**21[33-04, 41A60, 42C10, 65Q05].**—JET WIMP, *Computation with Recurrence Relations*, Pitman, Boston, 1984, xii + 310 pp., 24 cm. Price $50.00.

This monograph is a timely and welcome addition to the literature. The modern numerical theory of difference equations began with an inspiration of the late J. C. P. Miller while he was engaged upon the last table-making project to be completed by the British Association for the Advancement of Science. It proved to be impossible to compute the modified Bessel function $I_n(x)$ by forward recurrence on the order $n$ owing to severe instability. Miller realized, however, that this function could be generated easily and very accurately by backward recurrence on $n$, *with almost arbitrary starting values*, provided that the results were multiplied subsequently by an easily-found normalizing factor. This procedure, published eventually in the Introduction to the B. A. tables in 1952, is known now as Miller's algorithm. Since then, a steadily increasing stream of research papers on the whole subject has been issuing from many writers. Jet Wimp himself has made important contributions, and he now becomes the first author to attempt a book that treats all of these developments. The attempt is very successful. Virtually all of the more significant developments are described clearly and concisely; equally important, they are illustrated by carefully chosen examples drawn from various applications areas.

This book is not restricted to numerical methods, however. One of the appendices supplies a concise account of the main results in the asymptotic theory of linear difference equations, results that are illustrated by abundant applications in the main text. Almost anyone who has struggled with the operational version of this theory given in the textbook of Milne-Thomson, or has been discouraged by the massive papers of Birkhoff and Trjitzinsky, will be pleased by this easily-understood and easily-applied account by Wimp.

Another useful appendix—also included to make the book as self-contained as possible—sketches the principal results in the general analytic theory of linear difference equations.

Three-quarters of the main text, eleven chapters in all, is devoted to linear equations. The main topics covered are: Miller's algorithm, with extensions by Gautschi and others, for computing minimal solutions of inhomogeneous first-order equations and homogeneous second-order equations; an algorithm of the reviewer, with variations and extensions by other investigators, for computing intermediate solutions of inhomogeneous second-order equations; generalizations of these algorithms for equations of higher order; computations with orthogonal polynomials, especially Clenshaw's algorithm and its extensions for summing series expansions; the methods of Clenshaw, Elliott and Thacher for expanding solutions of ordinary differential equations in series of powers or orthogonal polynomials. Most of the algorithms are accompanied by proofs of convergence, under appropriate conditions. For some algorithms, particularly that of Miller, numerical and/or asymptotic methods are used to estimate the effects of truncation errors and demonstrate stability with respect to rounding errors. Particularly valuable, because of their inaccessibility in the published literature, are the accounts of the very general algorithm of Lozier for the stable computation of any solution of a broad class of linear difference equations, and the method of Lewanowicz for the construction of recurrence relations for the coefficients of series expansions in Gegenbauer polynomials.

Except for the expansions of solutions of ordinary differential equations, most of the applications are to difference equations satisfied by higher transcendental functions, particularly those belonging to the hypergeometric family. Another immense source of linear difference equations is the numerical discretization of ordinary differential equations. This area is not touched by Wimp, yet it is difficult to quarrel with this exclusion. The discretization process introduces its own convergence problems and sources of error, and a comprehensive treatment would have taken the subject matter well away from the main thrust of this reasonably-sized volume. Moreover, some aspects of this application have been treated in a recent book by J. R. Cash [1].

The remaining part of the main text, Chapters 12 to 14, treats systems of nonlinear difference equations. The aspects here are somewhat different. The emphasis is on convergence to fixed points of the corresponding operators, rather than on error analysis and stability. These chapters provide a brief introduction to convergence questions, invariants and divergence theory (strange attractors). Examples include arithmetic-harmonic means, arithmetic-geometric means, infinite products and generalizations of the algorithm of Gauss and Landen. Much of the applications centers on the computation of definite and indefinite integrals of elliptic type. Some aspects, for example, methods for the acceleration of convergence, are complemented by results given in an earlier book by the same author [2].

On the cover, the publisher claims "This book will be of interest to computer scientists, applied mathematicians, physicists and engineers. It contains a comprehensive, state-of-the-art, account of computational techniques based on the general recurrence relation $\mathbf{x}(n+1) = \mathbf{f}[\mathbf{x}(n), n]$". This reviewer endorses this claim wholeheartedly.

F. W. J. O.

1. J. R. Cash, *Stable Recursions, with Applications to the Numerical Solution of Stiff Systems*, Academic Press, London, 1979.

2. J. Wimp, *Sequence Transformations and Their Applications*, Academic Press, New York, 1981.

**22[46-01, 65J05].**—R. E. Moore, *Computational Functional Analysis*, Ellis Horwood Series, Mathematics and its Applications (G. M. Bell, Editor), Halsted Press, Wiley, Chichester, New York, 1985, 156 pp., $23\frac{1}{2}$ cm. Price $34.95.

Most areas of mathematics have their roots in the sciences. In fact, entire branches of mathematics arose from attempts to understand and analyze certain physical phenomena. As a discipline matures, however, it can often stray from its origins until it reaches a point in its evolution where it becomes self-sufficient. Its subsequent development can become so esoteric that students (even experts) have no inkling of its practical origins and applications, and indeed are amazed when informed of its usefulness in solving "real-world" problems. A frequently mentioned example is functional analysis.