The general theory relates solutions of equations

$$Au = w, \quad A_n u_n = w_n, \qquad n = 1, 2, \dots,$$

where A and A_n are maps between normed linear spaces. Thus,

$$A\colon E \to F, \qquad A_n\colon E_n \to F_n.$$

These spaces are connected by means of abstract restriction maps:

$$R_n^E\colon E \to E_n, \qquad R_n^F\colon F \to F_n,$$

which, for example, could be ordinary restrictions or projections onto subspaces.

Solutions of $Au = w$ and $A_n u_n = w_n$ are related by means of discrete convergence. This general concept was formulated primarily by F. Stummel, and developed further by R. D. Grigorieff and the author, H.-J. Reinhardt. Discrete convergence is a map, denoted by lim, from a set of sequences $u_n \in E_n$ to elements $u \in E$. It satisfies

$$\lim u_n = u, \qquad ||u_n - v_n||_n \to 0 \Leftrightarrow \lim u_n = \lim v_n.$$

For example,

$$\lim u_n = u \Leftrightarrow ||u_n - R_n^E u||_n \to 0.$$

Particular cases of discrete convergence are provided by continuous functions, L^p spaces, and weak convergence of measures.

Discrete convergence of mappings, $A_n \to A$, is defined by

$$u_n \to u \Rightarrow A_n u_n \to Au.$$

This is equivalent to stability and consistency. Discrete convergence $A_n \to A$ is used to obtain the convergence $u_n \to u$ of solutions of equations $A_n u_n = w_n$ and $Au = w$. The maps A and A_n are assumed to be equidifferentiable, or have discrete compactness properties, or have approximation regularity properties.

Applications include difference methods for boundary value problems via maximum principles or variational principles. Inverse stability, consistency, and convergence are obtained for initial value problems, using difference approximations or Galerkin methods.

This monograph presents an impressive array of theoretical results and a wealth of significant examples.

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16[34–02, 35–02, 47G05, 65N30].—JOHANNES ELSCHNER, *Singular Ordinary Differential Operators and Pseudodifferential Equations*, Lecture Notes in Math., vol. 1128, Springer-Verlag, Berlin, 1985, 200 pp., 24 cm. Price \$14.40.

A differential operator is called degenerate if its leading coefficient vanishes at some point. The first three chapters of the book deal with various properties, such as normal solvability, Fredholm property, and index, of ordinary degenerate differential operators in spaces of type C^∞, L_p, Sobolev (also weighted) and distributions. In Chapter 4 the results are extended to degenerate partial differential operators. Chapter 5 gives a very nice introduction (based on Fourier series) into the classical theory of pseudodifferential operators on closed curves, concentrating again on the degenerate case and in particular on the degenerate oblique derivative problem. Chapter 6 extends the well-known convergence analysis of the finite element method to the operators considered in Chapter 5.

In spite of its heavy mathematical content, the book is extremely readable, at least for readers who are familiar with the work of authors such as Triebel, Hörmander and Lions-Magenes. It should be useful to mathematicians who need a thorough treatment of the operator theory of degenerate operators, as well as to numerical analysts interested in numerical methods for degenerate operators.

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17[42–01, 42A16, 65T05].—VÁCLAV ČÍŽEK, *Discrete Fourier Transforms and Their Applications*, Adam Hilger, Bristol, Boston, 1986, 141 pp., 24 cm. Price \$28.00.

Preliminary Remarks. The advent of high-speed electronic computers and fast analog-to-digital converters have created not only an increased need for familiarity with Fourier methods but, just as important, it has shifted emphasis to different parts of Fourier theory.

To Fourier himself and many generations of mathematicians and engineers, Fourier analysis meant the use of an expansion of a piecewise continuous function on a finite interval into a series of discrete sines and cosines or, equivalently, complex exponentials, where, hopefully, the series converged rapidly. An important generalization of this has an integral instead of a series. Mathematicians have put the theory on a firm footing and practitioners have become skilled in the use of Fourier methods in analyzing and solving equations of electrical circuits, mechanical systems and analog devices of all kinds.

Owing to the great success of analytic methods and the great labor involved in the numerical application of Fourier methods, there was little emphasis on the latter until the advent of electronic computers and fast new algorithms. Since then, there has been a rapid shift from analytic to numerical methods.

While the object of interest, in analytic methods, is a piecewise continuous function having properties which ensure the convergence of the series or the existence of the integral, the computer must work with sequences of discrete sample values of the function. Consequently, the digital process must work with a Fourier transform which maps a discrete sequence into another discrete sequence. This transform has been called a "discrete" transform or a "finite" Fourier transform. In a sense these are equivalent since finiteness in one domain means discreteness in the other domain.