During recent years there has been a considerable research effort on the application of homotopy methods to this problem. The author is one of the most active contributors to this work and presents here an introduction to his methods for a general audience of potential users.

Briefly, let (1) $f(x) = 0$, $f \in \mathbb{C}^n \to \mathbb{C}^n$ be the given system and denote by $d_1, \ldots, d_n$ the degrees of the n components of f. The sum of these degrees is the total degree d of (1). Now an initial system (2) $g(x) = 0$ can be introduced with components $g_j(x) = p_j^{d_j} x_j^{d_j} - q_j^{d_j}$, $j = 1, \ldots, n$, where p_j and q_j are suitable complex constants. Then the desired homotopy is (3) $h(x,t) = (1-t)g(x) + tf(x)$, $0 \le t \le 1$, which permits the application of a continuation process to follow the d paths beginning at each of the solutions of (2). Clearly, the implementation of this concept requires much attention to various details, and that takes up a major part of the book.

There are two parts, covering the method and several applications, respectively. More specifically, after two introductory chapters introducing the basic ideas for one- and two-dimensional equations, Chapter 3 describes the general method, which is followed by a chapter on its implementation. The first part then ends with a chapter on scaling techniques and on some alternative continuation methods. The second part begins with Chapter 7, covering practical considerations of systems reduction, while the final Chapters 8 through 10 are devoted to case studies. More specifically, geometric intersection problems, chemical equilibrium systems, and kinematic problems of mechanisms are considered. There are also six appendices, namely five containing additional mathematical details and another one which presents some 200 pages of FORTRAN source code of all the procedures.

As the title already indicates, the book is intended primarily for engineers and scientists who wish to use these techniques. In line with this, an informal approach was adopted and the mathematical prerequisites were kept at the level of a working knowledge of multivariate calculus, linear algebra, and computer programming. The book certainly succeeds well in its aims and offers a nice introduction to these valuable new methods.

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29[65–01].—Annie Cuyt & Luc Wuytack, *Nonlinear Methods in Numerical Analysis*, North-Holland Mathematics Studies, vol. 136, North-Holland, Amsterdam, 1987, 278 pp., 24 cm. Price \$53.25/Dfl. 120.00.

This is a textbook based on a graduate course given at the University of Antwerp, a supplement to the many elementary books which mainly treat *linear* techniques. I opened the book with a sense of expectation and curiosity. What is it that can be found here, behind the rather general title?

Quite sensibly (and not unexpectedly) the authors have restricted their presentation to methods based on Padé approximation and "Padé-like" techniques like rational Hermite interpolation. A *Padé approximant* of order (m, n) to a function

$f(x)$ holomorphic at 0 is a rational function $r_{m,n}(x) = P_{m,n}(x)/Q_{m,n}(x)$, where $P_{m,n}$ and $Q_{m,n}$ are polynomials of degree m and n, respectively, such that

$$Q_{m,n}(x)f(x) - P_{m,n}(x) = \sum_{k=1}^{\infty} c_k x^{m+n+k} \tag{1}$$

in a neighborhood of 0. That is, it is basically interpolation at 0. Indeed, an alternative (but not quite equivalent) definition of a Padé approximant is that the derivatives evaluated at 0 satisfy $r_{m,n}^{(k)}(0) = f^{(k)}(0)$ for $k = 0, 1, \ldots, m+n$. A *Hermite interpolant* interpolates at more than one point. If f is holomorphic at the distinct interpolation points $\{x_i\}$, then the requirement is that

$$r_{m,n}^{(k)}(x_i) = f^{(k)}(x_i) \quad \text{for } k = 0, \ldots, t_i \text{ and } i = 1, \ldots, s \tag{2}$$

for some given constants $s, t_1, \ldots, t_s \in \mathbf{N} \cup \{0\}$, where $\sum_{i=1}^{s}(t_i + 1) = m + n + 1$.

These concepts are intimately connected with continued fractions, which are presented in a separate chapter in the book. The interpolation requirements (1) have their counterpart in *correspondence* of a continued fraction.

By presenting in turn continued fractions, Padé approximants and rational interpolants in separate, self-contained chapters, the authors manage to repeat the idea of interpolation in varying situations. This has a great pedagogical value and gives the book a unified and compact quality. The economy in the wording adds to this impression.

Within this framework the authors have also managed to present methods for the multivariate case—a considerably more complex task. By carefully following the same lines of presentation, they are able to give a logical and readable introduction to branched continued fractions and multivariate Padé approximation and interpolation. This important and useful contribution makes the book valuable also to users and workers in the field.

For teaching, the most valuable part of this book, perhaps, is the last chapter. It is devoted to applications of the techniques introduced. The headings are: convergence acceleration, nonlinear equations, initial value problems, numerical integration, partial differential equations and integral equations.

With few (rather randomly occurring) exceptions, stability analyses and error analyses are omitted, also in cases where error estimates could be obtained almost for free from arguments given in the text. Instead, the authors have chosen to present numerical examples where the exact answers are known, so that the accuracy obtained by various algorithms can be immediately seen from the output.

Speaking about examples, I would have liked to see some in which a numerical method is *needed* to solve a practical problem. Maybe also some more examples illustrating the advantages of nonlinear methods over linear methods. But these are minor objections. The book is a clear, well-written introduction to a fascinating and useful field. The extensive list of references and exercises is a valuable addition.

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