is not very wide, but at least the choice falls on methods and techniques which are very recent and effective.

Therefore, in a sense, all three aims of the book have been successfully achieved. Although the field itself is, I think, much wider than what is covered in the 376 pages of this book, I believe that the book can be very useful, both to experts and to beginners in the field.

The contents of the book are organized as follows: Chapter I: Basic laws and models for flow in porous media (Generalities, The geometry of the field, The basic laws for one- and two-phase flow, The basic models, Qualitative behavior of the solution in the no-diffusion and no-capillary pressure case). Chapter II: Slightly compressible monophasic fields (Construction of the pressure equation, Existence and uniqueness theorems, An alternative model of monophasic wells). Chapter III: Incompressible two-phase reservoirs (Introduction, Construction of the state equations, Summary of equations of two-phase flows for incompressible fluids and rock, An alternative model for diphasic wells, Mathematical study of the incompressible two-phase flow problems, The case of fields with different rock types). Chapter IV: Generalization to compressible, three-phase, black oil or compositional models (The two-phase compressible model, The three-phase compressible model, The black oil model, A compositional model). Chapter V: A finite element method for incompressible two-phase flow (Introduction, Approximation of the pressure-velocity equations, Resolution of the algebraic system for pressure-velocity, Approximation of the one-dimensional saturation equation: the case with neither capillary pressure nor gravity, Approximation of the one-dimensional saturation equation in the general case, Approximation of the saturation equation in two dimensions, Notes and remarks).

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15[65L60, 65M60, 65R20].—H.-J. REINHARDT, *Analysis of Approximation Methods for Differential and Integral Equations*, Applied Mathematical Sciences, Vol. 57, Springer-Verlag, New York, 1985, xi+390 pp., $23\frac{1}{2}$ cm. Price $45.00.

This is a major contribution to the literature on the approximate solution of differential and integral equations. Most of the material comes from the research of the author and colleagues during recent years. A unified theory yields general convergence results and error estimates for approximate solutions of linear and nonlinear problems. The theory is applied to finite difference approximations for initial and boundary value problems, projection methods for differential and integral equations, and quadrature methods for integral equations.

The book is divided into four parts: numerical methods and examples, general convergence theory; applications to boundary value problems and integral equations; inverse stability, consistency and convergence for initial value problems.

The general theory relates solutions of equations

$$Au = w, \quad A_n u_n = w_n, \qquad n = 1, 2, \ldots,$$

where A and A_n are maps between normed linear spaces. Thus,

$$A\colon E \to F, \qquad A_n\colon E_n \to F_n.$$

These spaces are connected by means of abstract restriction maps:

$$R_n^E\colon E \to E_n, \qquad R_n^F\colon F \to F_n,$$

which, for example, could be ordinary restrictions or projections onto subspaces.

Solutions of $Au = w$ and $A_n u_n = w_n$ are related by means of discrete convergence. This general concept was formulated primarily by F. Stummel, and developed further by R. D. Grigorieff and the author, H.-J. Reinhardt. Discrete convergence is a map, denoted by lim, from a set of sequences $u_n \in E_n$ to elements $u \in E$. It satisfies

$$\lim u_n = u, \qquad \|u_n - v_n\|_n \to 0 \Leftrightarrow \lim u_n = \lim v_n.$$

For example,

$$\lim u_n = u \Leftrightarrow \|u_n - R_n^E u\|_n \to 0.$$

Particular cases of discrete convergence are provided by continuous functions, L^p spaces, and weak convergence of measures.

Discrete convergence of mappings, $A_n \to A$, is defined by

$$u_n \to u \Rightarrow A_n u_n \to Au.$$

This is equivalent to stability and consistency. Discrete convergence $A_n \to A$ is used to obtain the convergence $u_n \to u$ of solutions of equations $A_n u_n = w_n$ and $Au = w$. The maps A and A_n are assumed to be equidifferentiable, or have discrete compactness properties, or have approximation regularity properties.

Applications include difference methods for boundary value problems via maximum principles or variational principles. Inverse stability, consistency, and convergence are obtained for initial value problems, using difference approximations or Galerkin methods.

This monograph presents an impressive array of theoretical results and a wealth of significant examples.

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16[34–02, 35–02, 47G05, 65N30].—JOHANNES ELSCHNER, *Singular Ordinary Differential Operators and Pseudodifferential Equations*, Lecture Notes in Math., vol. 1128, Springer-Verlag, Berlin, 1985, 200 pp., 24 cm. Price \$14.40.

A differential operator is called degenerate if its leading coefficient vanishes at some point. The first three chapters of the book deal with various properties, such as normal solvability, Fredholm property, and index, of ordinary degenerate