REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers is brackets are assigned according to the American Mathematical Society classification scheme. The 1980 Mathematics Subject Classification (1985 Revision) can be found in the December index volumes of Mathematical Reviews.

19[65-02, 65G10, 65J10].—EDGAR W. KAUCHER & WILLARD L. MIRANKER, *Self-Validating Numerics for Function Space Problems, Computation with Guarantees for Differential and Integral Equations*, Academic Press, Orlando, Fla., 1984, xiii + 255 pp., $23\frac{1}{2}$ cm. Price $28.00.

This report concerns an area which is situated half-way between classical numerical analysis and symbolic computation (computer algebra). On the one hand, functions and function space operators constitute explicit objects in the suggested algorithms, on the other hand all relations are eventually projected into some $\mathbf{R}^N$ so that only classical computational processes are executed. With a suitable programming system at hand, this projection should naturally be carried out automatically in the spirit of computer algebra; however, it seems that expert human interaction and guidance will be necessary for the achievement of reasonable results.

The authors treat two aspects of function space problems: The generation of an approximate solution, and the "validation" of such an approximate solution, i.e., the generation of a function strip about it such that the exact solution lies in this strip. The process is called self-validation because it may run automatically on a digital computer and because the successful completion of the algorithm is equivalent to a mathematical proof of the existence of the exact solution within the strip.

The basic ideas are straightforward: The coefficients of approximate solutions in a finite-dimensional subspace are found as solutions of algebraic problems which arise by "rounding" the infinite-dimensional problem into the subspace. The dimension of the approximation may be recursively increased in an iterative defect correction way. The great variety of possible roundings (Taylor rounding, Chebyshev rounding, spline rounding, etc.) offers a high flexibility. The defect-correction formulation is mandatory in the validation process which uses generalizations of classical fixed point theorems: The strip must be mapped into itself by a finite-dimensional interval extension of a mapping whose fixed point is the analytic solution. With a proper choice of the mapping and with sufficient effort, a narrow inclusion of the analytic solution may be obtained. Note that the into-ness of the mapping on a suitably generated strip can be discovered *by the computer* via the comparison of coefficient intervals.

The authors have initiated a direction of research and algorithmic design in numerical analysis which should become increasingly important (e.g., for use on high performance work-stations). They present the fundamental mechanisms of their approach quite clearly with the aid of a good number of well-chosen examples. Unfortunately, they have not always been successful in making the details of their processes transparent; a closer study of the report is made unnecessarily difficult by a notation which is often clumsy and inconsistent, by an excess of detail in some places coupled with a complete absence of analysis of important aspects, and a multitude of misprints and little errors. The crucial problem of how to start the validation process so that it may be successful is not analyzed. (The recipe given does not work in the elaborated examples, a fact which is covered up by "short cuts".) Also the choice of the approximate inverse in the validation iteration is left open. Nearly all interesting details are missing in the illustrative examples of Chapter 7.

The reviewer does not share some views of the authors: their analogy with floating-point computation may be more misleading than helpful, and their "block-relaxation" mechanism appears clearly inferior to simply solving larger-dimensional systems. However, he strongly feels that this volume points the way to important developments in numerical analysis; the many open questions and the interesting implementation problems should attract the attention of numerical and computational researchers.

H. J. S.

20[45-01].—ABDUL J. JERRI, *Introduction to Integral Equations with Applications*, Marcel Dekker, New York, Basel, 1985, x + 254 pp., $23\frac{1}{2}$ cm. Price $39.75.

This little text is simply written and easy to read, and it could easily be used as a one- or a two-semester undergraduate course for an introduction to integral equations. It could possibly also be used to teach a one-semester engineering mathematics course on integral equations, along with the one-semester ordinary and partial differential equations courses that are being taught at many universities. The subject matter is motivated with some applications, as well as with connections to ordinary and partial differential equations. The majority of the text is written in a nonrigorous style, the exception being Chapter 6, where the contraction mapping principle is presented; even there, an attempt is made at simplifying the functional analysis concepts. Only the simplest numerical methods of solution are discussed; for example, the only numerical integration techniques used are the midordinate, the trapezoidal and Simpson's rule.

In its table of contents one finds the following chapter headings:

F. S.