

describing the algorithms for these tasks and others such as computing the eigenvalues and eigenvectors of a matrix, interpolation and approximation, quadrature, etc. The treatment of these matters is on a very elementary level and is to a great extent superfluous. And if we were to accept the assumption of the author that a good knowledge of the algorithms is necessary in order to use them, then there are some serious omissions. Thus, in the discussion of the solution of linear equations, there is no mention of scaling, while in the description of the QR algorithm, the crucial point of using shifts is ignored. One serious mistake occurs on p. 225, where the definition that is given for a (convergent) improper integral is actually the definition of a Cauchy principal value integral.

Finally, since this book also deals with the writing of mathematical software, it provides at least one example of our thesis. In the chapter on quadrature, adaptive quadrature is described using a local strategy. Current practice prefers the use of a global strategy. Hence, anyone writing an adaptive quadrature routine based on the material in this book is not using the best strategy, and twenty years from now, he will still be writing such programs, instead of referring to a mathematical software library for a program using the latest techniques.

P. R.

5[46–01, 65J10].—COLIN W. CRYER, *Numerical Functional Analysis*, Oxford University Press, New York, 1982, iv + 568 pp., 24 cm. Price \$39.00.

This book is intended as the first volume of two. It is concerned with teaching the foundations of Functional Analysis and with its interplay with Numerical Analysis. The second volume will treat elliptic boundary value problems and nonlinear problems.

Special features of this text are as follows: In the systematic introduction of concepts of Functional Analysis frequent stops are made for applications relevant to Numerical Analysis. Thus the students immediately see the concepts in action. Many counterexamples are given to delineate the basic definitions and theory. There is plenty of exercises, and they come with solutions or references.

The choice of material is standard, as the following list of the first eight chapter headings indicates: Introduction, Topological vector spaces, Limits and convergence, Basic spaces and problems, The principle of uniform boundedness, Compactness, the Hahn-Banach theorem, Bases and projection. The ninth and last chapter covers approximate solution of linear operator equations (about a hundred pages) with emphasis on integral equations. This book also includes a list of notation, an excellent index and, still more excellent, a list of theorems, lemmas and corollaries. It ends with references and solutions, the latter covering a hundred and fifty pages.

I found the treatment very well suited to what I perceive as the “typical student” in Applied Mathematics. Concepts are thoroughly motivated, and interesting and enlightening applications to Numerical Analysis (and related fields) are given. In the long list of counterexamples I missed one of a linear map defined on the whole of a Banach space but discontinuous (one occurs implicitly on p. 273). In my experience in teaching Functional Analysis this is one counterexample students will be asking for if you withhold it for too long. It should be pointed out that it is not assumed

that the reader knows Measure Theory or Lebesgue Integration. These things are briefly summarized in an appendix to Chapter Four.

The only serious mistake I found is in connection with the Principle of Uniform Boundedness and its application to the Lax Equivalence Theorem. It is assumed, (5.47) and also p. 155, that the semigroup $E(t)$, the solution operator in L_2 to the homogeneous heat equation, is continuous with respect to t in the operator norm. This is false. However, for the Lax Equivalence Theorem it suffices that $E(t)v$ is continuous in t for each fixed v in L_2 , also known as “strong convergence”. The mistake is embarrassing since the proof of (5.47) is given as a problem, no. 5.29. The “solution” given to this problem ends by purporting to have shown a certain inequality. This inequality actually shows only “strong convergence”. However, the proof of even this weaker result is completely wrong, although the result is true.

I also found two problems that I regard as “marginally misleading”. Problem 8.27 might lead the unwary student to deduce a result which is true only in one space dimension. Interpreting the undefined and unlisted symbol $\mathcal{C}_0^2(\bar{\Omega})$ in the most charitable sense, one looks for solutions of a “smooth” second order elliptic problem $Lu = f \in \mathcal{C}(\bar{\Omega})$ with homogeneous Dirichlet conditions in $\mathcal{C}^2(\Omega) \cap \mathcal{C}^0(\bar{\Omega})$ which are zero on $\partial\Omega$ (classical solutions). In more than one space dimension the problem is correct only by virtue of its hypotheses always being false. (E.g., the problem $\Delta u = f$ in Ω a bounded domain in R^n , $n \geq 2$, $u = 0$ on $\partial\Omega$ nice, will *not* have a classical solution for every f in $\mathcal{C}(\bar{\Omega})$.) Problem 8.20 might lead the unsuspecting reader to believe that the Legendre series for a function in $\mathcal{C}^2([-1, 1])$ converges only to order $n^{-1/2}$ in the maximum norm; this is a full order n^{-1} off.

My impression is that in spite of minor flaws this is an excellent text for a stimulating one-year course in Functional Analysis with applications.

L. B. W.

6[65–01].—J. F. BOTHA & G. F. PINDER, *Fundamental Concepts in the Numerical Solution of Differential Equations*, Wiley, New York, 1983, xii + 202 pp., 24 cm. Price \$24.95.

This volume is intended for the novice practitioner who needs to read up quickly on basic practical numerical methods for (mainly) partial differential equations. It by and large bypasses theory, but some understanding and much practical advice is given. In the above respects it resembles von Rosenberg’s brief volume, [2]. von Rosenberg’s book treated finite difference methods, whereas the present exposition, written a decade and a half later, gives equal space to Galerkin Finite-Element methods, Collocation Finite-Element methods, and Boundary Element methods. It can be viewed as a pared-down version of Lapidus and Pinder [1]. I quote from the Preface: “This book is designed to provide an affordable reference on the methodology available for the solution of ordinary and partial differential equations in science and engineering.”

It is hard to judge whether the authors have succeeded in their goal. Certainly the treatment is brisk and to the point and always elucidated with examples showing the “how-to” of the method. A basic tenet is said to be the following: “The various