Ch. 5: Quadrature methods for Volterra equations of the second kind.

Here one finds a discussion of the implementation of classical numerical methods for solving ordinary differential equations to the numerical solution of one, and systems of Volterra integral equations.

Ch. 6: Eigenvalue problems and the Fredholm alternative.

The discussion of eigenvalues is related to arbitrary Fredholm equations in terms of degenerate systems. No error analysis is given.

Ch. 7: Expansion methods for Fredholm equations of the second kind.

The Galerkin and Ritz-Galerkin methods are discussed for reducing a linear Fredholm integral equation problem to a problem involving a system of linear algebraic equations.

Ch. 8: Numerical techniques for expansion methods.

Explicit methods are discussed for actually carrying out the details of the Galerkin methods of the previous section.

Ch. 9: Analysis of Galerkin method with orthogonal basis.

Truncation and roundoff errors of the Galerkin method with orthogonal basis are discussed.

Ch. 10: Numerical performance of algorithms for Fredholm equations of the second kind.

This chapter compares the time required of different existing computer algorithms for solving Fredholm integral equations.

Ch. 11: Singular integral equations.

The type of singularities discussed are: (i) an infinite, or semi-infinite range of integration in the integral operator; (ii) a discontinuous derivative in the kernel or in the nonhomogeneous term; and (iii) an infinite or nonexisting derivative of some finite order.

Ch. 12: Integral equations of the first kind.

Eigenfunction expansions, regularization, and computational methods are discussed.

Ch. 13: Integro-differential equations.

Several methods are discussed for solving Volterra and Fredholm-type integro-differential equations.

Appendix: Singular expansions.

Explicit formulas are tabulated, for the coefficients in the expansions of the solution in terms of Chebyshev polynomials, for the case when the kernel has various types of special singular forms.

F. S.

32[41-01, 34A40, 34A50].—RICHARD E. BELLMAN & ROBERT S. ROTH, *Methods in Approximation—Techniques for Mathematical Modelling*, Reidel, Dordrecht, 1986, xv + 224 pp., 23 cm. Price $49.00/Dfl. 120.00.

According to the editor of the series in which this book appears, it is a survey of the thoughts of R. Bellman on the how and why of approximation over the past twenty-five years. This seems an accurate enough description, provided that one takes the right definition of the word approximation.

The book consists of ten chapters. Aside from the first chapter, which serves to introduce some basic ideas, the book essentially divides into two parts: three chapters which deal with topics from classical Approximation Theory, and six chapters which deal with finding approximate solutions to several types of problems involving differential equations.

The chapters on classical approximation include material on polynomials, piecewise linear functions, splines, exponential sums, and even a special finite element. The treatment is fragmentary at best, and the reader who wants to know something more about these subjects (beyond their connection with dynamic programming) will have to look elsewhere. (The references at the ends of these chapters will not be of much help.) It is indeed the case that these approximation tools play a central role in the numerical solution of many problems in applied mathematics, but no systematic development of their use is given here. In fact, I could not find much of a connection between the three approximation chapters and the rest of the book.

The other six chapters of the book deal with quasilinearization, differential approximation, differential quadrature, and differential inequalities. These methods are applied to solve some problems in parameter identification, finding unknown initial conditions, to replace systems of nonlinear partial differential equations by systems of nonlinear ODE's, and to replace systems of nonlinear ODE's by linear ones. Several specific types of equations are considered, including the renewal equation and an equation arising in pharmacokinetics, as well as the Riccati, van der Pol, and Maxwell equations.

Each of the chapters in the book includes a bibliography, with almost all references being to papers and books of Bellman and his coauthors, mostly published in the 60's and 70's. The quality of exposition varies, and there are a fair number of problems with mangled sentences, missing or extra words, misspelled words, and notation. Some of these could cause some confusion for a novice reader, such as the use of the notation $(0, 1)$ for the closed unit interval, or the use of (f, g) for the norm of $f - g$. Some of the sections read as if they were typed directly from lecture notes with no further editing. It is hard for me to see how this book will be of much use to people in classical approximation theory. Other potential users such as numerical analysts, applied mathematicians, and engineers should form their own opinions.

L. L. S.