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Remark on a Lemma by R. Wong and J. P. McClure

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Abstract. A short proof is presented for a formula arising in the above-mentioned paper.

In [1] the authors prove the following lemma:

Let f and g be C^∞-functions, and let n be a nonnegative integer. Then

$$\begin{aligned}&[f(x)g^{n+1}(x)]^{(n+1)}\\&\quad=-\sum_{p=0}^{n}\binom{n+1}{p+1}[f(x)g^{n-p}(x)]^{(n+1)}(-g(x))^{p+1}\\&\qquad+(n+1)!f(x)\big(g'(x)\big)^{n+1}.\end{aligned}$$

This formula is important in deriving a Taylor series expansion for the Dirac δ-function. The proof of the lemma is rather complicated and covers two pages in print. Therefore the following simple proof may be of interest.

Proof. Obviously

$$g(t)-g(x)=(t-x)\big(g'(x)+\varepsilon(t,x)\big),$$

where for fixed x the function $\varepsilon(t, x)$ belongs to C^∞ with respect to t, and

$$\lim_{t\to x}\varepsilon(t,x)=0.$$

Then

$$\begin{aligned}&[f(x)g^{n+1}(x)]^{(n+1)}+\sum_{p=0}^{n}\binom{n+1}{p+1}[f(x)g^{n-p}(x)]^{(n+1)}(-g(x))^{p+1}\\&=\left(\frac{d^{n+1}}{dt^{n+1}}\Big[f(t)\big(g(t)-g(x)\big)^{n+1}\Big]\right)_{t=x}\\&=\left(\frac{d^{n+1}}{dt^{n+1}}\Big[(t-x)^{n+1}f(t)\big(g'(x)+\varepsilon(t,x)\big)^{n+1}\Big]\right)_{t=x}\\&=(n+1)!f(x)\big(g'(x)\big)^{n+1},\end{aligned}$$

where the first equality follows from the binomial theorem, and the last equality is obtained by using the Leibniz rule.

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1. R. WONG & J. P. MCCLURE, "On a method of asymptotic evaluation of multiple integrals," *Math. Comp.*, v. 37, 1981, pp. 509–521.