

## REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1980 Mathematics Subject Classification (1985 Revision) can be found in the December index volumes of *Mathematical Reviews*.

**36[65–01].**—KENDALL ATKINSON, *Elementary Numerical Analysis*, Wiley, New York, 1985, xii + 416 pp., 24cm. Price \$31.95.

This book is a first-rate textbook for a basic course in numerical analysis. As stated in the preface, the author has several objectives in teaching the course to students. “First, they should obtain an intuitive and working understanding of some numerical methods for the basic problems of numerical analysis (as specified by the chapter headings). Second, they should gain some appreciation of the concept of error and of the need to analyze and predict it. And third, they should develop some experience in the implementation of numerical methods using a computer.” The material presented in this textbook is consistent with these objectives. The range of topics dealt with in the book is illustrated by the chapter titles: Taylor Polynomials, Computer Representation of Numbers, Error, Rootfinding, Interpolation, Approximation of Functions, Numerical Integration and Differentiation, Solution of Systems of Linear Equations, and The Numerical Solution of Differential Equations. The author provides a wide range of problems at the end of each subsection and answers to selected problems. The book includes sample programs written in Fortran 77 (the formatting of the listing could be improved, but this is a minor issue). The text even includes an appendix describing sources of numerical software packages.

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**37[65N30, 65N15, 73K25].**—J. TINSLEY ODEN & GRAHAM F. CAREY, *Finite Elements, Mathematical Aspects*, Vol. IV, Prentice-Hall, Englewood Cliffs, N.J., 1983, viii + 195 pp., 23½cm. Price \$37.95.

Computer modelling of physical phenomena is a discipline that is developing rapidly and gaining name recognition (known variously as scientific computation or computational mathematics, with subfields such as computational mechanics or computational fluid dynamics). Some people identify this as the emergence of a third methodology in science and engineering, complementing experimental and theoretic-

cal (pencil and paper) studies. Among the various practitioners of the discipline, one finds a wide range of philosophical perspectives that guide the particular approach taken by individual investigators. One key variable characterizing different approaches is the role that mathematical rigor plays in the research. The subject can be described, in part, as the numerical approximation of solutions to abstract physical models. Thus one could, at one extreme, demand a proof that the numerical approximation reflects the properties of the mathematical model in the full detail that is actually programmed on the computer (and that the model itself be well posed in some sense). At the other extreme, algorithms could be developed based only on heuristic or physical arguments. It is useful to consider where the book under review fits into this variable's spectrum of possibilities.

Professor Oden's background is unusual in that he has done research at a rigorous level mathematically, as well as worked directly on the finite element method as a tool in engineering practice. The book under review is one volume of a six-volume series (the Texas Finite Element Series, all published by Prentice-Hall) reflecting this broader perspective, and the book of interest here represents the rigorous side of the subject. This series presents an integration of different approaches, and a reader interested in studying the possible interplay between theory and practice could consult the other volumes. However, this reviewer has not done a study of the other volumes in the series, and the remainder of the review will focus on Volume 4 only. Moreover, this volume is quite compact, and so it seems appropriate to make this review quite short as well.

The examples in the introduction used to motivate elliptic systems of partial differential equations are very good. The book begins with one of the simplest possible (Laplace's equation) and then describes the full three-dimensional equations of linear elasticity, thus giving a good feeling for the level of complexity that can arise in applications after starting with a simple example introducing the key ideas. The book assumes a high level of mathematical sophistication on the reader's part. For example, one of the first homework problems is to prove the Riesz representation theorem! Further, important results concerning Sobolev spaces that are later used are only quoted, not derived. Thus, the book would form the basis for an advanced graduate course only if the students were sufficiently well prepared or if the instructor supplemented the book with lectures on some fundamental functional analysis and function spaces.

To this reviewer's taste, there are both weak and strong points concerning the authors' selection of material on finite element discretizations, the heart of the book. These opinions are, of course, subjective, but they are included here for readers who might wish to know topics to emphasize, supplement, etc., in using the book as a text or reference for a course. Unisolvence of elements is not discussed as extensively as might be possible. For example, systematic techniques for constructing elements are not given. More examples like the Argyris element at the end of Section 2.3 would make this part of the subject richer. On the other hand, the general discussion of mixed methods is quite good, and one of the main selling points of the book. Additional detailed calculations like the verification of the inf-sup condition in Section 4.4.3 would also be useful. Finally, the information in the chapter on hybrid methods is not to be found elsewhere in book form, as far as we know.

The intent of the book seems to be to collect in one place some mathematical results directly or indirectly relevant to finite element methodology. It can serve as a compact reference to these results, but it would be hard for students of mathematics to use this as a course text, without extensive supplements.

R.S.

**38[65N99].**—DEREK B. INGHAM & MARK A. KELMANSON, *Boundary Integral Equation Analyses of Singular, Potential, and Biharmonic Problems*, Lecture Notes in Engineering (C. A. Brebbia and S. A. Orszag, Editors), Springer-Verlag, Berlin, Heidelberg, New York, 1984, xiv + 173 pp., 24 cm. Price \$12.50.

As the title of the book suggests, the purpose of this work is to extend the range of applicability of the boundary integral equation (BIE) method to include certain biharmonic problems and nonlinear potential problems, particularly problems which involve boundary singularities. The presence of these singularities greatly reduces the rate of convergence of standard BIE procedures. In their treatment of such problems, the authors modify the classical BIE method to take into account the analytic form of such a singularity, and demonstrate the efficacy of the modified method by comparing it with the classical BIE method on an appropriate test problem. In each problem discussed in this book, the approximate solution is piecewise constant, and, whenever possible, integrations are performed analytically, which, the authors claim, substantially reduces cpu time.

Chapter 1 is a brief introductory chapter. Chapter 2 is devoted to a discussion of the BIE solution of a biharmonic problem arising in fluid flow problems involving “stick-slip” boundary conditions, which give rise to a boundary singularity. In Chapter 3, methods similar to those developed in Chapter 2 are used to solve problems of flow near sharp corners, which involve corner singularities. Chapter 4 is concerned with certain nonlinear potential problems in which the differential equation can be linearized by applying the Kirchhoff transformation. The resulting problem not only has nonlinear boundary conditions but also boundary singularities. (It should be noted that Eq. (5) of this chapter is incorrect; the integral should be multiplied by  $\varphi^{-1}$ .) Viscous flow problems are also discussed in Chapters 5 and 6. In Chapter 5, problems with free surfaces, which are nonlinear, are considered, while Chapter 6 is devoted to a study of slow flow in bearings with arbitrary geometries. In Chapter 7, some conclusions are briefly stated. In Chapters 2 to 6, the results of numerical experiments are presented.

This book is an unusual publication. Not only is it a photo reproduction of the complete Ph.D. thesis of the second author, but five of its seven chapters, Chapters 2 to 6, have appeared in their entirety in five separate papers in refereed journals under the sole authorship of the second author. Since each of these chapters is self-contained, with its own abstract, introduction, conclusions, and references, there is substantial repetition, the elimination of which would considerably reduce the size of the book without any loss of information. Moreover, each of Chapters 2 to 6 is presented in the format of a preprint, with tables and figures gathered together after the references, and not inserted in the text where they are first mentioned. This,