experience be able to handle the exercises on convergence in Chapter 8 or have a chance of understanding the difficult and more advanced notion of Fréchet differentiation in Chapter 16? Even with good backgrounds in advanced calculus, most students must struggle to absorb the concepts of functional analysis.

In addition, a more extensive knowledge of applied mathematics is necessary before a student can appreciate the powerful methods of functional analysis and how they aid in the understanding and solution of applied problems. Undergraduate courses in differential equations usually do not include applications of sufficient complexity to require functional-analytic techniques in their solutions. A complex real-world problem, such as the fluid-flow problem discussed in Chapter 20, is probably beyond the grasp of a student whose applied mathematical experience consists of a single course in differential equations.

I am always attracted by analysis textbooks, and especially by those which purport to explore the rich and fruitful relationships between analysis and the applications. My on-going search will not end with this book.

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23[65B05, 65J05].—K. Böhmer & H. J. Stetter, Editors, *Defect Correction Methods—Theory and Applications*, Springer-Verlag, Wien, New York, 1984, vi + 242 pp., $24\frac{1}{2}$ cm. Price $20.00.

Numerical analysis is rich in iterative methods of diverse types, for example, Newton-Raphson, Gauss-Seidel, multigrid, iterative refinement, and deferred correction. A talk given at the 1973 Dundee Conference by P. E. Zadunaisky, which proposed estimating errors in the numerical solution of ODEs by determining the errors in the numerical solution of a neighboring problem with a known analytical (piecewise polynomial) solution, stimulated H. J. Stetter to propose yet another iterative method. It became apparent that this new method shared with so many other iterative methods the idea of computing a correction based on the computation of a relatively accurate residual, and hence Stetter formulated the "defect correction principle" in a paper which appeared in 1978. The use of the distinctive term "defect" for "residual" had been introduced by R. Frank and C. W. Ueberhuber and was probably helpful in attracting interest to this novel approach to iterative processes. Indicative of its rapid acceptance is the inclusion of a section entitled "Splitting methods and defect corrections" in the 1984 report of the NRC Committee on Applications of Mathematics.

This book is the proceedings of a 1983 Oberwolfach working conference on "Error Asymptotics and Defect Corrections." It is not an attempt to compile a book on defect correction. Rather it is a heterogeneous collection of papers tied together by the common thread of defect correction. Authors and titles follow:

Böhmer, Hemker, Stetter: Introduction: the defect correction approach.
Frank, Hertling, Lehner: Defect correction algorithms for stiff ODEs.

Reinhardt: On a principle of direct defect correction based on a posteriori error estimates.
Chatelin: Simultaneous Newton's iteration for the eigenproblem.
Mandel: On some two-level iterative methods.
Hackbusch: Local defect correction method and domain decomposition techniques.
McCormick: Fast adaptive composite grid methods.
Hemker: Mixed defect correction iteration for the solution of a singular perturbation problem.
Rump: Solution of linear and nonlinear algebraic problems with sharp, guaranteed bounds.
Kaucher, Miranker: Residual correction and validation in functoids.
Böhmer, Gross, Schmitt, Schwarz: Defect corrections and Hartree-Fock method.
Pereyra: Deferred corrections software and its application to seismic ray tracing.
Schönauer, Schnepf, Raith: Numerical engineering: experiences in designing PDE software with selfadaptive variable stepsize/variable order difference methods.

A number of these papers are condensations or revisions of earlier work. Some of them are very difficult to penetrate; others are highly readable, for example, the paper by Hemker, which brings together interesting results from previous papers on the use of alternating defect correction to create hybrid difference schemes for convection-dominated flow problems. Among the more novel papers were those of Hackbusch and McCormick on the construction of discretizations for composite grids. (Equation (3.10a) of the Hackbusch paper seems to have the inequality backwards.)

The introductory paper, written especially for the book, describes defect correction. It is not a popularized treatment of the subject useful to the nonspecialist, but rather a fairly precise and complete technical discussion. Unfortunately, the basic idea is to some extent obscured because of the several versions and extensions that are presented. Because these various manifestations of defect correction are so loosely related, it is better to regard it not as a method but as a pattern (a word twice used by the authors) for iterative methods for solving equations in vector and function spaces. The sole unifying theoretical concept is that of a contraction mapping, and even this idea must be modified in certain applications where a strictly limited number of iterations are performed. Such is sometimes the case for deferred correction, where one has at best a "pseudo-contraction" involving a sequence of progressively weaker, but more relevant, norms. (In this connection the reviewer would like to remark that the "crucial role" attributed to asymptotic expansions of the global error is an overstatement. They are useful but not necessary.) In defect correction a correction is computed from a residual, using a cheap approximation to the inverse of the operator. This may mean replacing a matrix by a "nearby" matrix which is easier to factor, or a high-order discretization by a low-order discretization, or a fine-grid discretization by a coarse-grid discretization, or an exact inverse by a finite-precision inverse. This simplified inverse may be linear or nonlinear. If it is linear, then defect correction amounts to nothing more than simplified Newton-Raphson, which is a pattern well established in numerical analysis. It is the possibility of doing nonlinear simplification that makes the defect correction principle interesting and worthwhile. Significant examples that come to mind are deferred correction and A. Brandt's FAS extension of the multigrid idea to nonlinear problems.

Simplified Newton iteration (including iterative refinement) is of especial importance in interval analysis because of the considerable pessimism of interval extensions of direct methods such as Gaussian elimination. It is better to do the initial computation using point values and to use intervals to compute corrections. Of crucial importance is the very accurate calculation of residuals. Often these residuals are inner products and in most other cases they can be so expressed by rewriting the problem. For this reason, U. Kulisch, W. Miranker, and others have advocated that in addition to the four arithmetic operations, there ought to be a built-in (microprogrammed) operation that delivers an inner product to the full precision of the computer. The paper by Rump describes algorithms for the solution of linear and nonlinear systems of equations based on this Kulisch/Miranker arithmetic, and these are implemented in the IBM program product ACRITH, on the market since March 1984. The paper of Kaucher and Miranker goes beyond this and considers the solution of equations in function space. The development in their paper is guided by an analogy between the digit-by-digit decimal expansion of a number and the term-by-term Chebyshev series expansion (for example) of a function. Both papers provide impressive examples, and together they seem to form a definitive condensation of the Kulisch/Miranker approach. However, the unfamiliar notation and terminology and the excessive formalisms are likely to deter any reader other than an interval analysis enthusiast. (This is typical of work in interval analysis and may be partly responsible for its unfortunate isolation from mainstream numerical analysis.) In addition, the substance of the Kulisch/Miranker approach has been criticized. The calculation of an inner product to full precision can be quite time-consuming because of the need for a Super-Accumulator in order to store the intermediate results to whatever precision is necessary. Also, examples have been given by W. Kahan/E. LeBlanc showing the ill effects of having to rewrite the problem so that the residuals are expressible as inner products; one such example is the rewriting of a continued fraction as the ratio of polynomials. Finally, it remains to be demonstrated that the goals of reliability and high accuracy could not be achieved instead with the use of double-precision interval arithmetic for selected intermediate results.

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24[65-02].—GENE H. GOLUB & CHARLES F. VAN LOAN, *Matrix Computations*, The Johns Hopkins University Press, Baltimore, Md., 1983, xvi + 476 pp., $23\frac{1}{2}$ cm. Price \$49.50 hardcover, \$24.95 paperback.

The authors admit to having taken 6 years to write this book. Those who have experienced the energy and enthusiasm which Professor Golub brings to everything