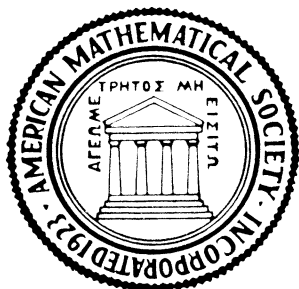


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January 1986

Volume 46, Number 173, Pages 1–360

**Published by the American Mathematical Society
Providence, Rhode Island USA**

ISSN 0025-5718

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