10[65-01].—DOUGLAS QUINNEY, *An Introduction to the Numerical Solution of Differential Equations*, Research Studies Press Ltd., Wiley, New York, 1985, xi + 283 pp., $23\frac{1}{2}$cm. Price $34.95.

This book is an attempt to introduce the theory and techniques of numerical solution of differential equations to the reader who is not mathematically accomplished. Instead of analysis, the author often uses computed examples for simple model problems to exhibit the phenomena he discusses. This lends concreteness to the presentation, which should be reassuring to the scientist who needs to solve practical problems.

The exposition begins with a chapter on background material relating to recurrence relations and iterative methods, including techniques for extrapolation and acceleration of convergence. The following chapter, which is also the longest one, concerns numerical methods for initial value problems for ordinary differential equations, and covers general single-step methods, such as Runge-Kutta type methods, and linear multi-step methods. The basic concepts of consistency, convergence, and a variety of different stability concepts are defined and illustrated by examples. The subsequent chapter on two-point boundary value problems describes shooting methods and finite difference methods and ends with a short introduction to finite element type techniques, based on the collocation, Galerkin and Rayleigh-Ritz methods.

The remaining chapters deal with partial differential equations. First comes a chapter on parabolic equations, which starts with a discussion of the forward Euler method for the standard heat equation in one space dimension and a statement of the Lax equivalence theorem. Examples are given to illustrate von Neumann stability analysis, explicit and implicit schemes and alternating direction methods. The next chapter deals with the method of characteristics and the most common finite difference schemes for hyperbolic equations, particularly first-order equations and the standard second-order wave equation in one space dimension. The final chapter on elliptic equations begins with the five-point difference method for Laplace's equation in the unit square, with Dirichlet boundary conditions, and proceeds to discuss modifications required near curved boundaries and the case of Neumann boundary conditions. The closing few pages are devoted to the finite element method.

A first appendix exhibits the eigenvalues and eigenvectors of a tridiagonal symmetric Toeplitz matrix and proves Gerschgorin's theorem, and a second appendix discusses the classification of second-order differential equations into elliptic, hyperbolic, and parabolic equations.

The main emphasis is thus on ordinary differential equations, and here a readable account is provided of the various phenomena and dangers confronting the user. Conspicuously missing is a serious discussion of step-size control. The section on partial differential equations is more superficial and is essentially restricted to finite difference methods for the simplest model problems. Less than ten pages are afforded the finite element method, and here one can find the somewhat surprising statement that this method suffers from "the lack of a simple error estimate".

The author has avoided mathematics to a degree which could sometimes result in misunderstandings. For example, the Lax equivalence theorem is stated without a proper presentation of the framework within which it is valid and is then somewhat too freely applied. In variable coefficient and nonlinear cases it is sometimes unclear what are established results and what are conjectures. In the elliptic chapter the author states incorrectly (p. 243) that the formally $O(h)$ Shortley-Weller approximation in the case of a curved boundary detracts from the overall $O(h^2)$ approximation of the five-point approximation. He further hints (p. 235) that higher-order approximation should be used in the case of nonsmooth solutions, although, in general, such methods require at least as much regularity to be competitive.

In conclusion, the reviewer feels that the book could be a useful introduction for the applied scientist with a weak mathematical background. It provides easy reading at the expense of generality, depth and precision. In its emphasis on finite differences, however, it does not properly account for the advances in computational techniques of the last few decades.

V. T.

11[65-01].—G. D. SMITH, *Numerical Solution of Partial Differential Equations, Finite Difference Methods*, 3rd ed., Clarendon Press, Oxford, 1985, xi + 337 pp., 22 cm. Price $19.95.

This book is the third, somewhat modified, edition of a text which first appeared in 1965. It presents the finite difference method in a manner which was standard at that time, with emphasis on formulation of finite difference equations, often motivated by manipulations with Taylor series. It describes the most basic explicit and implicit methods for model parabolic and hyperbolic equations in one space dimension and the usual five-point method for Poisson's equation, together with some common devices for increasing the accuracy. Special attention is paid to time discretization by means of Padé type schemes of equations which are already discretized in the space variable, and to the concept of stiff equations. Stability analysis is carried out for simple model problems in the time-dependent case, using von Neumann's approach, and Gerschgorin's matrix theorem plays a central role in the discussion of the matrix equations. In the elliptic part, the Jacobi, Gauss-Seidel, and SOR iterative procedures for solving the system of difference equations are also considered.

Although some additions have been made in the new edition, the exposition has been only slightly affected by the developments of the last three decades. Thus, the increasing impact of the theory of partial differential equations on the formulation and analysis of numerical methods is essentially ignored or inadequately represented. For instance, application of energy arguments and a discussion of the effect on the error of the regularity of the solution are missing. In the new section on stability for initial value problems the author is somewhat influenced by the Lax-Richtmyer theory, but does not present or apply it properly. Trying to illustrate the Lax equivalence theorem, he describes (p. 72) the existence part of the condition for correctness as: "A solution always exists for initial data that is arbitrarily close to initial data for which no solution exists." In the elliptic part he says (p. 248, unchanged from the first edition): "Although no useful general results concerning the magnitude of the discretization error as a function of the mesh lengths have yet been established, it seems reasonable to assume that this error will usually decrease as the mesh lengths are reduced." In spite of this statement he shows (in the new