1. K. BURRAGE & J. C. BUTCHER, "Stability criteria for implicit Runge-Kutta methods," *SIAM J. Numer. Anal.*, v. 16, 1979, pp. 46–57.

2. G. DAHLQUIST, "A special stability problem for linear multistep methods," *BIT*, v. 3, 1963, pp. 27–43.

3. P. HENRICI, *Discrete Variable Methods in Ordinary Differential Equations*, Wiley, New York, 1962.

**41[65L05, 65L20].**—RICHARD C. AIKEN (Editor), *Stiff Computation*, Oxford University Press, New York, 1985, xiv + 462 pp., 24 cm. Price $75.00.

The term *stiff differential systems* has been around for more than 30 years. It was introduced in 1952 by Curtiss and Hirschfelder [1]. In the intervening years, research on stiff systems has developed into different directions. We have learned to understand some of the mathematical properties of methods intended for such systems. $A$-stability is well known to all of us. In fact, several definitions of stability have been advanced. Around the same time as stiffness was born, the first code, based on the Kutta-Merson method, was written. From there on, we have witnessed a tremendous surge in the development of codes for stiff systems. These include codes intended both for numerical libraries and for use in larger simulation software, although the simulation researchers did not always adopt the best codes available.

The book under review contains the proceedings of a conference held April 12–14, 1982, in Park City, Utah. The purpose of this meeting was "to review the state of the art and practice of stiff computation, rather than to present latest research." Further, from the book's preface: "The speeches represented the spectrum of individuals involved in stiff computation from theoretical to software developer to end user." To me, this is important; researchers from all these aspects of stiff computation ought to be brought together to encourage maximum interaction.

In Chapter 1 Shampine describes stiffness. Most of us have a feeling for what stiffness is, but it is not easy to put it down in writing. This chapter clarifies the situation. However, there is still room for a precise definition, if that is possible.

Chapter 2 is devoted to application areas where stiff systems appear.

Newer methods for stiff systems are reviewed in Chapter 3, which is followed by a chapter on current software packages. Indeed, an interesting list of the most popular codes is found here. Naturally enough, a chapter on software tailored to specific applications is included.

It would be difficult to avoid a chapter on theoretical questions, presented by the field's "godfather", Germund Dahlquist. Let me cite Dahlquist: "Nothing is more practical than a good theory." The truth of this statement is deep. To write an efficient and robust code, we need theoretical insight.

The final chapter is most revealing. Cellier gives his opinion on where stiff computation is going. He points to very interesting open problems. Some of them are nearly solved, while others are wide open. Examples of the former are problems with discontinuities, examples of the latter, parallel methods. The chapter ends with a lively panel discussion. Everyone interested in stiff systems should read this part.

Let me close by citing Shampine from the last chapter: "... the theory is moving closer to practice...." To me, this is the ultimate goal of research. This book is a good contribution in that direction. Although a collection of individual papers, the book presents the material in a coherent way.

S. P. N.

1. C. F. CURTISS & J. O. HIRSCHFELDER, "Integration of stiff equations," *Proc. Nat. Acad. Sci. U.S.A.*, v. 38, 1952, pp. 235–243.

**42[65L05, 65L20].**—W. H. HUNDSDORFER, *The Numerical Solution of Nonlinear Stiff Initial Value Problems: An Analysis of One-Step Methods*, CWI Tract 12, Centre for Mathematics and Computer Science, Amsterdam, 1985, 138 pp., 24 cm. Price Dfl. 20.30.

This monograph is a reprint of the author's Ph.D. thesis written at the University of Leiden under the supervision of Professor M. N. Spijker. Broadly speaking, the topic considered is the use of one-step methods to solve nonlinear stiff initial-value problems of the form

$$y'(t) = f(y(t)), \qquad t > t_0,\ y(t_0) = y_0,$$

satisfying the one-sided Lipschitz inequality

$$\text{(1)} \qquad \operatorname{Re}\{(f(x) - f(y), x - y)\} \leqslant \beta(x - y, x - y),$$

where $t$, $\beta \in \mathbf{R}$ and $f, x, y \in \mathbf{C}^n$ (although sometimes restricted to $\mathbf{R}^n$).

However, as is the case for most good theses, this monograph examines in depth a much more narrowly defined topic. More specifically, the one-step methods that the author considers are restricted to implicit and semi-implicit Runge-Kutta methods, the latter being of the form

$$y_{n+1} = y_n + h \sum_{i=1}^{m} b_i(hJ(y_n))f(Y_i),$$

$$Y_i = y_n + h \sum_{j=1}^{i-1} a_{ij}(hJ(y_n))f(Y_j), \qquad 1 \leqslant i \leqslant m,$$

where $b_i$ and $a_{ij}$ are rational functions with real coefficients. Two classes of semi-implicit Runge-Kutta methods are examined in particular: the Rosenbrock methods for which $J(y_n) = f_y(y_n)$, and those for which $J$ is constant.

The two principal questions that the author addresses are:

(i) Assuming only that $f$ is continuous and satisfies (1), what conditions on the stepsize $h$ and the coefficients of a Runge-Kutta method ensure that the algebraic equations associated with the method are well defined and have a unique solution?

(ii) To what extent do the conclusions about the numerical approximations which can be drawn for the simple test problem $y' = \lambda y$, $\lambda \in \mathbf{C}$, $\operatorname{Re}\lambda \leqslant \beta$, carry over to nonscalar nonlinear problems satisfying inequality (1)?

In addressing question (ii), the author pays particular attention to developing useful stability bounds for methods that are strongly $A$-stable but not $B$-contractive. This is done by suitably restricting the class of nonlinear problems to which the results apply, without limiting the stiffness of the problems under consideration.

In addition to developing several new results in answer to these questions, the author presents a concise summary of preliminary material needed to address these topics, as well as a review of results presented earlier by himself and others.