TABLE ERRATA

609.—P. MOON & D. E. SPENCER, *Field Theory Handbook*, 2nd ed., Springer, Berlin, 1971.

Table 1.10, pp. 40–44, gives the separation of the Laplace equation in ellipsoidal coordinates (η, θ, λ) such that $\infty > \eta^2 > c^2 > \theta^2 > b^2 > \lambda^2 \geqslant 0$. Four of the six solutions given (p. 43) for $\alpha_2 = \alpha_3 = 0$, however, exhibit arguments of sn^{-1} greater than 1. They can, perhaps, be assigned a "formal" meaning; but we believe that, in a handbook, functions should have their standard significance, at least in the absence of clear warning. Accordingly, we believe the four solutions in question are, at best, misleading. We propose that they should be replaced, in sequence, by the following:

$$(1) \qquad H = A + B\,\mathrm{sn}^{-1}\left(\frac{c}{\eta}, \frac{b}{c}\right),$$

$$(2) \qquad \Theta = A + B\,\mathrm{sn}^{-1}\left(\left[\frac{c^2 - \theta^2}{c^2 - b^2}\right]^{1/2}, \left[\frac{c^2 - b^2}{c^2}\right]^{1/2}\right),$$

$$(3) \qquad \varphi = A + B\,\mathrm{sn}^{-1}\left(\frac{c}{\eta}, \frac{b}{c}\right),$$

$$(4) \qquad \varphi = A + B\,\mathrm{sn}^{-1}\left(\left[\frac{c^2 - \theta^2}{c^2 - b^2}\right]^{1/2}, \left[\frac{c^2 - b^2}{c^2}\right]^{1/2}\right).$$

Trivially, the misprint $dH/d\lambda$ should be corrected to $dH/d\eta$ in the first of the six separation equations listed under "If $\alpha_2 = \alpha_3 = 0$,".

Analogous problems arise with Table 1.09 (pp. 37–40) affecting separation of both the Laplace and Helmholtz equations in conical coordinates (r, θ, λ) with $\infty > c^2 > \theta^2 > b^2 > \lambda^2 \geqslant 0$. Of the separation solutions for the Laplace equation displayed on p. 39, the first shown for Θ is appropriately replaced by our (2) and the first shown for φ by our (4). Of the separation solutions for the Helmholtz equation displayed on p. 40, the last shown for Θ should be replaced by (2).

Table 1.11, pp. 44–46, involves a similar difficulty. This table sets out (p. 46) the separation of the Laplace equation in paraboloidal coordinates (μ, ν, λ) such that $\infty > \mu > b > \lambda > c > \nu \geqslant 0$. Here four of the six solutions given for $\alpha_2 = \alpha_3 = 0$ exhibit arguments of $\sin^{-1}$ with modulus greater than 1. We propose that these four misleading solutions be replaced, in sequence, by the following:

$$(5) \qquad M = A + B\ln\left[2\mu - b - c + 2(\mu - b)^{1/2}(\mu - c)^{1/2}\right],$$

$$(6) \qquad N = A + B\ln\left[b + c - 2\nu - 2(b - \nu)^{1/2}(c - \nu)^{1/2}\right],$$

$$(7) \qquad \varphi = A + B\ln\left[2\mu - b - c + 2(\mu - b)^{1/2}(\mu - c)^{1/2}\right],$$

$$(8) \qquad \varphi = A + B\ln\left[b + c - 2\nu - 2(b - \nu)^{1/2}(c - \nu)^{1/2}\right].$$

Related problems occur in Table 6.03. This table classifies separation equations for Laplace and Helmholtz equations according to the transformations reducing them to Bôcher form. On p. 153 the three Laplace separation equations for ellipsoidal coordinates (also certain Laplace and Helmholtz separation equations for conical coordinates) are all shown as generated by application of the transformation

$$z = \operatorname{sn}^{-1}\left(\frac{\zeta}{b}, \frac{b}{c}\right)$$

to the canonical form

$$\frac{d^2 Z}{dz^2} = 0, \quad \text{with solution } Z = A + Bz. \tag{9}$$

The solution to all three equations is then shown as

$$Z = A + B \operatorname{sn}^{-1}\left(\frac{\zeta}{b}, \frac{b}{c}\right).$$

From the present viewpoint, the foregoing is appropriate only for $c^2 > b^2 > \zeta^2 \geqslant 0$, and applies only to the separation equation

$$\frac{d}{d\zeta}\left[\left(b^2 - \zeta^2\right)^{1/2}\left(c^2 - \zeta^2\right)^{1/2} \frac{dZ}{d\zeta}\right] = 0.$$

With $\infty > \zeta^2 > c^2 > b^2 > 0$ and separation equation

$$\frac{d}{d\zeta}\left[\left(\zeta^2 - b^2\right)^{1/2}\left(\zeta^2 - c^2\right)^{1/2} \frac{dZ}{d\zeta}\right] = 0,$$

the appropriate transformation is

$$z = \operatorname{sn}^{-1}\left(\frac{c}{\zeta}, \frac{b}{c}\right),$$

giving the solution Z of form identical to the right side of (1) with η replaced by ζ.

With $\infty > c^2 > \zeta^2 > b^2 > 0$ and separation equation

$$\frac{d}{d\zeta}\left[\left(\zeta^2 - b^2\right)^{1/2}\left(c^2 - \zeta^2\right)^{1/2} \frac{dZ}{d\zeta}\right] = 0,$$

the appropriate transformation is

$$z = \operatorname{sn}^{-1}\left(\left[\frac{c^2 - \zeta^2}{c^2 - b^2}\right]^{1/2}, \left[\frac{c^2 - b^2}{c^2}\right]^{1/2}\right).$$

This gives the solution Z of form identical to the right side of (2) with θ replaced by ζ.

There are analogous troubles on p. 153 with the treatment of the three Laplace separation equations for paraboloidal coordinates. These are all shown as generated from (9) by the transformation

$$z = \sin^{-1}\left[\frac{2\zeta - (b + c)}{b - c}\right].$$

But this applies, from the present viewpoint, only for $b > \zeta > c > 0$ and separation equation

$$\frac{d}{d\zeta}\left[(b - \zeta)^{1/2}(\zeta - c)^{1/2} \frac{dZ}{d\zeta}\right] = 0.$$

For $b > c > \zeta \geqslant 0$ and separation equation

$$\frac{d}{d\zeta}\left[(b-\zeta)^{1/2}(c-\zeta)^{1/2}\frac{dZ}{d\zeta}\right]=0$$

the appropriate transformation is

$$z=\ln\left[b+c-2\zeta-2(b-\zeta)^{1/2}(c-\zeta)^{1/2}\right],$$

with the solution Z of form identical to the right side of (6) with ν replaced by ζ.

For $\infty > \zeta > b > c > 0$ and separation equation

$$\frac{d}{d\zeta}\left[(\zeta-b)^{1/2}(\zeta-c)^{1/2}\frac{dZ}{d\zeta}\right]=0$$

the appropriate transformation is

$$z=\ln\left[2\zeta-b-c+2(\zeta-b)^{1/2}(\zeta-c)^{1/2}\right],$$

with the solution Z of form identical to the right side of (5) with μ replaced by ζ.

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