

**30[33-01, 33-02, 33-04].**—JEROME SPANIER & KEITH B. OLDHAM, *An Atlas of Functions*, Hemisphere Publ. Corp., Washington, 1987, ix + 700 pp., 28½ cm. Price \$149.50.

The successor to Jahnke and Emde's tables of special functions [2], first published in 1909, was the 1964 handbook edited by Abramowitz and Stegun [1], often called AMS-55. An extensive supplement to the latter, written at an advanced level and emphasizing rational approximations, was published in 1975 by Luke [3]. AMS-55, now 24 years old, seems likely to be superseded by *An Atlas of Functions*, although the price, not unreasonable in view of glossy paper and multicolored graphs, may limit its popularity. Numerical tables have been replaced by algorithms for a programmable calculator that usually yield better than seven significant figures. Two or three significant figures can often be obtained directly from the many computer-generated graphs because of the admirable device of dotted coordinate lines. There is much more explanatory text and comment than in AMS-55, and the book seems aimed at a wider and mathematically less experienced audience. The whole volume represents an immense labor done with intelligence, skill, and great care.

The return to two-man authorship, as in Jahnke-Emde, has produced more uniform style and coverage, enhanced by rigid adherence to a system of dividing each of the 64 chapters into 13 sections on topics such as Expansions, Numerical Values, and Approximations (but not Inequalities). The section titled Operations of the Calculus gives the authors, who collaborated previously on a well-known book on fractional calculus, an opportunity to include representations by fractional derivatives and integrals. An occasional fourteenth section on Related Topics contains short accounts of subjects such as linear regression, orthogonal coordinate systems, fast Fourier transforms, and Laplace transforms. Like AMS-55, the book is not only a source of numerical values and a collection of graphs but also a formulary that ranks with the best and may prove easier to use than most because of good organization and many helpful comments.

Roughly half the book is devoted to functions on the integers, elementary algebraic functions, classical orthogonal polynomials, and elementary transcendental functions. The Heaviside step function and the Dirac delta function each have a chapter. The second half of the book treats the gamma function and its relatives, the indispensable roster of special confluent and Gaussian hypergeometric functions, the Struve function, elliptic integrals and functions, and the Hurwitz zeta function. One must still turn to AMS-55 for Coulomb wave functions, Mathieu functions, and spheroidal wave functions. The book ends with an appendix of utility algorithms, a second appendix with tables of physical units and constants, a list of references (not including Whittaker and Watson's *Modern Analysis* nor Watson's *Bessel Functions*), a subject index, and a symbol index.

Despite admiration and gratitude for a superb piece of work, one may still regret strongly a few of the authors' decisions. They treat special functions as functions of a real variable instead of a complex variable; in contrast with AMS-55, the reader will not learn here, for example, that the gamma function is analytic except for poles at the nonpositive integers nor that its reciprocal is entire. In each chapter there is a section titled Complex Argument, but it usually contains only a separation into real

and imaginary parts. In an age when complex function theory is an undergraduate course, when analytic continuation in the complex energy or angular-momentum plane is commonplace in physics, when engineers use analytic transfer functions to describe electrical and mechanical systems, it seems a step backward to survey the properties of special functions with no mention of analyticity, poles, or branch points.

Secondly, the authors have relegated to the background the one notational device,  ${}_pF_q$ , that has done more than any other to bring order and unity into the welter of redundant definitions and notations that have accumulated during two centuries. One must thank them for a long list (Table 18.14.2) identifying hypergeometric series having various parameters with functions discussed elsewhere in the book. However, they eschew the  ${}_pF_q$  notation for such series because they consider it more general (see Section 60:13 and p. 155) to specify the coefficient of  $x^n$  rather than  $x^n/n!$ . That is, they prefer to insert 1 as a parameter in the denominator when necessary rather than remove the  $n!$  when necessary by a parameter 1 in the numerator. This is really a question of taste, not generality. Their choice has the effect that closely related functions (for example, arcsin and arctan, special cases of  ${}_2F_1$ ) are listed with different numbers of parameters, and a linear transformation of  ${}_2F_1$  can change the number of parameters. Worse yet, it has the effect that the  ${}_pF_q$  notation is not used in the other chapters to show the reader that some order and simplicity underlie the chaotic throng of special hypergeometric functions. The shifted factorial or Pochhammer symbol,  $(a)_n$ , fares somewhat better with a chapter to itself, but in the rest of the book it gives way to semifactorials and even to the notations  $n!!!$  and  $n!!!!$ .

What one reader deplores, another may applaud; no book can satisfy everyone in all respects. This one should please nearly everyone in most respects and should have a deservedly bright future in the Citation Index.

B. C. C.

1. M. ABRAMOWITZ & I. A. STEGUN, eds., *Handbook of Mathematical Functions*, National Bureau of Standards Applied Mathematics Series, No. 55, U. S. Government Printing Office, Washington, D. C., 1964. Reprinted by Dover Publications, New York, 1965.

2. E. JAHNKE & F. EMDE, *Tables of Higher Functions*, 6th ed. revised by F. Lösch, McGraw-Hill, New York, 1960.

3. Y. L. LUKE, *Mathematical Functions and their Approximations*, Academic Press, New York, 1975.

**31[62M15].** S. LAWRENCE MARPLE, JR., *Digital Spectral Analysis with Applications*, Prentice-Hall Signal Processing Series (Alan. V. Oppenheim, Editor), Prentice-Hall, Englewood Cliffs, N. J., 1987, xx + 492 pp., 24 cm. Price \$43.95.

Applied spectral analysis has undergone significant changes in recent years. While traditional spectral analysis has evolved around Fourier methods, the modern approach emphasizes parametric modeling and makes extensive use of matrix analysis. The advent of powerful hardware for numerical processing, together with the ever increasing demand for speed, accuracy and complexity, are largely responsible for this shift of emphasis.