together with the lack of consistency in the notation, makes the book awkward to read.

There is no question that the material presented in this book is of much interest to researchers concerned with the development and application of BIE methods, particularly those interested in solving slow viscous flow problems. But it ought not to have been published in this form, when it has already appeared verbatim in the open literature.

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39[93B40, 93C20, 93C75].—K. L. TEO & Z. S. WU, *Computational Methods for Optimizing Distributed Systems*, Mathematics in Science and Engineering, Vol. 173, Academic Press, Orlando, Fla., 1984, xiii + 317 pp., $23\frac{1}{2}$cm. Price \$68.50.

With the rapid decrease of cost of computer power, the possibilities of using mathematical modelling in science and engineering have dramatically increased during the last decade. In particular, it is possible today to use computer-implemented numerical methods to solve complicated optimal control problems which may not be solved by classical analytical or ad hoc methods. This is particularly true for optimal control problems involving partial differential equations, and there is thus a great practical interest in having efficient numerical methods for such problems. There is a comparatively rich literature on theoretical-mathematical aspects of optimal control of partial differential equations, but much less so concerning numerical methods. The present book aims at partly filling this gap and is concerned with computational methods for optimal control of partial differential equations or distributed parameter systems. This problem area is very vast, including, in addition to analysis of the continuous problem, also discretization using finite element or finite difference methods and application of optimization methods for finite-dimensional problems, together with related convergence questions. The emphasis of the book in this wide spectrum is towards the continuous problem, particular attention being given to a mathematical convergence theory for certain gradient type methods for some optimal control problems involving (linear) parabolic equations. Discretized problems occur in the numerical examples, but are not treated theoretically. The material is based mainly on research of the authors and their associates.

A brief outline of the contents of the book is as follows: Chapters 1 and 2 contain background material from functional analysis and existence and regularity theory for linear parabolic partial differential equations. Chapter 3 is concerned with a class of optimal control problems involving linear parabolic equations with the controls occurring (nonlinearly) in lower-order derivative terms and in the forcing term. The controls are supposed to belong to a compact and convex subset of $\mathbb{R}^m$, and the cost functional is essentially a linear functional of the state at a given terminal time. This is a typical example of a stochastic optimal control problem with Markov terminal time. For this problem the authors consider a gradient type method involving, as is usual, the solution of an adjoint problem. An algorithm for constructing a sequence of controls with improving cost is presented. Theoretical aspects of the convergence of the algorithm are discussed and two numerical examples from stochastic population dynamics, involving one-dimensional parabolic problems, are presented. In Chapter 4, results of the previous chapter are extended, e.g., to problems involving also parameter selection, and two applications are made to one-dimensional parabolic problems arising in a study of choosing an optimal level of advertising in a marketing problem. Chapter 5 is concerned with the theoretical problem of existence of optimal controls, using suitable weak topologies and corresponding relaxed controls. Finally, Chapter 6 contains material similar to that of Chapter 3, now with Neumann boundary conditions occurring in the parabolic problem, and an application using a finite element discretization to a problem of optimally heating a slab of metal is made.

As indicated, the emphasis of the book is on theoretical aspects of computational procedures. The presentation is fairly technical and requires a relatively good mathematical background. The book addresses an important problem area of potentially great practical significance and presents several interesting contributions, together with brief overviews of earlier literature and an extensive bibliography.

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40[65L05, 65L20, 65M20].—K. Dekker & J. G. Verwer, *Stability of Runge-Kutta Methods for Stiff Nonlinear Differential Equations*, CWI Monograph 2, North-Holland, Amsterdam, 1984, ix + 307 pp., $24\frac{1}{2}$cm. Price $36.50.

This book is concerned with aspects of the numerical solution of ordinary differential equations.

The work of Dahlquist, as presented in the classical book of Henrici [3], culminated in the so-called equivalence theorem: Convergence is equivalent to consistency and stability. This result covers the case when the stepsize tends to zero. However, more often we are interested in using a stepsize as large as possible. The stability behavior for a fixed step sequence is important.

Based on this vague requirement, Dahlquist [2] introduced the fundamental concept of A-stability for linear multistep methods. Later, A-stability was defined for other classes of methods, amongst them the family of Runge-Kutta methods. Contrary to linear multistep methods, whose order under the constraint of A-stability is bounded by two, A-stable Runge-Kutta methods of arbitrarily high order were shown to exist. But the implementation in efficient software was more difficult.

A-stability is based on the simple test equation $y' = \lambda y$, $\lambda \in \mathbf{C}$. In order to understand how a Runge-Kutta method would behave on a nonlinear problem, Burrage and Butcher [1] studied B-stability, i.e., the stability of the method on monotone differential systems. B-stability is a stronger requirement than A-stability. As a consequence, fewer methods are B-stable than A-stable.

The theme of this book is stability of Runge-Kutta methods for nonlinear stiff problems. As the title indicates, the exposition is largely theoretical. The book is