differential operators in spaces of type C^∞, L_p, Sobolev (also weighted) and distributions. In Chapter 4 the results are extended to degenerate partial differential operators. Chapter 5 gives a very nice introduction (based on Fourier series) into the classical theory of pseudodifferential operators on closed curves, concentrating again on the degenerate case and in particular on the degenerate oblique derivative problem. Chapter 6 extends the well-known convergence analysis of the finite element method to the operators considered in Chapter 5.

In spite of its heavy mathematical content, the book is extremely readable, at least for readers who are familiar with the work of authors such as Triebel, Hörmander and Lions-Magenes. It should be useful to mathematicians who need a thorough treatment of the operator theory of degenerate operators, as well as to numerical analysts interested in numerical methods for degenerate operators.

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17[42–01, 42A16, 65T05].—VÁCLAV ČÍŽEK, *Discrete Fourier Transforms and Their Applications*, Adam Hilger, Bristol, Boston, 1986, 141 pp., 24 cm. Price $28.00.

Preliminary Remarks. The advent of high-speed electronic computers and fast analog-to-digital converters have created not only an increased need for familiarity with Fourier methods but, just as important, it has shifted emphasis to different parts of Fourier theory.

To Fourier himself and many generations of mathematicians and engineers, Fourier analysis meant the use of an expansion of a piecewise continuous function on a finite interval into a series of discrete sines and cosines or, equivalently, complex exponentials, where, hopefully, the series converged rapidly. An important generalization of this has an integral instead of a series. Mathematicians have put the theory on a firm footing and practitioners have become skilled in the use of Fourier methods in analyzing and solving equations of electrical circuits, mechanical systems and analog devices of all kinds.

Owing to the great success of analytic methods and the great labor involved in the numerical application of Fourier methods, there was little emphasis on the latter until the advent of electronic computers and fast new algorithms. Since then, there has been a rapid shift from analytic to numerical methods.

While the object of interest, in analytic methods, is a piecewise continuous function having properties which ensure the convergence of the series or the existence of the integral, the computer must work with sequences of discrete sample values of the function. Consequently, the digital process must work with a Fourier transform which maps a discrete sequence into another discrete sequence. This transform has been called a "discrete" transform or a "finite" Fourier transform. In a sense these are equivalent since finiteness in one domain means discreteness in the other domain.

The theory of the finite Fourier transform has theorems which are counterparts of almost all of the theorems in the Fourier theory of integral transforms. Of course, continuity, convergence, and analyticity, are absent in the discrete Fourier transform. However, beyond that, the finite transform has theorems and properties which are strictly dependent on discreteness. Thus, theorems of number theory and group theory become involved. The notable consequences of these are the fast Fourier transform algorithm, the prime factor algorithms, and many other algorithms which are used in numerical processes.

The present book was originally published in the Czech language as a textbook for graduate students and practicing engineers. It treats the very important transition between the older analytic methods to the new digital processes and describes the theorems, properties and algorithms with a rather uneven degree of success.

1. *Introduction.* The first problem in the book is in the first line of the introduction, which says "Discrete Fourier transforms, better known to the general public as fast Fourier transforms, represent one of the computational methods,.... Discrete Fourier transforms as a means of computing Fourier transforms...". To most of us, the discrete Fourier transform (DFT) is a mathematically defined object: the discrete sequence of coefficients of sines and cosines or complex exponentials. (A third form, also defined in the book is in terms of amplitude and phase.) The name "DFT" says nothing about an algorithm for computing it. On the other hand, common usage is to drop the word "algorithm" from the name "fast Fourier transform" (FFT) when we mean a particular class of algorithms for computing the DFT. This as well as the problems of translation of the book may have contributed to the confusion of terms.

2. *Fourier Series.* The start of the second chapter states "Discrete Fourier transforms are related to Fourier series and Fourier transforms." This is still a little confusing but is closer to the truth. The remainder of Chapter 2 is a good description of Fourier series, convergence, Gibbs phenomena, and the minimum mean square error property of the truncated Fourier series. One section heading says "Fourier series of distribution" but the section says very little about the theory of distributions. The only distribution it treats is the delta function which is given in an acceptable fashion, so the only complaint is about the section heading. Fourier integral transforms and their properties are then described.

3. *Practical Methods of Computing Fourier Transforms.* This chapter starts by describing the calculation of Fourier integral transforms. It is hard to see why the author started with the integral transform. It would seem more logical to start with the finite integral for the coefficients of the Fourier series. In any case, he suggests a kind of semianalytic method for approximating the integrals of the tails. Then, for the integral over a finite domain, he describes various methods, such as integrating by parts, in order to express the integral in terms of derivatives, and later, differences. Examples are given for which this is a good method. Of course, where there are discontinuities, one will get better results by doing this, but, where the function is continuous, it gains nothing. Furthermore, this puts the frequency variable in the denominator which, for low frequencies will amplify errors. There is not enough emphasis on the usual or common situations and too much on special methods.

4. *Discrete Fourier Transforms.* The first sentence says "Discrete Fourier transforms are transforms of finite sequences of complex or real numbers." It may be mentioned here that some prefer the terminology "finite Fourier transform" since finite in one domain implies discrete in the other. On the other hand, the use of "DFT" may be better since the abbreviation of "finite Fourier transform" is already in use.

Chapter 4 proceeds with a very good and complete description of properties of the finite (discrete) Fourier transform, including the inversion theorem, relations involving symmetry, the effects of shifting, stretching, padding with zeros, repeating a sequence within a period, and of course, the most important theorem of all, the convolution theorem. This chapter covers an important part of Fourier theory which, as mentioned before, was much neglected before computers and the FFT algorithm.

5. *Other Properties of Discrete Fourier Transforms and Their Use in Computing Fourier Transforms.* Here, the book describes the next aspect of Fourier theory which requires more emphasis in numerical Fourier theory. When programmers were faced with the task of computing Fourier integrals, they often evaluated the integral with Simpson's rule or Newton-Cotes formulas, using the error estimates of those formulas. Chapter 5 gives a proper treatment which shows that sampling a function produces aliasing and that errors in Fourier integrals should really be expressed in terms of this aliasing. This not only yields more accurate error estimates but, to the engineer, it is more intuitive and suggestive of ways to reduce or avoid the errors.

The only problem in this chapter is the one cited earlier with respect to Chapter 3 where the author suggests converting the integral to a sum of differences. The implementation is described here with no word of caution about the frequency variable in the denominator.

6. *Methods of Computation of Discrete Fourier Transforms.* In a brief survey in the introduction to this chapter, a number of methods are mentioned without relations to each other or chronology. For example, the Goertzel algorithm is mentioned after the FFT and others. Actually it came long before. However, a good description of this important algorithm is given later in the chapter.

The FFT algorithms for the radix 2 are described very well with a good and liberal use of the signal flow graphs which have always been popular with engineers. This is followed by the mixed radix algorithms with an effective presentation and good flow graphs. Results of error analysis are stated concisely as they should be in a short book such as this.

A section titled "Winograd algorithm" gives a very brief summary of the prime factor algorithms and the Winograd transform. However, the distinction is not quite made clear. It is unfortunate that the author did not continue the type and quality of presentation of the previous section. Even engineering students who do not plan to program FFT's would find prime factor and Winograd algorithms interesting. Furthermore, there will be occasions when they will want to know what considerations go into making good choices of algorithms and subroutines.

The basic FFT algorithm is for complex to complex transforms. For data having special symmetries, such as being real, conjugate even, symmetric, etc., special algorithms are given. There are many good and important papers on these special algorithms which are not mentioned or referred to.

7. *Some Applications of Discrete Fourier Transforms.* A very brief but useful sketch of the application of the FFT algorithm to convolution calculations is given. This can also go under the names of correlation, covariance, digital filtering, and so on. The author fails to point out how his treatment can handle all of these and what alterations must be made in the basic techniques in order to handle the special cases. There is very little said about the power spectrum except to state that it is the magnitude of the Fourier transform. The two-dimensional transform is simply described as a row-column iteration of the FFT algorithm. Nothing about special considerations such as how to treat real data is given.

8. *Discrete Hilbert Transforms.* Although this topic is important in systems and signal processing, it has a disproportionate amount of attention in this book. A student who has to study at the level presented here will not only fail to see the significance but will have difficulty going through the intricate derivations. Furthermore, it does not seem pedagogically wise to present, in the first full-scale application, a case where the integrals do not even exist in the sense in which the reader has understood them throughout the book. He should have some understanding of principle value definitions of integrals and of the theory of distributions. He should also know something about the problems in treating such integrals numerically. Of course, it goes without saying that he should have a better understanding of why the Hilbert transform is important.

Summary. The book has a good introduction to the Fourier theory needed for understanding its numerical applications. Either as a textbook or as source of information for a practicing engineer, the book is somewhat uneven in quality. The FFT sections are very good, but as mentioned above, they should contain more about prime factor algorithms. It is a rather short book of 141 pages. The bibliography is extensive and well referenced in the text. However, as one may expect from a book written in Eastern Europe, one is often disappointed to find references, which one would like to read, published in Russian.

The book probably served its purpose very well when written and used where access to books on the subject were perhaps limited and where one had the author to teach and explain. The English language book could probably be used effectively with supporting material. It has very few examples, it has no problems or exercises for the student and surprisingly, it gives no program listings. Among the many books on the subject in the English language, there are far more useful books for teaching or for reference.

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18[33–00, 65A05].—MILTON ABRAMOWITZ & IRENE A. STEGUN (Editors), *Pocketbook of Mathematical Functions—Abridged edition of Handbook of Mathematical Functions, Milton Abramowitz and Irene A. Stegun (eds.),* Material