usual, the solution of an adjoint problem. An algorithm for constructing a sequence of controls with improving cost is presented. Theoretical aspects of the convergence of the algorithm are discussed and two numerical examples from stochastic population dynamics, involving one-dimensional parabolic problems, are presented. In Chapter 4, results of the previous chapter are extended, e.g., to problems involving also parameter selection, and two applications are made to one-dimensional parabolic problems arising in a study of choosing an optimal level of advertising in a marketing problem. Chapter 5 is concerned with the theoretical problem of existence of optimal controls, using suitable weak topologies and corresponding relaxed controls. Finally, Chapter 6 contains material similar to that of Chapter 3, now with Neumann boundary conditions occurring in the parabolic problem, and an application using a finite element discretization to a problem of optimally heating a slab of metal is made.

As indicated, the emphasis of the book is on theoretical aspects of computational procedures. The presentation is fairly technical and requires a relatively good mathematical background. The book addresses an important problem area of potentially great practical significance and presents several interesting contributions, together with brief overviews of earlier literature and an extensive bibliography.

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40[65L05, 65L20, 65M20].—K. DEKKER & J. G. VERWER, *Stability of Runge-Kutta Methods for Stiff Nonlinear Differential Equations*, CWI Monograph 2, North-Holland, Amsterdam, 1984, ix + 307 pp., $24\frac{1}{2}$cm. Price \$36.50.

This book is concerned with aspects of the numerical solution of ordinary differential equations.

The work of Dahlquist, as presented in the classical book of Henrici [3], culminated in the so-called equivalence theorem: Convergence is equivalent to consistency and stability. This result covers the case when the stepsize tends to zero. However, more often we are interested in using a stepsize as large as possible. The stability behavior for a fixed step sequence is important.

Based on this vague requirement, Dahlquist [2] introduced the fundamental concept of A-stability for linear multistep methods. Later, A-stability was defined for other classes of methods, amongst them the family of Runge-Kutta methods. Contrary to linear multistep methods, whose order under the constraint of A-stability is bounded by two, A-stable Runge-Kutta methods of arbitrarily high order were shown to exist. But the implementation in efficient software was more difficult.

A-stability is based on the simple test equation $y' = \lambda y$, $\lambda \in \mathbf{C}$. In order to understand how a Runge-Kutta method would behave on a nonlinear problem, Burrage and Butcher [1] studied B-stability, i.e., the stability of the method on monotone differential systems. B-stability is a stronger requirement than A-stability. As a consequence, fewer methods are B-stable than A-stable.

The theme of this book is stability of Runge-Kutta methods for nonlinear stiff problems. As the title indicates, the exposition is largely theoretical. The book is devoted to the central question of stability. The purpose is to give a sound mathematical treatment of the development in the stability of Runge-Kutta methods from the A-stability childhood to today's B-stability manhood.

In the book's title there is one very important word, stiff. The usual way to define stiffness is through the eigenvalues of the system Jacobian. That definition links stability and stiffness. However, it has become more and more clear that there is more to stiffness than that. In the first chapter the authors therefore try to describe and identify stiff systems. As they phrase it on the first page, "The essence of stiffness is that the solution to be computed is slowly varying but that perturbations exist which are rapidly damped." Basically, that is correct, but the problem is how to describe stiffness in a firm mathematical way. As far as I can see, the authors are not able to give a precise definition. My conclusion is that we still do not know what a stiff system is.

Other related concepts, such as dissipativity and logarithmic norm, are introduced here.

In Chapters 2, 4, and 6 contractivity is defined and discussed. It is here that the results on B-stability are collected. However, although the book's title says nonlinear stability, I feel that it would have been natural to also include a discussion of A-stability, all the more so since the order star theory essentially characterizes that concept. There is no book yet that gives a treatment of this important work from 1978.

Stable Runge-Kutta methods are implicit. In Chapter 5 recent work on the existence of solutions to the respective nonlinear systems is described. Under the condition of a one-sided Lipschitz constant strictly smaller than zero the existence of a unique solution can be guaranteed.

Chapter 7 is devoted to B-convergence. The idea is to give an equivalence theorem for stiff systems.

A chapter on Rosenbrock methods and a chapter on stiff systems from semidiscretizations of partial differential equations are also included.

In summary, I will formulate my views as follows:

(1) The book is an up-to-date monograph on nonlinear stability of Runge-Kutta methods. However, too often the authors only tell where proofs can be found. In that way one needs a good collection of journals if one really wants to know how the results are obtained.

(2) The important work on order stars is mentioned only briefly. The book would have been much more complete with a chapter on that subject.

(3) The construction of Runge-Kutta methods is hardly mentioned. The implementation of the methods into good software is not considered at all. (I am aware of the fact that a discussion of that deserves a book on its own.)

(4) The book contains recent results on nonlinear stability. As such, it deserves to be read by all interested in that topic. It cannot be used as a textbook, but serves as a good reference text to the latest work in the field of stability for Runge-Kutta methods. The authors deserve our thanks for a valuable piece of work.

S. P. N.

1. K. BURRAGE & J. C. BUTCHER, "Stability criteria for implicit Runge-Kutta methods," *SIAM J. Numer. Anal.*, v. 16, 1979, pp. 46–57.

2. G. DAHLQUIST, "A special stability problem for linear multistep methods," *BIT*, v. 3, 1963, pp. 27–43.

3. P. HENRICI, *Discrete Variable Methods in Ordinary Differential Equations*, Wiley, New York, 1962.

41[65L05, 65L20].—RICHARD C. AIKEN (Editor), *Stiff Computation*, Oxford University Press, New York, 1985, xiv + 462 pp., 24 cm. Price $75.00.

The term *stiff differential systems* has been around for more than 30 years. It was introduced in 1952 by Curtiss and Hirschfelder [1]. In the intervening years, research on stiff systems has developed into different directions. We have learned to understand some of the mathematical properties of methods intended for such systems. A-stability is well known to all of us. In fact, several definitions of stability have been advanced. Around the same time as stiffness was born, the first code, based on the Kutta-Merson method, was written. From there on, we have witnessed a tremendous surge in the development of codes for stiff systems. These include codes intended both for numerical libraries and for use in larger simulation software, although the simulation researchers did not always adopt the best codes available.

The book under review contains the proceedings of a conference held April 12–14, 1982, in Park City, Utah. The purpose of this meeting was "to review the state of the art and practice of stiff computation, rather than to present latest research." Further, from the book's preface: "The speeches represented the spectrum of individuals involved in stiff computation from theoretical to software developer to end user." To me, this is important; researchers from all these aspects of stiff computation ought to be brought together to encourage maximum interaction.

In Chapter 1 Shampine describes stiffness. Most of us have a feeling for what stiffness is, but it is not easy to put it down in writing. This chapter clarifies the situation. However, there is still room for a precise definition, if that is possible.

Chapter 2 is devoted to application areas where stiff systems appear.

Newer methods for stiff systems are reviewed in Chapter 3, which is followed by a chapter on current software packages. Indeed, an interesting list of the most popular codes is found here. Naturally enough, a chapter on software tailored to specific applications is included.

It would be difficult to avoid a chapter on theoretical questions, presented by the field's "godfather", Germund Dahlquist. Let me cite Dahlquist: "Nothing is more practical than a good theory." The truth of this statement is deep. To write an efficient and robust code, we need theoretical insight.

The final chapter is most revealing. Cellier gives his opinion on where stiff computation is going. He points to very interesting open problems. Some of them are nearly solved, while others are wide open. Examples of the former are problems with discontinuities, examples of the latter, parallel methods. The chapter ends with a lively panel discussion. Everyone interested in stiff systems should read this part.

Let me close by citing Shampine from the last chapter: "... the theory is moving closer to practice...." To me, this is the ultimate goal of research. This book is a