

Duffing's equation and to calculating eigenvalues for Mathieu's equation. New iterative methods with quadratic convergence properties are then used as successive approximation schemes to effectively obtain periodic solutions to differential equations.

The authors finally present their approach to numerical-analytic solutions, which emphasizes the use of a computer for intermediate steps in obtaining analytical solutions, as in using iterative methods to solve algebraic equations. Much of such necessary algebraic details might also benefit from symbolic computation. I cannot report that this monograph is preferable to all others available in English (such as Arnold [1], Guckenheimer and Holmes [2], and Sanders and Verhulst [3]). It does, however, present valuable material and a unique perspective on an important, though specialized, class of problems.

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1. V. I. ARNOLD, *Geometrical Methods in the Theory of Ordinary Differential Equations*, Springer-Verlag, New York, 1983.
2. J. GUCKENHEIMER & P. HOLMES, *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields*, Springer-Verlag, New York, 1983.
3. J. A. SANDERS & F. VERHULST, *Averaging Methods in Nonlinear Dynamical Systems*, Springer-Verlag, New York, 1985.

15[45D05, 65R20].—PETER LINZ, *Analytical and Numerical Methods for Volterra Equations*, SIAM Studies in Appl. Math., SIAM, Philadelphia, Pa., 1985, xiii + 227 pp., 23½ cm. Price \$32.50.

This book contains an elementary but thorough and self-contained introduction to the theory and the numerical solution of Volterra integral equations; as stated in the preface, "The audience for which this book is intended is a practical one with an immediate need to solve real-world problems." Thus, the chosen mathematical setting is that of (continuous) real-valued functions of one or several real variables, and proofs are often either just sketched or omitted entirely (the reader is then directed to an appropriate reference).

The first part of the book (six chapters, covering some 90 pages) deals with the classical quantitative theory of linear and nonlinear Volterra equations. It includes a brief chapter on some typical applications of Volterra integral and integro-differential equations, and it introduces some elementary results on the asymptotic behavior of solutions to certain second-kind integral equations. These chapters are particularly valuable, as most books on integral equations focus on Fredholm equations and treat Volterra equations only in a passing manner.

Numerical methods are discussed in the second part of the book, comprising about 110 pages. Chapter 7 covers direct quadrature methods (including a convergence analysis, asymptotic error estimates, and numerical stability), block-by-block methods, and explicit Runge-Kutta methods for second-kind equations with bounded kernels. Various product integration methods for second-kind equations with unbounded (or otherwise poorly behaved) kernels form the contents of Chapter 8. The

next two chapters are devoted to the solution of integral equations of the first kind having either differentiable or Abel-type kernels; here, the discussion focuses on the midpoint method and the trapezoidal method (and their product analogues), as well as on certain block-by-block methods. Chapter 11 is on numerical methods for first-order integro-differential equations; in addition to a description of linear multistep methods and block-by-block methods, we also find brief remarks on numerical stability and on more general Volterra functional equations.

Motivated partly by the lack of Volterra subroutines in software libraries, the author gives, in Chapter 12, listings (in Pascal) of a number of simple programs for (systems of) first-kind and second-kind integral equations. These algorithms are based on the midpoint rule and the trapezoidal rule and employ a fixed step size. Finally, Chapter 13 contains three case studies, involving the problem of error estimation, a nonstandard system of integral equations arising in polymer rheology, and the solution of a first-kind integral equation with nonexact data (this complements remarks, made in Chapters 9 and 10, on the ill-posed nature of first-kind equations). An extensive bibliography (some 280 references) concludes the book.

The book is well written and contains numerous examples which serve to illuminate the general discussion. The only slight shortcoming is that, in Chapter 8, the convergence orders are derived under the assumption that the solution of the second-kind Volterra integral equation with weakly singular factor $(t - s)^{-1/2}$ in its kernel have continuous derivatives of sufficiently high order (the order of, e.g., the block-by-block method based on quadratic interpolation is then $p = 7/2$). This is somewhat misleading since, typically, the solution of such an equation has derivatives which are unbounded at the left endpoint of the interval of integration (thus reducing the order of convergence on uniform meshes to $p = 1/2$). However, this minor criticism is greatly outweighed by the overall quality of this book, which is a most welcome addition to the literature on integral equations and their numerical solution.

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16[65-02, 65-04, 65F15, 65F20].—JANE K. CULLUM & RALPH A. WILLOUGHBY, *Lanczos Algorithms for Large Symmetric Eigenvalue Computations*, Vol. I: *Theory*, Progress in Scientific Computing, Vol. 3, Birkhäuser, Boston, 1985, xiv + 268 pp., 23 cm. Price \$29.95. Vol. II: *Programs*, Progress in Scientific Computing, Vol. 4, Birkhäuser, Boston, 1985, vii + 496 pp., 23 cm. Price \$49.95.

Cornelius Lanczos (another brilliant Hungarian) was a student of Albert Einstein. During World War II he put aside his studies in General Relativity and turned his abundant energy to the struggle against Nazi Germany. This led him into problems of engineering and scientific computation, and he never lost his interest in them, even after he was settled in Dublin in the Institute for Theoretical Physics.

He contributed a number of seminal ideas for applying classical analysis to the standard mathematical tasks where computation is vital. It is a pleasure to see a