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The Subgroup Structure of the Mathieu Group M_{12}

By Francis Buekenhout and Sarah Rees

Abstract. We present a full description of the lattice of subgroups of the Mathieu group M_{12}.

1. Introduction. The recently fast developing search for geometries invariant by a finite simple group G (see [1], [9], [7], [2]) makes it suitable to dispose of various portions of the subgroup lattice of G. Hence it may be interesting to determine once and for all the full subgroup lattice of G. Of course, this lattice may be too large for a useful description. It seems more realistic to look for the *subgroup pattern*, in which the elements are conjugacy classes of subgroups and order is provided by inclusion. Even so, the task may look too difficult for any of the sporadic groups, but the result is surprisingly elegant for groups such as M_{11} and J_1 (see [2]) while the method of search is obviously unpleasant. Large portions of the subgroup pattern of the sporadic group Mc have been worked out by O. Diawara [5]. The purpose of the present paper is to discuss the subgroup pattern of the Mathieu group M_{12}. In calculating this, some use was made of the information available in [4], [8] and [6] and of the Cayley programming language ([3]).

2. Results. It turns out that there are 147 conjugacy classes of subgroups in M_{12}. We denote these classes by $K_1, K_2, \ldots$ and list them in increasing order, somewhat as Cayley does. We take the outer automorphism group of M_{12}, namely $M_{12}.2$, into account as follows. When two classes of M_{12} are fused under $M_{12}.2$, we denote them by K_i and $K_{\dot{i}}$. In view of this twinning procedure, the labels i run from 1 to 107 instead of 147. If no confusion can arise, the class K_x may be called "class x".

For each class K_x, we give one or more descriptions of the isomorphism type of its members, using fairly standard notation for this, in particular, the conventions of the Atlas [4]. So "6" stands for a cyclic group of order six, D_8 for a dihedral group of order 8, $A.B$ for a group G having a normal subgroup of type A with $G/A \sim B$, etc. We list (see table in Appendix) the classes K_x, the length or cardinality of K_x, the class K_y of the normalizer of a member of K_x, the number of maximal subgroups and of minimal overgroups in each possible class for a given member of K_x. Since M_{12} has a particularly interesting representation on the 12 left cosets of some subgroup M_{11}, we give the lengths of the orbits of a member H of K_x on these 12 points by a notation such as $[4,2,2,2]$, $[4 \times 3]$ or $[2^6]$. These mean that

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H has an orbit of 4 points and 4 orbits of 2 points in the first case, that H has an orbit of 12 points with 2 systems of imprimitivity blocks of sizes 4 and 3 respectively in the second case and that H has an orbit of 12 points with 6 blocks of imprimitivity of size 2 in the third case.

Département de Mathématique
Campus Plaine C.P. 216
Université Libre de Bruxelles
1050 Bruxelles, Belgium

Department of Mathematics
University of Illinois at Chicago
Box 4348
Chicago, Illinois 60680

1. F. BUEKENHOUT, "Diagram geometries for sporadic groups," in *Finite Groups—Coming of Age*, Conf. Proceedings (J. McKay, ed.), Contemporary Mathematics, vol. 45, Amer. Math. Soc., Providence, RI, 1985.

2. F. BUEKENHOUT, "The geometry of the finite simple groups," in *Buildings and the Geometry of Diagrams*, Como, 1984 (L. Rosati, ed.), Lecture Notes in Math., vol. 1181, Springer-Verlag, Berlin, Heidelberg, New York.

3. J. CANNON, *A Language for Group Theory*, Univ. of Sydney, 1982.

4. J. H. CONWAY, R. T. CURTIS, S. P. NORTON, R. A. PARKER & R. A. WILSON, *Atlas of Finite Groups*, Oxford Univ. Press, 1985.

5. O. DIAWARA, *Sur le Groupe Simple de J. McLaughlin*, Thesis, Université Libre de Bruxelles, 1984.

6. V. FELSCH & G. SANDLÖBER, "An interactive program for computing subgroups," *Computational Group Theory* (M. Atkinson, ed.), Academic Press, New York, 1984.

7. M. RONAN & G. STROTH, "Minimal parabolic geometries for the sporadic groups," *European J. Combin.*, v. 5, 1984, pp. 59–92.

8. A. D. THOMAS & G. V. WOOD, *Group Tables*, Shiva Publ. Limited, 1980.

9. J. TITS, "Buildings and Buekenhout geometries," in *Finite Simple Groups* II (M. Collins, ed.), Academic Press, New York, 1981, pp. 309–320.

Appendix: Subgroups of M_{12}. See table on the following pages.

Class number	Description of group	Orbits on 12 points	Length of class	Normalizers	Maximal subgroups	Minimal overgroups
1	Trivial		1	$K107$	–	$396K2, 495K3, 1320K4, 880K5, 2376K11, 1728K32$
2	2	2, 2, 2, 2, 2, 2	396	$K97$	$1K1$	$10K7, 15K10, 10K13, 30K14, 6K29, 30K30$
3	2	2, 2, 2, 2, 1, 1, 1, 1	495	$K95$	$1K1$	$3K6, 3K\overline{6}, 3K8, 12K9, 6K10, 16K12, 8K15, 16K16, 16K\overline{16}, 24K31$
4	3	3, 3, 3, 3	1320	$K85$	$1K1$	$3K13, 3K14, 1K15, 4K27, 6K37, 8K38$
5	3	3, 3, 3, 1, 1, 1	880	$K89$	$1K1$	$9K12, 3K16, 3K\overline{16}, 2K27, 1K28, 1K\overline{28}, 6K39, 9K40$
6	4	4, 4, 1, 1, 1, 1	1485	$K81$	$1K3$	$1K17, 2K18, 1K19, 1K23, 2K24, 2K25, 1K26$ $8K52, 4K69$
$\overline{6}$	4	4, 4, 2, 2	1485	$K81$	$1K3$	$1K\overline{17}, 2K18, 1K19, 1K\overline{23}, 2K\overline{24}, 2K\overline{25}, 1K\overline{26}, 8K\overline{52}, 4K\overline{69}$
7	2^2	$2^2, 2^2, 2^2$	1320	$K85$	$3K2$	$3K20, 1K33, 9K35, 2K38, 1K39$
8	2^2	2, 2, 2, 2, 2, 2	495	$K94$	$3K3$	$3K19, 3K21, 1K22, 6K24, 6K\overline{24}, 4K37$
9	2^2	2^2, 2, 2, 2, 1, 1	1980	$K73$	$3K3$	$1K21, 3K25, 3K\overline{25}, 12K36, 1K40$
10	2^2	4, 4, 2, 2	2970	$K62$	$2K2, 1K3$	$1K18, 1K20, 1K22, 1K23, 1K\overline{23}, 1K24, 1K\overline{24}, 4K34, 4K53$
11	5, Sylow 5 group	5, 5, 1, 1	2376	$K70$	$1K1$	$1K29, 1K30, 1K31, 8K77$
12	6	$1, 2, 3, 2 \times 3$	7920	$K36$	$1K3, 1K5$	$1K36, 2K49, 1K50, 1K\overline{50}, 1K57, 1K\overline{57}, 2K59, 1K60$
13	6	$2 \times 3, 2 \times 3$	3960	$K54$	$1K2, 1K4$	$1K33, 1K34, 1K35, 2K58$

Class number	Description of group	Orbits on 12 points	Length of class	Normalizers	Maximal subgroups	Minimal overgroups
14	S_3	$2 \times 3, 2 \times 3$	3960	$K54$	$3K2, 1K4$	$1K34, 2K35, 2K55, 2K\overline{55}, 4K79$
15	S_3	$3, 3, 3, 3$	1320	$K85$	$3K3, 1K4$	$3K34, 4K49, 3K78$
16	S_3	$3^2, 3, 1, 1, 1$	2640	$K\overline{68}$	$3K3, 1K5$	$3K36, 1K\overline{50}, 1K51, 3K56, 3K80$
$\overline{16}$	S_3	$3, 3, 3, 2, 1$	2640	$K68$	$3K3, 1K5$	$3K36, 1K50, 1K\overline{51}, 3K\overline{56}, 3K\overline{80}$
17	8	$4^2, 2, 1, 1$	1485	$K81$	$1K6$	$2K45, 2K47, 2K48, 4K83$
$\overline{17}$	8	$4^2, 4$	1485	$K81$	$1K\overline{6}$	$2K\overline{45}, 2K\overline{47}, 2K\overline{48}, 4K\overline{83}$
18	2×4	$4^2, 2, 2$	2970	$K62$	$1K6, 1K\overline{6}, 1K10$	$1K42, 1K46, 1K\overline{46}, 4K70$
19	2×4	$4, 4, 2, 2$	1485	$K81$	$1K6, 1K\overline{6}, 1K8$	$1K41, 1K43, 1K44, 1K46, 1K\overline{46}, 1K48, 1K\overline{48}$
20	2^3	$4 \times 2, 4$	990	$K88$	$4K7, 3K10$	$3K42, 4K54, 4K59$
21	2^3	$4, 2, 2, 2, 2$	495	$K95$	$3K8, 4K9$	$3K43, 6K44, 4K60$
22	2^3	$4, 4, 4$	495	$K94$	$1K8, 6K10$	$6K42, 3K43, 4K58$
23	D_8	$4^2, 2, 2$	1485	$K81$	$1K6, 2K10$	$2K42, 2K45, 1K46$
$\overline{23}$	D_8	$4^2, 4$	1485	$K81$	$1K\overline{6}, 2K10$	$2K42, 2K\overline{45}, 1K\overline{46}$
24	D_8	$4, 4, 2, 2$	2970	$K62$	$1K6, 1K8, 1K10$	$1K42, 1K43, 1K46, 2K55$
$\overline{24}$	D_8	$4, 4, 4$	2970	$K62$	$1K\overline{6}, 1K8, 1K10$	$1K42, 1K43, 1K\overline{46}, 2K\overline{55}$
25	D_8	$4, 4, 2, 1, 1$	2970	$K65$	$1K6, 2K9$	$1K44, 1K45, 1K47, 2K56, 2K82$
$\overline{25}$	D_8	$4, 4, 2, 2$	2970	$K\overline{65}$	$1K\overline{6}, 2K9$	$1K44, 1K\overline{45}, 1K\overline{47}, 2K\overline{56}, 2K\overline{82}$

26	Q_8	$4^2, 1, 1, 1, 1$	495	$K95$	$3K6$	$3K46, 6K47, 4K57, 4K84$
$\overline{26}$	Q_8	$4, 4, 4$	495	$K95$	$3K\overline{6}$	$3K\overline{46}, 6K\overline{47}, 4K\overline{57}, 4K\overline{84}$
27	3^2	$3 \times 3, 3$	1760	$K76$	$3K4, 1K5$	$3K49, 1K61, 3K67$
28	3^2	$9, 1, 1, 1$	220	$K99$	$4K5$	$12K50, 1K51, 4K61$
$\overline{28}$	3^2	$3, 3, 3, 3$	220	$K\overline{99}$	$4K5$	$12K\overline{50}, 1K\overline{51}, 4K61$
29	10	$10, 2$	2376	$K70$	$1K2, 1K11$	$1K53$
30	D_{10}	$10, 2$	2376	$K70$	$5K2, 1K11$	$1K53, 4K79$
31	D_{10}	$5, 5, 1, 1$	2376	$K70$	$5K3, 1K11$	$1K52, 1K\overline{52}, 1K53, 1K78, 2K80, 2K\overline{80}$
32	11, Sylow 11 group	$11, 1$	1728	$K77$	$1K1$	$1K77$
33	$2^2 \times 3$	3×4	1320	$K85$	$1K7, 3K13$	$1K54, 1K67$
34	D_{12}	$2 \times 3, 2 \times 3$	3960	$K54$	$3K10, 1K13, 1K14, 1K15$	$1K54, 1K91, 1K\overline{91}, 1K92$
35	D_{12}	3×2^2	3960	$K54$	$3K7, 1K13, 2K14$	$1K54, 2K72, 2K101$
36	D_{12}	$6, 2, 3, 1$	7920	$K36$	$3K9, 1K12, 1K16, 1K\overline{16}$	$1K68, 1K\overline{68}, 1K73, 1K74, 1K\overline{74}, 1K90, 1K\overline{90}, 1K100$
37	A_4	$2^3, 2^3$	1980	$K72$	$4K4, 1K8$	$1K55, 1K\overline{55}, 1K58, 1K71, 1K78$
38	A_4	3×4	2640	$K67$	$4K4, 1K7$	$1K67, 3K79$
39	A_4	$4, 4, 4$	1320	$K85$	$4K5, 1K7$	$3K59, 1K67$

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40	A_4	$6, 4, 1, 1$	1980	$K73$	$4K5, 1K9$	$1K56, 1K\overline{56}, 1K60, 2K80, 2K\overline{80}$
41	4×4	$4, 4, 4$	495	$K94$	$3K19$	$3K63, 3K\overline{63}, 1K64, 1K71$
42	$2 \times D_8$	$4^2, 4$	2970	$K62$	$1K18, 1K20, 1K22, 1K23, 1K\overline{23}, 1K24, 1K\overline{24}$	$1K62, 2K72$
43	$2 \times D_8$	$4, 4, 4$	1485	$K81$	$1K19, 1K21, 1K22, 2K24, 2K\overline{24}$	$1K62, 1K64, 1K66$
44	$2 \times D_8$	$4, 4, 2, 2$	1485	$K81$	$1K19, 2K21, 2K25, 2K\overline{25}$	$1K64, 1K65, 1K\overline{65}, 4K73$
45	D_{16}	$4^2, 2, 2$	2970	$K65$	$1K17, 1K23, 1K25$	$1K65, 1K103$
$\overline{45}$	D_{16}	$4^2, 4$	2970	$K\overline{65}$	$1K\overline{17}, 1K\overline{23}, 1K\overline{25}$	$1K\overline{65}, 1K\overline{103}$
46	$Q_8.2$	$4^2, 2, 2$	1485	$K81$	$2K18, 1K19, 1K23, 2K24, 1K26$	$1K62, 1K\overline{63}, 1K65$
$\overline{46}$	$Q_8.2$	$4^2, 4$	1485	$K81$	$2K18, 1K19, 1K\overline{23}, 2K\overline{24}, 1K\overline{26}$	$1K62, 1K63, 1K\overline{65}$
47	$D_8.2$	$4^2, 2, 1, 1$	2970	$K65$	$1K17, 1K25, 1K26$	$1K65, 2K74, 2K93, 1K104$
$\overline{47}$	$D_8.2$	$4^2, 2, 2$	2970	$K\overline{65}$	$1K\overline{17}, 1K\overline{25}, 1K\overline{26}$	$1K\overline{65}, 2K\overline{74}, 2K\overline{93}, 1K\overline{104}$
48	8.2	$4^2, 2, 2$	1485	$K81$	$2K17, 1K19$	$1K63, 1K65, 1K66$
$\overline{48}$	8.2	$4^2, 4$	1485	$K81$	$2K\overline{17}, 1K19$	$1K\overline{63}, 1K\overline{65}, 1K66$
49	$3 \times S_3$	$3 \times 3, 3$	5280	$K49$	$3K12, 1K15, 1K27$	$1K76, 1K85$
50	$3 \times S_3$	$9, 2, 1$	2640	$K68$	$3K12, 1K\overline{16}, 1K28$	$1K68, 1K\overline{75}, 1K76$

$\overline{50}$	$3 \times S_3$	$3^2, 3, 3$	2640	$K\overline{68}$	$3K12, 1K16, 1K\overline{28}$	$1K\overline{68}, 1K75, 1K76$
51	$3^2.2$	$9, 1, 1, 1$	220	$K99$	$12K16, 1K28$	$6K68, 3K69, 4K75$
$\overline{51}$	$3^2.2$	$3, 3, 3, 3$	220	$K\overline{99}$	$12K\overline{16}, 1K\overline{28}$	$6K\overline{68}, 3K\overline{69}, 4K\overline{75}$
52	5.4	$5, 5, 1, 1$	2376	$K70$	$5K6, 1K31$	$1K70, 2K\overline{90}, 1K91, 1K104$
$\overline{52}$	5.4	$5^2, 2$	2376	$K70$	$5K\bar{6}, 1K31$	$1K70, 2K90, 1K\overline{91}, 1K\overline{104}$
53	$2 \times D_{10}$	$5^2, 2$	2376	$K70$	$5K10, 1K29, 1K30, 1K31$	$1K70, 1K92, 1K103, 1K\overline{103}$
54	$2^2 \times S_3$	3×2^2	1320	$K85$	$3K20, 1K33, 3K34, 3K35$	$1K85, 3K97$
55	S_4	$2^3, 2^3$	1980	$K72$	$4K14, 3K24, 1K37$	$1K72, 1K87, 1K91$
$\overline{55}$	S_4	$2^3 \times 2$	1980	$K72$	$4K14, 3K\overline{24}, 1K37$	$1K72, 1K\overline{87}, 1K\overline{91}$
56	S_4	$6, 4, 1, 1$	1980	$K73$	$4K16, 3K25, 1K40$	$1K73, 2K\overline{90}, 1K98$
$\overline{56}$	S_4	$6, 4, 2$	1980	$K73$	$4K\overline{16}, 3K\overline{25}, 1K40$	$1K73, 2K90, 1K\overline{98}$
57	$SL_2(3)$	$8, 3, 1$	1980	$K74$	$4K12, 1K26$	$1K74, 1K88, 1K96$
$\overline{57}$	$SL_2(3)$	$8, 4$	1980	$K\overline{74}$	$4K12, 1K\overline{26}$	$1K\overline{74}, 1K88, 1K\overline{96}$
58	$2 \times A_4$	$2^3 \times 2$	1980	$K72$	$4K13, 1K22, 1K37$	$1K72, 1K86, 1K92$
59	$2 \times A_4$	$8, 4$	3960	$K59$	$4K12, 1K20, 1K39$	$1K85, 1K88$
60	$2 \times A_4$	$6, 4, 2$	1980	$K73$	$4K12, 1K21, 1K40$	$1K73, 1K88$
61	3^{1+2}, Sylow 3 group	$3^3, 3$	880	$K89$	$2K27, 1K28, 1K\overline{28}$	$1K75, 1K\overline{75}, 1K76$
62	$(Q_8.2)2$	$4^2, 4$	495	$K95$	$6K42, 3K43, 3K46, 3K\overline{46}$	$3K81, 1K88$

Class number	Description of group	Orbits on 12 points	Length of class	Normalizers	Maximal subgroups	Minimal overgroups
63	$(4 \times 4)2$	$4^2, 4$	1485	$K81$	$1K41, 1K\overline{46}, 1K48$	$1K81, 1K87$
$\overline{63}$	$(4 \times 4)2$	$4^2, 4$	1485	$K81$	$1K41, 1K46, 1K\overline{48}$	$1K81, 1K\overline{87}$
64	$(2 \times D_8)2$	$4, 4, 4$	495	$K94$	$1K41, 3K43, 3K44$	$3K81, 1K86$
65	$D_{16}.2$	$4^2, 2, 2$	1485	$K81$	$1K44, 2K45, 1K46, 2K47, 1K48$	$1K81, 2K105$
$\overline{65}$	$D_{16} \cdot 2$	$4^2, 4$	1485	$K81$	$1K44, 2K\overline{45}, 1K\overline{46}, 2K\overline{47}, 1K\overline{48}$	$1K81, 2K\overline{105}$
66	$(8.2)2$	$4^2, 4$	1485	$K81$	$1K43, 1K48, 1K\overline{48}$	$1K81$
67	$3 \times A_4$	3×4	1320	$K85$	$4K27, 1K33, 2K38, 1K39$	$1K85$
68	$S_3 \times S_3$	$9, 2, 1$	1320	$K82$	$6K36, 2K50, 1K51$	$1K82, 2K89$
$\overline{68}$	$S_3 \times S_3$	$3^2, 3, 3$	1320	$K\overline{82}$	$6K36, 2K\overline{50}, 1K\overline{51}$	$1K\overline{82}, 2K89$
69	$3^2.4$	$9, 1, 1, 1$	660	$K93$	$9K6, 1K51$	$1K82, 1K83, 1K84, 1K98$
$\overline{69}$	$3^2.4$	$3^2, 3^2$	660	$K\overline{93}$	$9K\overline{6}, 1K\overline{51}$	$1K\overline{82}, 1K\overline{83}, 1K\overline{84}, 1K\overline{98}$
70	2×5.4, Sylow -5 normalizer	$5^2, 2$	2376	$K70$	$5K18, 1K52, 1K\overline{52}, 1K53$	$1K97, 1K105, 1K\overline{105}$
71	$(4 \times 4)3$	4^3	495	$K94$	$4K37, 1K41$	$1K86, 1K87, 1K\overline{87}$
72	$2 \times S_4$	$2^3 \times 2$	1980	$K72$	$4K35, 3K42, 1K55, 1K\overline{55}, 1K58$	$1K94, 1K97$
73	$2 \times S_4$	$6, 4, 2$	1980	$K73$	$4K36, 3K44, 1K56, 1K\overline{56}, 1K60$	$1K95, 1K102, 1K\overline{102}$
74	$GL_2(3) = 2.S_4$	$8, 3, 1$	1980	$K74$	$4K36, 3K47, 1K57$	$1K95, 1K99, 1K106$

$\overline{74}$	$GL_2(3) = 2.S_4$	$8, 4$	1980	$K\overline{74}$	$4K36, 3K\overline{47}, 1K\overline{57}$	$1K95, 1K\overline{99}, 1K\overline{106}$
75	$3^2.S_3$	$3^3, 3$	880	$K89$	$3K\overline{50}, 1K51, 1K61$	$1K89, 1K96$
$\overline{75}$	$3^2.S_3$	$3^3, 3$	880	$K89$	$3K50, 1K\overline{51}, 1K61$	$1K89, 1K\overline{96}$
76	$3^2.S_3$	$3^3, 3$	880	$K89$	$6K49, 3K50, 3K\overline{50}, 1K61$	$1K89$
77	11.5, Sylow 11 normalizer	$11, 1$	1728	$K77$	$11K11, 1K32$	$1K100, 1K101$
78	A_5	$6, 6$	396	$K97$	$10K15, 6K31, 5K37$	$1K91, 1K\overline{91}, 1K92$
79	A_5	12	1584	$K79$	$10K14, 6K30, 5K38$	$2K101$
80	A_5	$10, 1, 1$	792	$K90$	$10K16, 6K31, 5K40$	$1K90, 1K98, 2K100$
$\overline{80}$	A_5	$6, 5, 1$	792	$K\overline{90}$	$1K\overline{16}, 6K31, 5K40$	$1K\overline{90}, 1K\overline{98}, 2K100$
81	Q_8D_8, Sylow 2 group	$4^2, 4$	1485	$K81$	$1K62, 1K63, 1K\overline{63}, 1K64, 1K65$ $1K\overline{65}, 1K66$	$1K94, 1K95$
82	3^2D_8	$9, 2, 1$	660	$K93$	$9K25, 2K68, 1K69$	$1K93, 1K102$
$\overline{82}$	3^2D_8	$3^2, 3^2$	660	$K\overline{93}$	$9K\overline{25}, 2K\overline{68}, 1K\overline{69}$	$1K\overline{93}, 1K\overline{102}$
83	$3^2.8$	$9, 2, 1$	660	$K93$	$9K17, 1K69$	$1K93, 1K103$
$\overline{83}$	$3^2.8$	3^4	660	$K\overline{93}$	$9K\overline{17}, 1K\overline{69}$	$1K\overline{93}, 1K\overline{103}$
84	$3^2.Q_8 = M_9$	$9, 1, 1, 1$	220	$K99$	$9K26, 1K69$	$3K93, 1K96, 3K104$
$\overline{84}$	$3^2.Q_8 = M_9$	3^4	220	$K\overline{99}$	$9K\overline{26}, 1K\overline{69}$	$3K\overline{93}, 1K\overline{96}, 3K\overline{104}$
85	$A_4 \times S_3$	3×4	1320	$K85$	$4K49, 1K54, 3K59, 1K67$	$1K107$

Class number	Description of group	Orbits on 12 points	Length of class	Normalizers	Maximal subgroups	Minimal overgroups
86	$2^{2+3}.3$	4^3	495	$K94$	$4K58, 1K64, 1K71$	$1K94$
87	$(4\times 4)S_3$	4^3	495	$K94$	$4K55, 3K63, 1K71$	$1K94$
$\overline{87}$	$(4\times 4)S_3$	4^3	495	$K94$	$4K\overline{55}, 3K\overline{63}, 1K71$	$1K94$
88	2^3A_4	8, 4	495	$K95$	$4K57, 4K\overline{57}, 8K59, 4K60, 1K62$	$1K95$
89	3^2D_{12}, Sylow 3 normalizer	$3^3, 3$	880	$K89$	$3K68, 3K\overline{68}, 1K75, 1K\overline{75}, 1K76$	$1K99, 1K\overline{99}$
90	S_5	10, 2	792	$K90$	$10K36, 65K\overline{52}, 5K\overline{56}, 1K80$	$1K102, 1K\overline{106}$
$\overline{90}$	S_5	6, 5, 1	792	$K\overline{90}$	$10K36, 6K52, 5K56, 1K\overline{80}$	$1K\overline{102}, 1K106$
91	S_5	6, 6	396	$K97$	$10K34, 6K52, 5K55, 1K78$	$1K97$
$\overline{91}$	S_5	6×2	396	$K97$	$10K34, 6K\overline{52}, 5K\overline{55}, 1K78$	$1K97$
92	$2\times A_5$	6×2	396	$K97$	$10K34, 6K53, 5K58, 1K78$	$1K97$
93	$M_9.2$	9, 2, 1	660	$K93$	$9K47, 1K82, 1K83, 1K84$	$1K99, 1K105, 1K106$
$\overline{93}$	$M_9.2$	3^4	660	$K\overline{93}$	$9K\overline{47}, 1K\overline{82}, 1K\overline{83}, 1K\overline{84}$	$1K\overline{99}, 1K\overline{105}, 1K\overline{106}$
94	$2^{2+3}S_3$	4^3	495	$K94$	$4K72, 3K81, 1K86, 1K87, 1K\overline{87}$	$1K107$
95	M_8S_4	8, 4	495	$K95$	$4K73, 4K74, 4K\overline{74}, 3K81, 1K88$	$1K107$
96	$M_9.3$	9, 3	220	$K99$	$9K57, 4K75, 1K84$	$1K99$
$\overline{96}$	$M_9.3$	3^4	220	$K\overline{99}$	$9K\overline{57}, 4K\overline{75}, 1K\overline{84}$	$1K\overline{99}$
97	$2\times S_5$	2×6	396	$K97$	$10K54, 6K70, 5K72, 1K91, 1K\overline{91}, 1K92$	$1K107$

98	A_6	$10,1,1$	66	$K105$	$30K56, 10K69, 12K80$	$1K102, 1K103, 1K104$
$\overline{98}$	A_6	$6,6$	66	$K\overline{105}$	$30K\overline{56}, 10K\overline{69}, 12K\overline{80}$	$1K\overline{102}, 1K\overline{103}, 1K\overline{104}$
99	$M_9S_3 = AGL_2(3)$	$9,3$	220	$K99$	$9K74, 4K89, 3K93, 1K96$	$1K107$
$\overline{99}$	$M_9S_3 = AGL_2(3)$	3^4	220	$K\overline{99}$	$9K\overline{74}, 4K89, 3K\overline{93}, 1K\overline{96}$	$1K107$
100	$L_2(11)$	$11,1$	144	$K100$	$55K36, 12K77, 11K80, 11K\overline{80}$	$1K106, 1K\overline{106}$
101	$L_2(11)$	12	144	$K101$	$55K35, 12K77, 22K79$	$1K107$
102	S_6	$10,2$	66	$K105$	$30K73, 10K82, 12K90, 1K98$	$1K105$
$\overline{102}$	S_6	$6,6$	66	$K\overline{105}$	$30K73, 10K\overline{82}, 12K\overline{90}, 1K\overline{98}$	$1K\overline{105}$
103	$PGL_2(9)$	$10,2$	66	$K105$	$45K45, 36K53, 10K83, 1K98$	$1K105$
$\overline{103}$	$PGL_2(9)$	6^2	66	$K\overline{105}$	$45K\overline{45}, 36K53, 10K\overline{83}, 1K\overline{98}$	$1K\overline{105}$
104	M_{10}	$10,1,1$	66	$K105$	$45K47, 36K52, 10K84, 1K98$	$1K105, 2K106$
$\overline{104}$	M_{10}	6^2	66	$K\overline{105}$	$45K\overline{47}, 36K\overline{52}, 10K\overline{84}, 1K\overline{98}$	$1K\overline{105}, 2K\overline{106}$
105	$M_{10}.2 = P\Gamma L_2(q)$	$10,2$	66	$K105$	$45K65, 36K70, 10K93, 1K102, 1K103, 1K104$	$1K107$
$\overline{105}$	$M_{10}.2 = P\Gamma L_2(q)$	6^2	66	$K\overline{105}$	$45K\overline{65}, 36K70, 10K\overline{93}, 1K\overline{102}, 1K\overline{103}, 1K\overline{104}$	$1K107$
106	M_{11}	$11,1$	12	$K106$	$165K74, 66K\overline{90}, 55K93, 12K100, 11K104$	$1K107$
$\overline{106}$	M_{11}	12	12	$K\overline{106}$	$165K\overline{74}, 66K90, 55K\overline{93}, 12K100, 11K\overline{104}$	$1K107$
107	M_{12}	12	1	$K107$	$1320K85, 495K94, 495K95, 396K97$ $220K99, 220K\overline{99}, 144K101, 66K105$ $66K\overline{105}, 12K106, 12K\overline{106}$	