

examples. There are two versions of the *Example Book*, one in Fortran and the other in Pascal. The text of the two appears to be identical, except that routine names are all capitals in the Fortran version and lower case in the Pascal version. The example programs listed in the Pascal version appear to be translations into Pascal of the Fortran examples.

As the title implies, the programs are intended more to serve as guides to usage than to be thorough test routines. It is a little annoying that in most cases in which comparison values are included in the program, they are merely printed for a visual check by the user. It would be better to have the *computer* do the comparison and print the difference or "ok". The book is also excessively repetitive. In those cases where essentially the same program is used to test several *NR* routines, each is still listed in complete detail.

The example programs are available on DOS diskettes for IBM PC and compatible computers at \$19.95 each. It is this reviewer's opinion that the printed programs are worth very little without their machine-readable counterparts. Since the *Example Book* contains very little text besides the programs, the authors could have incorporated the text into the programs and saved the users \$18.95!

We conclude by noting a few errors. In the text describing D3R12, the second derivative of the test function is $2x_1^2$, not $2x_1x_2$. (It is given correctly in the program itself.) The functions HEX2IN and IN2HEX, included in D7R13 to convert characters representing a hexadecimal number to/from its internal representation, are not quite as machine independent as the text claims. They assume that the characters '0' through '9' have internal representations that are consecutive integers; the same applies to 'A' through 'F'. The text before D12R1 is at least misleading. The program does not actually perform the four listed tests; it prints the results and expects the user to do a visual verification. The statement "if a data array is Fourier transformed twice in succession, the resulting array should be identical to the original" is false: the second transform must be the *inverse*, and one needs to include the factor $1/N$ that appears in (12.1.9) of *NR*, as in the program.

FREDERICK N. FRITSCH

Computing & Mathematics Research Division
Lawrence Livermore National Laboratory
Livermore, California 94550

5[65A05].—A. P. PRUDNIKOV, YU. A. BRYČKOV & O. I. MARIČEV, *Integrals and Series (Supplementary Chapters)* (in Russian), "Nauka", Moscow, 1986, 800 pp., 22 cm. Price 4 Rubles, 50 Kopecks.

Within the remarkably short period of five years, the authors have succeeded in the formidable task of preparing and presenting to the scientific community three volumes of integrals and series, each of about 800 pages. The volumes of integrals and series of elementary functions [1], reviewed in [2], and of integrals and series of special functions [3], reviewed in [4], have in the meantime been published in English [5], [6], and some errors or misprints have been corrected in [5]. The present table is the last volume of this collection. It consists of exactly 800 pages of formulas for

integrals, sums, infinite series and functional relations of (mainly) "higher" special functions.

Perhaps even more than in [1] and [3], the formulas in this volume are characterized by their unusual complexity, often involving several functions of different type, and by the number of their parameters: in other words, by their generality. As is mentioned in the abstract of the table, many of these formulas are new, having been developed by the authors themselves and are here presented for the first time.

The main part of this volume is divided into eight chapters, with each chapter divided into many sections and subsections. As in [1] and [3], the notation is standard, and a knowledge of Russian (required only for the few short sections of text) is not essential. Chapter 1 (29 pages) deals with indefinite integrals, including integrands involving polylogarithms, generalized Fresnel integrals, Struve, Anger, Weber, Lommel, Kelvin and Airy functions, integral Bessel functions J_i , etc., elliptic integrals, Legendre functions, Whittaker functions, confluent, Gaussian, and generalized hypergeometric functions, Meijer's G -, MacRobert's E -, and Fox's H -functions, and the elliptic functions of Jacobi and Weierstrass. The long Chapter 2 (317 pages) consists of definite integrals. It is divided into sections for integrands containing the gamma function, the generalized zeta function, Bernoulli and Euler polynomials, polylogarithms, generalized Fresnel integrals, Struve, Anger, Weber, Lommel, Kelvin, Airy, integral Bessel, Laguerre and Bateman functions, elliptic integrals, Legendre functions, Whittaker functions, confluent, Gaussian, and generalized hypergeometric functions with one or more variables, G -, E -, and H -functions, theta functions and Mathieu functions. Many of the integrands contain combinations of these functions or combinations with other functions, e.g., elementary functions or Bessel functions.

The short Chapter 3 (7 pages) contains formulas for the Laplace transform of step functions or other piecewise continuous functions. Chapter 4 (6 pages) is composed of double integrals of Struve, Kelvin, Anger, Lommel functions, of confluent, Gaussian and generalized hypergeometric functions, with elementary functions as factors. It also contains multiple integrals of generalized hypergeometric functions and integrals on a sphere. Chapter 5 (13 pages) deals with finite sums of Bernoulli and Euler polynomials and numbers, Legendre functions, generalized hypergeometric functions and G -functions. Chapter 6 (34 pages) presents infinite series with terms composed of the generalized zeta function, Bernoulli and Euler polynomials and numbers, Legendre functions, confluent, Gaussian and generalized hypergeometric functions, E -, and G -functions. As in the case of definite integrals, many formulas contain several types of functions, including elementary functions.

Chapter 7 (186 pages) presents in a concise form properties, representations and—probably for the first time so extensively—tables of special cases for several kinds of hypergeometric functions. These tables have about 2800 entries for ${}_pF_q(a_1, \dots, a_p; b_1, \dots, b_q; z)$, where $p = q + 1$ ($q = 0, 1, 2, \dots, 8$), $p = 0$ ($q = 1, 2, 3$), $p = q$ ($q = 1, 2$), $p = q - 1$ ($q = 2, 3$). The arguments a_i and b_j are special expressions or rational numbers. The results are sometimes given as functions of z , and sometimes for special values of this variable, e.g., $z = 1, -1, \frac{1}{2}$, and others. There are also formulas for general p and q .

The last chapter, Chapter 8 (117 pages), consists of two main sections. The first section (14 pages) consists of a concise description of the properties of Meijer's G - and of Fox's H -function. The much larger second section (103 pages) consists of tables of functions whose Mellin transform is (essentially) of the form

$$\prod_{j,k,l,m} \frac{\Gamma(a_j + A_j s)}{\Gamma(c_l + C_l s)} \frac{\Gamma(b_k - B_k s)}{\Gamma(d_m - D_m s)}$$

where $A_j, B_k, C_l, D_m > 0$. This table is characterized by the fact that it gives not only the image function of a given original function, but also expresses the latter as a special case of the G - or (in some cases) H -function. It can therefore also be used to find expressions which represent certain special cases of G or H . A similar table without the representation in terms of G or H , has been issued previously as part of a book [7] (under a somewhat misleading title) by one of the authors of the present table.

There are two appendices. Appendix I discusses general properties of integrals, series, products, and operations upon them, in particular, convergence criteria and the manipulation of formal power series. Appendix II lists definitions and properties of some special functions. A small dictionary of notation completes the table.

There is a bibliography of 65 items, but there are no references given with the formulas.

The printing and binding of this volume are good. The professional skill of the typographers deserves special mention: The formulas are set out with great care. As is the case with the two previous volumes, this table is an important reference book for mathematicians, physicists, theoretically interested engineers, and others working in fields where such formulas are likely to occur. Especially in view of its modest price, this table, as well as those for the elementary and special functions, ought to be available in the libraries of all relevant institutes and in the private libraries of those working with such formulas. Unfortunately, the number of copies printed (20 thousand) is even smaller than for the two other (Russian) tables. It is likely that there will be an English edition in due course.

K. S. KÖLBIG

Data Handling Division
European Organization for Nuclear Research
CERN
CH-1211 Geneva 23
Switzerland

1. A. P. PRUDNIKOV, YU. A. BRYČKOV & O. I. MARIČEV, *Integrals and Series of Elementary Functions*, "Nauka", Moscow, 1981. (Russian)
2. Y. L. LUKE, Review 4, *Math. Comp.*, v. 40, 1983, pp. 413-414.
3. A. P. PRUDNIKOV, YU. A. BRYČKOV & O. I. MARIČEV, *Integrals and Series of Special Functions*, "Nauka", Moscow, 1983. (Russian)
4. K. S. KÖLBIG, Review 2, *Math. Comp.*, v. 44, 1985, pp. 573-574.
5. A. P. PRUDNIKOV, YU. A. BRYČKOV & O. I. MARIČEV, *Integrals and Series*, v. 1: *Elementary Functions*, Gordon and Breach, New York, 1986.
6. A. P. PRUDNIKOV, YU. A. BRYČKOV & O. I. MARIČEV, *Integrals and Series*, v. 2: *Special Functions*, Gordon and Breach, New York, 1986.

7. O. I. MARICHEV, *Handbook of Integral Transforms of Higher Transcendental Functions*, Ellis Horwood, Chichester, 1982.

6[62Hxx].—FIONN MURTAGH & ANDRÉ HECK, *Multivariate Data Analysis*, Astrophysics and Space Science Library, vol. 131, D. Reidel Publishing Co., Dordrecht, 1987, xvi + 210 pp., 24½ cm. Price \$49.50/Dfl. 120.00.

This book gives a basic introduction to selected methods of multivariate statistical analysis. It is aimed at students and researchers in the astrophysical sciences, and its main strength is an extensive, carefully annotated bibliography of research papers in astronomy where multivariate methods have been applied. Because of its specialized audience and narrow coverage, the book is rather unlikely to appeal to statisticians or numerical analysts.

The topics covered include principal component analysis, cluster analysis and discriminant analysis. Some other techniques are briefly discussed. Most chapters are supplemented by illustrative examples and by listings of FORTRAN programs. Since some of the listings are fairly long and difficult to copy without error, the reviewer would have preferred appropriate references to subroutine libraries like NAG, IMSL and EISPACK.

BERNHARD FLURY

Institute of Statistics and Operations Research
Victoria University of Wellington
Private Bag Wellington
New Zealand

7[11F11, 11Y60, 33A25, 33A70, 65D15].—JONATHAN M. BORWEIN & PETER B. BORWEIN, *Pi and the AGM—A Study of Analytic Number Theory and Computational Complexity*, Canadian Mathematical Society Series of Monographs and Advanced Texts, Wiley, New York, 1987, xv + 414 pp., 24 cm. Price \$49.95.

When the reviewer was a teenager, he and three friends, after a high school basketball game, would frequently get in a car and drive over the labyrinth of country roads in the rural area in which they lived. The game we played was to guess the name of the first village we would enter. Since there were many meandering roads and countless small hamlets that dotted the rural landscape, since it was dark, and since we were not blessed with keen senses of direction, we were often surprised when the signpost identified for us the town that we were entering.

For one not too familiar with the seemingly disparate topics examined by the Borweins in their book, one might surmise that the authors were travelling along mathematical byways with the same naivete and lack of direction as the reviewer and his friends. However, the authors travel along well-lit roads that are marked by the road signs of elegance and usefulness and that lead to beautiful results. They do not take gravel-surfaced roads that lead to dead ends in cow pastures. But sometimes the destinations are surprising—at least to those not familiar with the landscape.