However, the prospective reader should note that this book does not contain a wide-ranging review of stability for stiff nonlinear initial-value problems. Nor is it a guide for the practicing scientist or engineer seeking to find advice on the numerical solution of stiff nonlinear problems. But it should not be faulted for failing to address these subjects: This book is exactly what it purports to be—a well-written research monograph on the narrowly focused topic that it addresses.

KENNETH R. JACKSON

Department of Computer Science
University of Toronto
Toronto, Ontario, Canada M5S 1A4

43[41-02, 46-02].—ALLAN PINKUS, *n-Widths in Approximation Theory*, Springer-Verlag, Berlin, Heidelberg, New York, 1985, x + 291 pp., 25 cm. Price $39.00.

The n-width measures how well a subset of a normed linear space can be approximated by n-dimensional subspaces. A typical, and very important, example is the approximation of smooth functions. With this application in mind, Kolmogoroff [2] introduced the concept of n-width in 1936. For a long time, little progress has been made, except in a Hilbert space setting. But in the past 20 years, remarkable results have been obtained on the n-width of Sobolev spaces. Micchelli and Pinkus [3] were able to characterize the optimal subspaces in many cases. Kashin [1] showed the existence of approximation processes which converge at a substantially better rate than any of the standard approximation methods. To date, there are still many basic open problems; an example is the asymptotic order of the n-width of the Sobolev space W_1^1 in L_q for $2 < q < \infty$.

The book under review gives for the first time a comprehensive and up-to-date description of the theory of n-width and related s-numbers [4]. A major part of the book is devoted to the results on Sobolev spaces. In Chapter 5 and part of Chapter 4 the optimality of spline interpolation is shown as an application of a more general theory for the approximation of integral operators. A prerequisite is Chapter 3 which reviews the basic facts about total positivity and Chebyshev systems. As the diagrams on p. 233 (which do not yet represent the most complete description) indicate, the asymptotic results, which are described in Chapter 7, are fairly complicated. While not all proofs for the upper bounds are given, the basic techniques are covered and several illustrative special cases are discussed in detail. In addition to n-width for Sobolev spaces and related topics, the author considers n-width of matrices (Chapter 6), n-width in Hilbert spaces (Chapter 4), and n-width of algebraic functions (Chapter 9).

The book is well written and very systematically organized. It is an excellent text for the specialist. However, it might be difficult to read for anyone looking for an introduction to the subject.

K. HÖLLIG

Computer Sciences Department
University of Wisconsin
Madison, Wisconsin 53706

1. B. S. KASHIN, "Diameters of some finite-dimensional sets and classes of smooth functions," *Izv. Akad. Nauk SSSR*, v. 41, 1977, pp. 334–351.

2. A. KOLMOGOROFF, "Über die beste Annäherung von Funktionen einer gegebenen Funktionenklasse," *Ann. of Math.*, v. 37, 1936, pp. 107–110.

3. C. A. MICCHELLI & A. PINKUS, "Some problems in the approximation of functions of two variables and the n-widths of integral operators," *J. Approx. Theory*, v. 24, 1978, pp. 51–77.

4. A. PIETSCH, "s-numbers of operators in Banach spaces," *Studia Math.*, v. 51, 1974, pp. 201–223.

44[65D20].—C. G. VAN DER LAAN & N. M. TEMME, *Calculation of Special Functions: The Gamma Function, the Exponential Integrals and Error-Like Functions*, CWI Tract 10, Centrum voor Wiskunde en Informatica (Centre for Mathematics and Computer Science), 1984, iv + 231 pp., $23\frac{1}{2}$ cm. Price Dfl. 33.30.

This book is the first of a projected series intended to supplement AMS 55, the *Handbook of Mathematical Functions* [1], by providing detailed information for preparing and testing computer software for special functions. AMS 55 remains unsurpassed as a no-nonsense compilation of the properties of special functions, but it was prepared too early to contain the software information supplied here.

This book contains five chapters. The first is an introduction for the entire projected series. It contains an annotated bibliography on the computation of elementary and special functions, and a survey of major sources of function software. The latter includes a brief summary of the design criteria (e.g., whether portability is emphasized and how it is achieved) for a given collection whenever those criteria are known.

Chapter 2 is by far the longest in the book. It discusses topics that are fundamental to the following chapters. These include a brief discussion of error analysis, a particularly thorough discussion of linear recurrences, and quick overviews of continued fractions and hypergeometric functions.

Each of the remaining three chapters is dedicated to a different family of functions. Chapter 3 concerns the gamma family; Chapters 4 and 5 are dedicated, respectively, to the exponential integral family and the error function family. The overall plan of these three chapters is first to summarize important analytic properties, emphasizing those that are useful in numerical evaluation, and then to discuss algorithms and existing software in detail. The text is liberally sprinkled with references to recent work on practical convergence and utility of expansions, problems of range reduction, tables of coefficients, error analysis, etc. Thus, each chapter informs the potential designer or user of function software which methods might be useful, which have already been tried, which have succeeded, and where the software is to be found.

If there is a criticism of this book, it is in the binding. The paperback binding is not sturdy enough for a volume this useful. One of the two copies I own is already losing pages.

In summary, this book, together with those by the late Yudell Luke [2], [3], is an essential companion to AMS 55 and should be on the shelf of anyone concerned about the computation of special functions. We can all hope that Van der Laan and