next two chapters are devoted to the solution of integral equations of the first kind having either differentiable or Abel-type kernels; here, the discussion focuses on the midpoint method and the trapezoidal method (and their product analogues), as well as on certain block-by-block methods. Chapter 11 is on numerical methods for first-order integro-differential equations; in addition to a description of linear multistep methods and block-by-block methods, we also find brief remarks on numerical stability and on more general Volterra functional equations.

Motivated partly by the lack of Volterra subroutines in software libraries, the author gives, in Chapter 12, listings (in Pascal) of a number of simple programs for (systems of) first-kind and second-kind integral equations. These algorithms are based on the midpoint rule and the trapezoidal rule and employ a fixed step size. Finally, Chapter 13 contains three case studies, involving the problem of error estimation, a nonstandard system of integral equations arising in polymer rheology, and the solution of a first-kind integral equation with nonexact data (this complements remarks, made in Chapters 9 and 10, on the ill-posed nature of first-kind equations). An extensive bibliography (some 280 references) concludes the book.

The book is well written and contains numerous examples which serve to illuminate the general discussion. The only slight shortcoming is that, in Chapter 8, the convergence orders are derived under the assumption that the solution of the second-kind Volterra integral equation with weakly singular factor $(t - s)^{-1/2}$ in its kernel have continuous derivatives of sufficiently high order (the order of, e.g., the block-by-block method based on quadratic interpolation is then $p = 7/2$). This is somewhat misleading since, typically, the solution of such an equation has derivatives which are unbounded at the left endpoint of the interval of integration (thus reducing the order of convergence on uniform meshes to $p = 1/2$). However, this minor criticism is greatly outweighed by the overall quality of this book, which is a most welcome addition to the literature on integral equations and their numerical solution.

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16[65-02, 65-04, 65F15, 65F20].—JANE K. CULLUM & RALPH A. WILLOUGHBY, *Lanczos Algorithms for Large Symmetric Eigenvalue Computations*, Vol. I: *Theory*, Progress in Scientific Computing, Vol. 3, Birkhäuser, Boston, 1985, xiv + 268 pp., 23 cm. Price $29.95. Vol. II: *Programs*, Progress in Scientific Computing, Vol. 4, Birkhäuser, Boston, 1985, vii + 496 pp., 23 cm. Price $49.95.

Cornelius Lanczos (another brilliant Hungarian) was a student of Albert Einstein. During World War II he put aside his studies in General Relativity and turned his abundant energy to the struggle against Nazi Germany. This led him into problems of engineering and scientific computation, and he never lost his interest in them, even after he was settled in Dublin in the Institute for Theoretical Physics.

He contributed a number of seminal ideas for applying classical analysis to the standard mathematical tasks where computation is vital. It is a pleasure to see a book with the word Lanczos in the title, and it is fitting that a work of this length is needed to elaborate a clever idea he presented in a couple of papers in 1950.

Lanczos showed how to reduce any symmetric matrix A to a similar tridiagonal matrix T without applying any explicit similarity transformations to A. However, our way of looking at this scheme has undergone a number of significant transformations since 1950. An ambitious implementation of the method is a far harder task than an outsider might suspect.

The book under review is an excellent account of the present state of affairs. In fact, it is a lot more than that. The authors, both with IBM Research Center at Yorktown Heights, have been in the forefront of research on how best to implement, on a computer, the famous three-term recurrence presented in the original papers of Lanczos. They address the more general problem of how to compute some or all of A's eigenvalues, either with or without the corresponding eigenvectors, in the important case when A has many rows and many zero elements in them.

The exposition is exceptionally clear, in part because the authors wish to reach a broadly based readership: physicists, engineers, chemists, as well as applied mathematicians. Moreover, great care has been taken with references, with descriptions of work by others in this field, and with notation (see Chapter 0). Chapter 1 presents necessary background material and could serve as a model for textbook writers, particularly the list of applications. Chapter 2 presents what is now known about the behavior of the Lanczos algorithm both in exact and in finite precision arithmetic. New material appears here when the authors undertake a thorough review of all relevant research known to them. This is a daunting task because there is a certain amount of rivalry: different groups have espoused different strategies. Chapter 3 will tell you more than you ever wanted to know about tridiagonal matrices but, it turns out, this is the key to good implementations and to theoretical analysis of what is called 'convergence'.

The heart of the book is Chapter 4 wherein the authors develop and justify their implementation of the Lanczos recursion. The remaining chapters are concerned with extensions (such as block algorithms) and further applications (singular values and complex matrices). Volume II contains programs (in FORTRAN) and further detailed documentation. The references are impeccable and bear witness to the high scholarship practiced by Cullum and Willoughby. An example to us all.

For the only mildly interested reader let me try to sketch the reason why all this activity is necessary. After all, we do not really want a book associated with every nontrivial program.

Roundoff error wrecks the very pretty theory but it does not wreck the method. The explanations of this fact was the valuable contribution of C. C. Paige in 1971/72. In practice, the method becomes iterative whereas in exact arithmetic it terminates in at most n steps, where n is the order of A. We have as yet no theory which puts a precise bound on the number of steps needed so that the associated tridiagonal matrix yields every eigenvalue of A. The authors' response is to establish, in finite precision, the connection between the Lanczos recursion and the conjugate gradient (CG) algorithm for solving the system $Ax = v$, where v is the starting vector for the Lanczos scheme. Then they use the monotone decline in a measure of the error in the CG algorithm to give credence to the eventual appearance of each eigenvalue of A among the eigenvalues of T. It must be said that the clear, careful pursuit of this approach does not make for easy reading. The wood gets lost in the trees. Here is an example of how hard it is to keep a global perspective. The authors make use of Paige's error bounds, and these bounds are based on the model that the multiplication $w = Av$ is what it seems to be, that is, $w_k = \sum a_{kj} v_j$ where the sum is over nonzero elements in row k of A. Consequently, the results do not apply to some important applications where Av is merely shorthand for the solution of a set of equations of the form $(K - \sigma M)w = Mv$ and so $A = (K - \sigma M)^{-1}M$. This example is not to be construed as a blemish in the book but as an illustration of the quandary faced by the designers of an algorithm that succeeds but cannot be formally proved to succeed. What is to be done?

Apart from 'Convergence' theory, an important contribution made by the authors is the test they have gradually developed and refined for discriminating between desirable and undesirable eigenvalues of the tridiagonal matrix T. This is a fascinating topic but too technical for description here. After all, if T's order is three times that of A then some selection has to be made.

In their careful discussion of other ways of implementing the Lanczos algorithm, Cullum and Willoughby clearly believe that there is a "best", or preferred, way to do the job. Certain turns of phrase suggest that there may have been intellectual skirmishes between rival research groups. It is all rather tantalizing. To me, it seems far more likely that efficient execution of different tasks will require different implementations of the Lanczos recursion. Sophisticated structural engineers may well stick to their own nearly orthogonal set of Lanczos vectors. On the other hand, those scientists who need half or all of the spectrum of a conventional sparse symmetric matrix A will find the Cullum/Willoughby algorithm very hard to beat. And this book's description of it is a lesson to us all.

B. P.

17[30-02, 65E05, 42-XX, 30E20, 30C30, 30C50].—PETER HENRICI, *Applied and Computational Complex Analysis*, Vol. 3: *Discrete Fourier Analysis—Cauchy Integrals—Construction of Conformal Maps—Univalent Functions*, Wiley, New York, 1968, xiii + 637 pp., $23\frac{1}{2}$ cm. Price \$59.95.

This text is another excellent, long awaited, important and welcome addition to the previous two volumes written by the same author. The titles and chapter headings of the now available three volumes are:

Vol. 1: Title, "Power Series—Integration—Conformal Mapping—Location of Zeros"; Chapters, 1. Formal Power Series, 2. Functions Analytic at a Point, 3. Analytic Continuation, 4. Complex Integration, 5. Conformal Mapping, 6. Polynomials, 7. Partial Fractions.

Vol. 2: Title, "Special Functions—Integral Transforms—Asymptotics—Continued Fractions"; Chapters, 8. Infinite Products, 9. Ordinary Differential Equations, 10. Integral Transforms, 11. Asymptotic Methods, 12. Continued Fractions.

Vol. 3: Title, given above; Chapters, 13. Discrete Fourier Analysis, 14. Cauchy Integrals, 15. Potential Theory in the Plane, 16. Construction of Conformal Maps: Simply Connected Regions, 17. Construction of Conformal Maps for Multiply