figures and tables. The computer outputs are hardly structured, which makes them quite annoying to read and to interpret.

An appendix describes a magnetic tape containing all programs. The tape can be ordered from the publisher.

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47[03E72, 03H15, 60A05, 65G10].—ARNOLD KAUFMANN & MADAN M. GUPTA (Editors), *Introduction to Fuzzy Arithmetic, Theory and Applications*, Van Nostrand Reinhold, New York, 1985, xvii + 351 pp., $23\frac{1}{2}$ cm. Price $44.95.

Fuzzy numbers are one way to describe the vagueness and lack of precision of data. The theory of fuzzy numbers is based on the theory of fuzzy sets which was introduced in 1965 by L. Zadeh who wrote the foreword for this book. The concept of a fuzzy number was first used by Nahmias in the United States, and by Dubois and Prade in France in the late seventies. The present book by Arnold Kaufmann and Madan M. Gupta is the first introduction to the theory of fuzzy numbers; it is primarily aimed at the beginner who wants to learn these concepts from the start. But it contains numerous novel definitions and illustrative examples as well, and therefore it can be used as a collection of ideas for new research in the field of fuzzy numbers and for the application of these concepts. The authors are experienced researchers in the field of fuzzy sets and have written four books and numerous research papers on the subject before.

A fuzzy number represents an approximation of an unknown real or integer value. To every "level of presumption" between zero and one an "interval of confidence" is attributed which is believed to contain the true value with the corresponding degree of certainty. At least one value has to possess the highest level of presumption, i.e., one. The lower the level of presumption gets, the larger is the interval of confidence. Operations for fuzzy numbers are defined using the so-called max-min convolution. The authors illustrate the concept of a fuzzy number with the following example:

A certain job is known to be completed between May 15 and May 31; possibly it is completed on May 22. Then we can assign two levels of confidence in this situation, namely 1 for the interval [May 22, May 22] and 0 for [May 15, May 31]. Of course, an appropriate interval can be assigned for every level between 0 and 1.

In a certain way, this concept is a generalization of interval arithmetic, where every level of presumption could be assigned the same interval of confidence. An interval (as an approximation of a real number) contains the unknown true value with certainty and nothing is known about the precise location of the true value inside the interval. Fuzzy numbers are similar to random numbers as well, but they should not be confused: A random number is associated with the error in the measurement of a theoretically precise value, whereas a fuzzy number is a way to describe the uncertainty of human thought; it is a "subjective valuation assigned by one or more human operators."

A large number of generalizations of the concept of a fuzzy number are considered. Fuzzy numbers of higher dimensions can be introduced. Fuzzy complex numbers, fuzzy relative integers modulo n, fuzzy reals modulo one, and other concepts are considered. Fuzzy numbers of type two can be defined as fuzzy numbers where the intervals of confidence are not precisely known, being fuzzy numbers themselves. Numerous notions of statistics can be adapted to, or combined with the theory of fuzzy numbers; this, for example, leads to the novel concept of a hybrid number.

All of the concepts and operations are illustrated by a large number of examples which are in most cases given in the form of easy-to-understand figures. Many of these examples make use of triangular fuzzy numbers which are the easiest to compute with. In addition to these illustrative examples, the book contains some applications, including an optimization problem and an application of fuzzy numbers in catastrophe theory. The reviewer would like to make two recommendations about future research in the field of fuzzy numbers: First, fuzzy numbers are described by a function which tends to get more and more complicated after every operation; for example, the sum of triangular fuzzy numbers is a triangular fuzzy number again, but the product is not; is there a class of fuzzy numbers which can be easily represented in a computer and which is, in addition, closed, under all operations under consideration? Second, more applications should be investigated where fuzzy arithmetic is essential to obtain clear, reliable, detailed results that interval arithmetic, for example, could not deliver.

In summary, the present book is an excellent introduction for anybody who wants to get acquainted with the theory of fuzzy numbers. The numerous illustrative examples make it ideal for self-instruction as well as for courses on the subject. For the more advanced reader it contains a large number of novel ideas and incentives for application and research.

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48[68-02, 68P05, 68Q25, 68U05].—FRANCO P. PREPARATA & MICHAEL IAN SHAMOS, *Computational Geometry—An Introduction*, Texts and Monographs in Computer Science, Springer-Verlag, New York, 1985, xii + 390 pp., 24 cm. Price $45.00.

A book like this was badly needed by the scientific community, especially in view of the rapid growth and increasing popularity of the area of computational geometry, and its importance from both a theoretical and practical point of view. The book