568790331501283098925929529
257.	22988734297 * 10073811610622418028425741738319757818107396980605471702450570926313
267.	5 * 123794003928545064364330189 : 3401264941 * 11221454641 * 10038055841545956979111137292020661
269.	5 : 2153 * 3229 * 5381 * 4273873 * 1633401082697 * 3918695179304214327885157 * 185382112947811828276076281
273.	13^2 * 53 * 113 * 157 * 313 * 1249 * 1429 * 4733 * 556338525912325157 : 503413 * 467811806281 * 275700717951546566946854497
275.	5^3 * 397 * 268501 * 3630105520141 : 12101 * 35201 * 698617420601 * 18735216413769901 * 225117233926884384606401
289.	137 * 953 : 7698961 * 21886549 * 113478990853 * 398410160527221094178749181184472290805236187881699426313

TABLE 6. $2^n + 2^{(n+1)/2} + 1$, n odd

n	Factorization of $2^n + 2^{(n+1)/2} + 1$
137.	5 : 168434085820849 * 206875670104957744917147613
151.	5 : 4373689270176379261201 * 130530323901899210670077
167.	5 : 75005713 * 27395325377910797 * 18208260781190156536114609
169.	5 * 1613 : 180201997 * 1259036730797 * 408946876729703992293841657
173.	7152893721041 * 1673815085186574700322174232069942181681
183.	5 * 733 * 1709 * 368140581013 : 12836737570021 * 414194958733796530899181
187.	137 * 953 * 2113 : 5237 * 551353793 * 1819762572673 * 135322045917118601273437
191.	5 : 3821 * 89618875387061 * 1833085153842665442652283234165143433597
193.	5 : 3089 * 148997 * 14402030644704405877 * 378791300027089635677652285973
203.	113 * 536903681 : 9810958633253 * 21597468549493958664902504331670645757
209.	5 * 229 * 397 * 457 : 6689 * 2039731321 * 149832750683283097 * 1937385241416564065603093
217.	5 * 29 * 8681 * 49477 : 31249 * 776729668507005203702993 * 139335546032913681584758997
219.	13 * 9444732965601851473921 : 371335727233 * 18478609113710122023550126425157
237.	13 * 604462909806215075725313 : 18890331057055511701 * 1487840558911519281039078769
239.	5 : 77852679293 * 2269474963255693085711432948387582114817557263546457947501201
243.	13 * 37 * 279073 * 3618757 * 4977454861 : 2917 * 4861 * 26129603777437 * 15778453094691989880197773477
249.	5 * 13063537 * 148067197374074653 : 1993 * 80485166514184335373 * 583117579691967491546961181
253.	277 * 2113 * 30269 : 25301 * 109297 * 756550961 * 2569737193 * 9623862953 * 156296877661 * 101027360307659633
263.	5 : 119929 * 731141 * 99972364781 * 338153229347093487293402061645864051641494661202651405269
265.	5^2 * 1801439824104653 : 51941 * 24082141 * 31213331016701 * 33716583668208510447368101472499412321
285.	13 * 41 * 61 * 761 * 131101 * 160969 * 525313 * 2416923620660807201 * 1457772869697961 * 64326196787727903551977150861
297.	5 * 109 * 397 * 42373 * 246241 * 4327489 * 15975607282273 : 2377 * 22573 * 155399494141 * 4712151755917 * 41523259994275786297957
299.	53 * 157 * 277 * 30269 : 20333 * 956801 * 15595841 * 19294368341 * 6339840806910833 * 393345821366273907459718331839045409

4. Acknowledgements. The author wishes to thank K. Andersen, P. Kornerup, J. K. Kjærgård, J. W. Nielsen, I. H. Sørensen, S. M. Sørensen, F. Wibroe, and O. Østerby for their generous support and guidance in various phases of this work. Many thanks are also due to D. H. Lehmer and S. S. Wagstaff for making the Cunningham Project tables available to the author.

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