7. O. I. MARICHEV, *Handbook of Integral Transforms of Higher Transcendental Functions*, Ellis Horwood, Chichester, 1982.

6[62Hxx].—FIONN MURTAGH & ANDRÉ HECK, *Multivariate Data Analysis*, Astrophysics and Space Science Library, vol. 131, D. Reidel Publishing Co., Dordrecht, 1987, xvi + 210 pp., $24\frac{1}{2}$ cm. Price \$49.50/Dfl. 120.00.

This book gives a basic introduction to selected methods of multivariate statistical analysis. It is aimed at students and researchers in the astrophysical sciences, and its main strength is an extensive, carefully annotated bibliography of research papers in astronomy where multivariate methods have been applied. Because of its specialized audience and narrow coverage, the book is rather unlikely to appeal to statisticians or numerical analysts.

The topics covered include principal component analysis, cluster analysis and discriminant analysis. Some other techniques are briefly discussed. Most chapters are supplemented by illustrative examples and by listings of FORTRAN programs. Since some of the listings are fairly long and difficult to copy without error, the reviewer would have preferred appropriate references to subroutine libraries like NAG, IMSL and EISPACK.

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7[11F11, 11Y60, 33A25, 33A70, 65D15].—JONATHAN M. BORWEIN & PETER B. BORWEIN, *Pi and the AGM—A Study of Analytic Number Theory and Computational Complexity*, Canadian Mathematical Society Series of Monographs and Advanced Texts, Wiley, New York, 1987, xv + 414 pp., 24 cm. Price \$49.95.

When the reviewer was a teenager, he and three friends, after a high school basketball game, would frequently get in a car and drive over the labyrinth of country roads in the rural area in which they lived. The game we played was to guess the name of the first village we would enter. Since there were many meandering roads and countless small hamlets that dotted the rural landscape, since it was dark, and since we were not blessed with keen senses of direction, we were often surprised when the signpost identified for us the town that we were entering.

For one not too familiar with the seemingly disparate topics examined by the Borweins in their book, one might surmise that the authors were travelling along mathematical byways with the same naivete and lack of direction as the reviewer and his friends. However, the authors travel along well-lit roads that are marked by the road signs of elegance and usefulness and that lead to beautiful results. They do not take gravel-surfaced roads that lead to dead ends in cow pastures. But sometimes the destinations are surprising—at least to those not familiar with the landscape.

Instead of beginning at the high school gymnasium, the authors begin with the arithmetic-geometric mean and the contributions to it by Gauss. This leads to elliptic integrals, especially the complete elliptic integrals of the first and second kinds. Now we arrive at one of the main destinations, the calculation of π by the employment of the arithmetic-geometric mean and elliptic integrals. The authors return to the principal city of π several times, but they next take the road leading from elliptic integrals to their inversion and the big city of elliptic functions. The city's leading citizens are the theta functions, and the complete elliptic integral of the first kind makes another appearance when it is evaluated in terms of theta functions. The prominent theta functions then lead us to "squares." But we are not talking about unattractive lower class citizens, we have in mind beautiful formulas for the number of representations of positive integers as sums of certain numbers of squares. Another roadway from elliptic functions leads to the subject of partitions. In particular, Bressoud's elegant proof of the Rogers-Ramanujan identities is detailed.

We return now to elliptic integrals and their transformations, for which that of Landen is perhaps the paradigm. It is then a short journey to modular equations. The authors' discussion of modular equations is very enlightening and a focal point of the book. They begin with an algebraic approach before letting the principal townspeople, the theta functions, demonstrate their superiority. Modular equations then lead us again to algorithms for the calculation of π. These algorithms provide improvements in the rapidity of convergence.

A main interstate highway leads from elliptic functions to modular forms. The discussion of modular forms is very brief, but the absolute modular invariant is introduced, and we are led again to modular equations.

The authors next return to approximations to π. In fact, the subsequent work is more closely related to modular forms than is indicated by the authors. The approximations of π in question are due to Ramanujan and arise from formulas for Eisenstein series (not identified as such by the authors), which are modular forms. However, rightly so, the requisite formulas and the subsequent approximations to π are derived within the context of modular equations. These ideas naturally lead to work of Ramanujan and G. N. Watson on singular moduli. The authors are to be commended for their careful presentation of much of the content of Ramanujan's famous paper, "Modular Equations and Approximations to π". This material has not heretofore appeared in book form. However, more importantly, Ramanujan provided no proofs for many of the claims that he made, and so the authors provided many of the missing details.

We now return to the town from which we started our journey. Various variants and generalizations of the arithmetic-geometric mean are presented. If the arithmetic-geometric mean were useful for the computation of only π, its reputation would dwindle. The authors show how it can be used to rapidly calculate elementary functions. Other methods to approximate elementary functions are also explored. There is also a lengthy discussion of computational complexity and the fast Fourier transform.

At the end of their journey, the authors return to π. In an unusual feature for a book of this type, they provide an interesting historical account of attempts to calculate π. Then they conclude the tour with a discussion of the transcendence of π and irrationality measures.

As we have indicated, our visits with the theta functions were the highlights of this picturesque excursion. Frobenius echoed the thoughts of many mathematicians when he declared that

> "In der Theorie der Thetafunctionen ist es leicht, eine beliebig grosse Menge von Relationen aufzustellen, aber die Schwierigkeit beginnt da, wo es sich darum handelt, aus diesem Labyrinth von Formeln einen Ausweg zu finden."

The Borweins, indeed, have helped us to find the right roads.

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8[11A51, 68P25, 11K45].—EVANGELOS KRANAKIS, *Primality and Cryptography*, Wiley Teubner Series in Computer Science, Wiley, Chichester, 1986, xv + 235 pp., $23\frac{1}{2}$ cm. Price $41.95.

Some number theorists consider number theory as the exclusive domain of a select group of scholars, and they regard the appearance of strangers as an unwelcome intrusion. These strangers, they say, have a tendency to walk outside the marked pathways, to give different names to the roses that they encounter, and to introduce a lot of noise that disturbs the pure and quiet atmosphere; moreover, if they are caught in one of the many traps that the Queen of Mathematics has set, they do not seem to notice.

Other number theorists acclaim the arrival of modern times in their underpopulated area. They welcome visitors from outside as bringing in fresh air and financial resources. They had always believed that oil could be found in their lot of land, and they are happy to make their visitors believe this as well.

The present tour bus visiting the community is not likely to cause major excitement. It contains a collection of quiet *theoretical computer scientists* that came to see the *primality testing* plant, and among themselves they carry on a cultured conversation on abstract properties of *pseudorandom generators* and *public key cryptosystems.* They appear to be more profoundly interested in primality testing than is justified by the application that they have in mind. Should they not spend some time at the *factorization* facility as well? Their experienced driver, who was in the area before, determined otherwise. But he has an enthusiastic and original way of explaining to his passengers what they do see, and it is sure that at the end of the trip they will be better and wiser computer scientists.

Unfortunately, the zealous driver does in a technical sense not obey the rules as carefully as is traditional in this region. The eulogy on *mathematical rigor* that he recited at the border inspired confidence. But then, the very first exercise: *Let G be a finite abelian group. Show that all equations of the form* $x^2 = a$, *where* $a \in G$, *have exactly the same number of solutions in* G. If this is one of numerous typographical sins, here is another exercise (Section 2.10): *Show that every finite abelian group*