good contribution in that direction. Although a collection of individual papers, the book presents the material in a coherent way.

S. P. N.

1. C. F. CURTISS & J. O. HIRSCHFELDER, "Integration of stiff equations," *Proc. Nat. Acad. Sci. U.S.A.*, v. 38, 1952, pp. 235–243.

42[65L05, 65L20].—W. H. HUNDSDORFER, *The Numerical Solution of Nonlinear Stiff Initial Value Problems: An Analysis of One-Step Methods*, CWI Tract 12, Centre for Mathematics and Computer Science, Amsterdam, 1985, 138 pp., 24 cm. Price Dfl. 20.30.

This monograph is a reprint of the author's Ph.D. thesis written at the University of Leiden under the supervision of Professor M. N. Spijker. Broadly speaking, the topic considered is the use of one-step methods to solve nonlinear stiff initial-value problems of the form

$$y'(t) = f(y(t)), \qquad t > t_0,\ y(t_0) = y_0,$$

satisfying the one-sided Lipschitz inequality

$$\text{(1)} \qquad \operatorname{Re}\{(f(x) - f(y), x - y)\} \leqslant \beta(x - y, x - y),$$

where $t, \beta \in \mathbf{R}$ and $f, x, y \in \mathbf{C}^n$ (although sometimes restricted to $\mathbf{R}^n$).

However, as is the case for most good theses, this monograph examines in depth a much more narrowly defined topic. More specifically, the one-step methods that the author considers are restricted to implicit and semi-implicit Runge-Kutta methods, the latter being of the form

$$y_{n+1} = y_n + h \sum_{i=1}^{m} b_i(hJ(y_n))f(Y_i),$$

$$Y_i = y_n + h \sum_{j=1}^{i-1} a_{ij}(hJ(y_n))f(Y_j), \qquad 1 \leqslant i \leqslant m,$$

where b_i and a_{ij} are rational functions with real coefficients. Two classes of semi-implicit Runge-Kutta methods are examined in particular: the Rosenbrock methods for which $J(y_n) = f_y(y_n)$, and those for which J is constant.

The two principal questions that the author addresses are:

(i) Assuming only that f is continuous and satisfies (1), what conditions on the stepsize h and the coefficients of a Runge-Kutta method ensure that the algebraic equations associated with the method are well defined and have a unique solution?

(ii) To what extent do the conclusions about the numerical approximations which can be drawn for the simple test problem $y' = \lambda y$, $\lambda \in \mathbf{C}$, $\operatorname{Re}\lambda \leqslant \beta$, carry over to nonscalar nonlinear problems satisfying inequality (1)?

In addressing question (ii), the author pays particular attention to developing useful stability bounds for methods that are strongly A-stable but not B-contractive. This is done by suitably restricting the class of nonlinear problems to which the results apply, without limiting the stiffness of the problems under consideration.

In addition to developing several new results in answer to these questions, the author presents a concise summary of preliminary material needed to address these topics, as well as a review of results presented earlier by himself and others.

However, the prospective reader should note that this book does not contain a wide-ranging review of stability for stiff nonlinear initial-value problems. Nor is it a guide for the practicing scientist or engineer seeking to find advice on the numerical solution of stiff nonlinear problems. But it should not be faulted for failing to address these subjects: This book is exactly what it purports to be—a well-written research monograph on the narrowly focused topic that it addresses.

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43[41-02, 46-02].—ALLAN PINKUS, *n-Widths in Approximation Theory*, Springer-Verlag, Berlin, Heidelberg, New York, 1985, x + 291 pp., 25 cm. Price $39.00.

The n-width measures how well a subset of a normed linear space can be approximated by n-dimensional subspaces. A typical, and very important, example is the approximation of smooth functions. With this application in mind, Kolmogoroff [2] introduced the concept of n-width in 1936. For a long time, little progress has been made, except in a Hilbert space setting. But in the past 20 years, remarkable results have been obtained on the n-width of Sobolev spaces. Micchelli and Pinkus [3] were able to characterize the optimal subspaces in many cases. Kashin [1] showed the existence of approximation processes which converge at a substantially better rate than any of the standard approximation methods. To date, there are still many basic open problems; an example is the asymptotic order of the n-width of the Sobolev space W_1^1 in L_q for $2 < q < \infty$.

The book under review gives for the first time a comprehensive and up-to-date description of the theory of n-width and related s-numbers [4]. A major part of the book is devoted to the results on Sobolev spaces. In Chapter 5 and part of Chapter 4 the optimality of spline interpolation is shown as an application of a more general theory for the approximation of integral operators. A prerequisite is Chapter 3 which reviews the basic facts about total positivity and Chebyshev systems. As the diagrams on p. 233 (which do not yet represent the most complete description) indicate, the asymptotic results, which are described in Chapter 7, are fairly complicated. While not all proofs for the upper bounds are given, the basic techniques are covered and several illustrative special cases are discussed in detail. In addition to n-width for Sobolev spaces and related topics, the author considers n-width of matrices (Chapter 6), n-width in Hilbert spaces (Chapter 4), and n-width of algebraic functions (Chapter 9).

The book is well written and very systematically organized. It is an excellent text for the specialist. However, it might be difficult to read for anyone looking for an introduction to the subject.

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