

## Supplement to Computation of Independent Units in Number Fields by Dirichlet's Method

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TABLE 1

n = 6								
D	P	R	$X_1$	$X_2$	$X_3$	$X_4$	$X_5$	$X_6$
2	2	8.07	1 1 -1	1 1 0	0 1 0	0 1 1	0 1 0	0 1 0
3	3	77.99	-2 -1 1	0 -1 -1	0 1 -1	1 -1 -1	0 0 0	0 1 1
5	3	105.92	2 -2 -2	0 1 -1	0 0 0	1 -1 1	0 1 1	0 0 0
6	9	666.30	1 1 -53	0 0 -36	-3 -3 0	0 0 18	0 0 12	1 -1 -2
7	10	1140.68	44 -3 -4593	0 3 -3331	23 11 -2405	0 11 -1736	12 6 -1259	0 1 -909
10	17	2037.05	-181 -101683 181	0 69276 82	-84 -47197 -21	0 32155 -50	-39 -21907 -21	0 14925 10
11	10	4269.68	-1475 -175 353330	988 0 -237006	-662 52 158800	444 0 -106533	-298 12 71460	200 0 -47880
12	22	5314.04	7 -78961825 -38135	0 -52185932 -38758	0 -34489736 41756	2 -22794312 -10528	0 -15064792 -10963	0 -9956338 11993

TABLE 1 (continued)

n = 6									
D	P	R	X <sub>1</sub>	X <sub>2</sub>	X <sub>3</sub>	X <sub>4</sub>	X <sub>5</sub>	X <sub>6</sub>	
13	4	747.04	34 -4 22	22 0 -25	14 -3 29	9 0 -6	6 2 7	4 0 -8	
14	34	3445.52	-29 -2199 -403201	0 1424 0	-12 -919 0	0 590 107760	-5 -378 0	0 243 0	
15	18	11794.33	31 -290524 -290524	0 19422 19422	0 105372 105372	-8 -74971 74971	0 5040 5040	0 27206 -27206	
17	18	9939.50	-33 -465121	0 -461148	0 468540	-6 -112824	0 -111870	0 113628	COEFFICIENTS > 1.E8
18	3	275.76	-1 -1 17	-1 1 0	1 1 0	0 0 4	0 0 0	0 0 0	
19	16	13960.50	-12304 -13216 -19651490	0 8116 0	-4611 -4965 0	0 3028 4508361	-1728 -1848 0	0 1132 0	
20	12	1880.76	-361 -45741 -11	0 -27763 -53	-133 -16851 -21	0 -10228 6	-49 -6208 14	0 -3768 6	
21	29	8285.91	-1705 5149 -665335	0 3096 0	-618 1851 0	0 1108 145188	-224 668 0	0 405 0	
22	23	14442.96	-793 30580901	0 0	-283 0	0 -6519870	-101 0	0 0	COEFFICIENTS > 1.E8
23	29	8233.13	-24 14332267 -373912	0 8498816 106947	0 5039686 66785	5 2988480 -77827	0 1772138 22749	0 1050856 14143	

TABLE 2

n = 7

D P	R	X <sub>1</sub>	X <sub>2</sub>	X <sub>3</sub>	X <sub>4</sub>	X <sub>5</sub>	X <sub>6</sub>	X <sub>7</sub>
2 5	53.57	1 -1 -1	1 -2 0	1 1 -2	1 1 -1	1 -1 0	1 0 -1	1 1 -1
3 7	116.79	1 10 7	1 9 5	0 8 5	0 7 4	1 6 3	0 5 3	0 4 2
4 12	214.27	-13 -1 -3	-11 -1 7	-9 2 -1	-7 0 -4	-6 -2 2	-5 0 2	-4 1 -2
5 20	591.07	1 4 11	1 5 -8	2 -3 -3	1 -1 5	-1 -3 1	-1 0 -4	0 1 1
6 19	881.17	-11 -19 -1	22 -11 -1	-1 -10 -3	-13 -9 -5	5 -6 -5	6 -4 -3	-5 -4 -1
7 41	21045.34	-587 1469 -2738021	490 -329 -3554159	28 35 -1786064	-375 250 351695	308 -504 1356453	-110 347 1079259	49 -289 241250
8 18	857.09	-25 1689 57	-19 1255 45	-13 932 47	-11 692 29	-7 514 16	-6 382 17	-4 284 14
9 72	6306.77	-19 712 -1873	11 -428 2041	-13 184 2859	3 -14 1512	0 -80 -147	0 113 -940	3 -106 -779
10 32	1748.37	-9 61 -1063861	5 46 -765644	-2 37 -551025	1 29 -396563	1 21 -285402	-1 14 -205399	1 8 -147623
11 51	5989.02	-16644 10	-4470 -27	9722 4	-507 12	-4424 -6	1776 -4	1618 4
COEFFICIENTS > 1.E8								
12 39	7406.40	-2327 -12239 -8735	135 138039 5149	1290 -74750 -2165	1045 -75562 202	271 118056 786	-266 -57373 -1111	-365 4362 1013

TABLE 2 (continued)

13 24	1370.92	1 12 157	0 -1 595	1 -6 -259	2 -6 -206	1 -1 188	0 1 41	0 2 -103
14 90	378243.8029			COEFFICIENTS	>	1.E8		
				COEFFICIENTS	>	1.E8		
				COEFFICIENTS	>	1.E8		
15 95	16480.76	-61 -629 -97379	-5 -1412 -71262	-8 722 -7890	29 439 31313	2 -463 33639	5 -63 16409	-5 231 -15
16 89	13713.42	-15 -4143 -63617	70 3310 -45262	-3 -2103 -28977	-28 1036 -18634	7 -315 -13545	7 -77 -9156	-7 238 -5755

TABLE 3

n = 8

D	P	R	X <sub>1</sub>	X <sub>2</sub>	X <sub>3</sub>	X <sub>4</sub>	X <sub>5</sub>	X <sub>6</sub>	X <sub>7</sub>	X <sub>8</sub>
-2	5	300.06	-1 1 1	0 0 0	-2 2 2	-2 -2 0	0 0 0	0 0 0	0 0 0	1 1 1
-3	5	385.26	-4 4 -2	-1 -1 1	2 -2 2	1 1 1	-1 1 -1	-1 -1 -1	0 0 0	1 1 1
-5	12	4522.72	2 -16298 -2	2 30522 2	-2 27664 2	0 -3440 0	1 -18297 -1	-2 -10968 -2	0 5410 0	0 11572 0
-6	28	16952.36	-575 -95 95	572 -76 -76	-294 -70 70	-266 -30 -30	312 -8 8	-114 2 2	-220 12 -12	203 21 21
-7	18	1217.82	-6 6 34	-3 -3 -1	2 -2 26	2 2 -3	0 0 9	-1 -1 -3	-1 1 -2	0 0 -1
-8	9	2400.45	1 1 -1	0 0 4	-2 -2 -2	-4 4 -4	-2 -2 2	1 -1 1	1 1 -3	0 0 2
-9	27	16014.99	435 -53 2000539	0 30 -32060362	198 9 43857521	0 -20 -43065604	0 12 35142668	0 0 -24476132	-66 -1 14074695	0 -4 -5629476
-10	18	8330.27	1 -173 1	16 -216 -16	-6 192 -6	8 26 -8	-4 -127 -4	-2 56 2	0 45 0	-3 -64 3
-11	46	46376.61	2078 1 1	-24603 201 -201	23604 156 156	-3645 -15 15	-19719 72 72	9693 33 -33	7854 -24 -24	-6591 -9 9
-12	26	15649.07	25 1585 -25	-10 -266 -10	30 -1176 -30	-44 -424 -44	36 456 -36	-15 377 -15	-1 12 1	4 -178 4
-13	52	32271.41	1221 -1221 473650	316 316 421212	-830 830 317434	290 290 202828	276 -276 103251	-304 -304 31172	26 -26 -11896	148 148 -31842

TABLE 3 (continued)

-14	32	47055.93	7143361	8494380	981576	-3851200	-2626784	545434	1658132	630504
			-7143361	8494380	-981576	-3851200	2626784	545434	-1658132	630504
			-36770047	-29676652	-20418088	-11784224	-5100448	-684204	1727888	2649312
-15	19	1591.59	4	3	-5	5	-4	3	-2	1
			4	-3	-5	-5	-4	-3	-2	-1
			-46	9	12	-21	21	-17	12	-7
-16	23	40006.37	-7361	4760	-2540	952	0	-476	635	-595
			7361	1904	-2540	-2380	0	1190	635	-238
			COEFFICIENTS > 1.E8							
-17	30	40341.69	13072	0	-8861	0	3001	0	59	0
			-931	-516	-251	-24	101	131	124	93
			817240	7758303	3764591	-1798801	-2820137	-628853	1051095	874251

TABLE 4

n = 8

D	P	R	X <sub>1</sub>	X <sub>2</sub>	X <sub>3</sub>	X <sub>4</sub>	X <sub>5</sub>	X <sub>6</sub>	X <sub>7</sub>	X <sub>8</sub>
2	2	49.17	-1	1	0	0	0	0	0	0
			1	1	0	0	0	0	0	0
			1	0	1	0	0	1	0	0
			1	0	1	0	0	0	0	0
3	8	6455.46	2	-14	19	-6	-9	10	-5	2
			-2	-14	-19	-6	9	10	5	2
			4	-4	1	1	-1	1	-1	0
			1	0	-1	0	0	0	1	0
5	8	10515.62	-6	-7	16	-5	-5	5	-2	1
			126	22	28	66	-3	-82	-54	14
			-126	22	-28	66	3	-82	54	14
			3	0	2	0	0	0	0	0
6	22	26423.50	5	-32	16	8	-6	-3	-4	7
			-721	4	330	4	-120	96	130	-64
			721	4	-330	4	120	96	-130	-64
			293	0	260	0	-110	0	-100	0
7	20	117945.70	-3032	-790	1448	1832	1147	298	-548	-692
			-3032	790	1448	-1832	1147	-298	-548	692
			-27	8	12	-10	1	0	1	2
			-1	0	1	0	1	0	0	0
8	9	3147.02	-3	0	0	0	1	0	0	0
			-65	52	-40	30	-24	18	-14	11
			1	-2	0	1	0	0	1	1
			-1	-2	0	1	0	0	-1	1
10	60	371303.57	COEFFICIENTS > 1.E8							
			-319843	84510	201568	-15722	-91831	13779	44776	-17805
			319843	84510	-201568	-15722	91831	13779	-44776	-17805
			-1	0	-14	0	-4	0	2	0
11	27	136464.85	10	-3	-8	2	5	-2	-2	1
			-10	-3	8	2	-5	-2	2	1
			80884	65099	-45965	-33870	24127	18734	-12779	-10853
			98	0	-81	0	-3	0	39	0
12	6	25368.50	-1	-2	-4	0	0	1	1	0
			-1	2	-4	0	0	-1	1	0
			1	-8	8	-4	0	2	-2	1
			-7	0	-8	0	-6	0	-2	0
13	87	3845635.63	1119	490	-206	-382	-312	-136	58	106
			1119	-490	-206	382	-312	136	58	-106
			COEFFICIENTS > 1.E8							
			298	0	306	0	-81	0	-84	0
14	7	100723.38	-15	-12	0	14	0	-8	2	2
			15	32	64	80	68	40	16	4
			15	-32	64	-80	68	-40	16	-4
			15	0	16	0	12	0	4	0

TABLE 5

n = 9

D	P	R	X <sub>1</sub>	X <sub>2</sub>	X <sub>3</sub>	X <sub>4</sub>	X <sub>5</sub>	X <sub>6</sub>	X <sub>7</sub>	X <sub>8</sub>	X <sub>9</sub>
2	5	165.95	-1	1	0	0	0	0	0	0	0
			-1	0	0	1	0	0	0	0	0
			1	-1	-1	1	1	0	-1	0	1
			1	0	0	1	1	0	-1	-1	0
3	11	1056.70	2	0	1	-1	0	-1	0	0	0
			-1	0	-1	-2	-1	0	0	-1	-1
			4	3	1	-1	-2	-2	-1	0	1
			-13	-6	8	7	-4	-7	1	6	1
4	11	1327.61	-17	-15	-13	-11	-9	-8	-7	-6	-5
			1	0	-1	1	0	0	0	0	0
			5	0	0	3	0	0	2	0	0
			-7	3	1	-3	5	-5	5	-4	3
5	27	182467.79	-1	0	0	4	0	0	-2	0	0
			-51	17	-4	-21	24	-27	28	-20	17
			-9	63	-56	34	55	-34	5	13	-32
			199	-884	686	-670	865	-641	285	-233	192
6	38	154638.34	1	0	0	-6	0	0	3	0	0
			-5	9	3	-6	-2	3	2	0	-2
			-1	-24	-15	-12	6	3	9	7	-1
			-13236993	18838326	32542473	28206786	13560254	-1917096	-11513318	-13167747	-8800581



7	16	10421.08	-2	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			5	-3	-1	-1	-3	0	0	1	0	0	0	0	0	0	0	0	0	0
			-26	0	19	7	-9	-6	5	5	6	6	-1	1						
			18	36	6	-38	3	11	12	12	-12	-5	-5							
9	12	9511.00	-2	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			-28	24	-18	12	-7	3	0	0	0	0	0	0	0	0	0	0	0	0
			28	39	30	12	-4	-12	-12	-12	-7	-1	-1							
			1	24	-15	0	9	-9	0	0	5	-4	-4							
10	56	372546.87	-1	5	-7	0	2	-1	0	0	2	2	-1	0	0	0	0	0	0	0
			1	0	0	6	0	0	-3	0	0	0	0	0	0	0	0	0	0	0
			3529	-3791	3407	-2686	1859	-1097	488	488	-51	-22	-22							
			10500761	-12251295	-9592912	4759591	7028411	-967272	-4474559	-4474559	-621305	2516329	2516329							
11	67	1606649.22	-1	0	0	-4	0	0	2	2	0	0	0	0	0	0	0	0	0	0
			-21	-34	27	5	-16	10	-2	-2	-5	7	7							
						COEFFICIENTS > 1.E8														
						COEFFICIENTS > 1.E8														
12	58	229134.44	-323	-105	66	141	126	63	0	0	-35	-40	-40							
			61	123	-138	18	55	-57	13	13	27	-24	-24							
			107	0	0	-33	0	0	-6	-6	0	0	0							
			2863	1452	-1326	-1182	449	801	-49	-49	-473	-97	-97							

TABLE 6

n = 10

D	P	R	X <sub>1</sub>	X <sub>2</sub>	X <sub>3</sub>	X <sub>4</sub>	X <sub>5</sub>	X <sub>6</sub>	X <sub>7</sub>	X <sub>8</sub>	X <sub>9</sub>	X <sub>10</sub>
-2	7	10268.18	-1	-5	-5	1	5	2	-1	-2	-3	-2
			1	0	1	0	0	0	0	0	0	0
			1	-5	5	1	-5	2	1	-2	3	-2
			-5	6	9	2	-7	6	8	-4	-4	2
-3	14	744293.19	-180532	-159775	83070	150133	-7182	-60725	15651	3786	-57494	-21169
			1	0	-2	0	-1	0	1	0	0	0
			-2	144	-84	39	38	-57	47	-30	-27	11
			2	144	84	39	-38	-57	-47	-30	27	11
-4	9	45232.36	-1	6	-1	-2	2	0	0	0	-1	1
			1	6	1	-2	-2	0	0	0	1	1
			-231	-19	-11	-39	97	60	43	107	48	36
			231	-19	11	-39	-97	60	-43	107	-48	36
-5	33	351452.45	1	0	-5	-10	-5	0	-2	-2	0	0
			1	0	0	0	0	0	-1	0	-1	0
			1	0	-5	10	-5	0	-2	2	0	0
			-5863401	74626500	3580705	-55000445	-1549720	38869305	633629	-27697493	-1245674	19974611

-6 34 10397094.11	-9926569	-6761011	-185929	3912968	3528759	783692	-1468612	-1942930	-789117	870067
	1	0	-60	0	20	0	15	0	-20	0
	26191	15569	4313	-979	-8398	-9185	-10885	-10190	-8369	-6743
	-26191	15569	-4313	-979	8398	-9185	10885	-10190	8369	-6743
-7 52 16096856.94	-28167	-14695	7640	5742	9190	3856	-7425	-4780	-2112	-470
	1	0	-8	0	-3	0	-1	0	-2	0
	-28167	14695	7640	-5742	9190	-3856	-7425	4780	-2112	470
			COEFFICIENTS > 1.E8							
-8 21 2409999.67	1	0	-5	0	-3	0	2	0	1	0
	-22244921	14926038	14714445	-9868620	-9688762	6505490	6391101	-4290501	-4225545	2838260
	-425	162	-41	-164	29	0	54	34	-52	49
	-425	-162	-41	164	29	0	54	-34	-52	-49

TABLE 7

n = 10

D P	R	$X_1$	$X_2$	$X_3$	$X_4$	$X_5$	$X_6$	$X_7$	$X_8$	$X_8$	$X_{10}$	
2 5	6359.86	-1 1 3 -1 -3	1 1 -1 -1 -1	0 0 -2 5 2	0 0 1 1 1	0 0 1 3 -1	0 0 -2 2 -2	0 0 1 2 -1	0 0 1 2 -1	0 0 1 0 1	0 0 -2 -1 2	0 0 0 1 1 0
3 8	564471.86	-1 -1 -478 -478 1	1 -2 683 -683 -2	-1 2 -628 -628 -2	-2 -1 245 -245 -1	4 -1 57 57 1	0 0 -83 83 0	0 -1 234 234 1	-1 1 -433 433 1	-1 -1 313 313 1	2 -1 -9 9 -1	2 -1 -1 -9 9 -1
5 17	46674845.69	-9 9 -2643 185488 -2643	24 24 903 255685 -903	-37 37 -719 14400 -719	32 32 899 -187240 -899	-4 4 2287 -102590 2287	-22 -22 111 84098 -111	23 -23 617 105650 617	-10 -10 -971 10080 971	2 -2 -642 -79494 -642	0 0 51 -50750 -51	0 0 51 -50750 -51

6 22	286112723.68	-13	20	33	-3	-16	-5	8	10	-2	-5
				COEFFICIENTS > 1.E8							
		7279	-4959	-2885	4381	4926	-5781	-2178	2956	1485	-2162
		-7279	-4959	2885	4381	-4926	-5781	2178	-2956	-1485	-2162
7 54	315740274.75	13	-6	-5	7	1	-2	-1	3	-1	-2
		-13	-6	5	7	-1	-2	1	3	1	-2
		-65038	-23465	65130	-49179	46686	-22339	-9646	22072	-21456	15550
		-65038	23465	-65130	49179	46686	22339	-9646	-22072	-21456	-15550
				COEFFICIENTS > 1.E8							
8 27	91174894.77	-17	4	19	8	-9	-10	-2	5	3	-2
		-11	18	11	-6	-13	-3	4	7	-2	-2
		18113939	-3412416	-6814162	4944916	-1051381	-3694959	4249980	1514683	-3821396	-164516
		-11	-18	11	6	-13	3	4	-7	-2	2
		-18113939	-3412416	6814162	4944916	1051381	-3694959	-4249980	1514683	3821396	-164516

TABLE 8

n = 11

D P	R	X <sub>1</sub>	X <sub>2</sub>	X <sub>3</sub>	X <sub>4</sub>	X <sub>5</sub>	X <sub>6</sub>	X <sub>7</sub>	X <sub>8</sub>	X <sub>9</sub>	X <sub>10</sub>	X <sub>11</sub>
2 9	6602.07	1 -1 -3 1 -1	1 1 -4 2 0	0 1 5 0 0	0 -1 1 -1 -1	0 0 5 -1 0	0 1 0 -2 0	0 0 -1 0 -1	0 0 2 1 0	0 0 -2 1 1	0 0 2 1 0	0 0 0 1 0
3 14	48549.91	1 -130 -1 4 1	6 -112 1 3 -2	-4 131 0 0 -1	-3 115 0 0 0	8 -122 0 -2 -2	9 -70 0 -3 -1	-1 63 1 -3 1	-6 53 0 -4 0	1 -11 0 -3 0	9 -52 0 -3 1	7 -16 0 -2 0
4 39	422532.74	-11 -5 -5 135 5	22 3 0 -913 -2	-9 1 0 545 -1	-10 2 3 404 1	15 -1 2 -825 1	-4 -3 0 394 2	-8 2 -1 304 0	8 3 -3 -518 -2	1 -3 -1 169 1	-7 -2 0 222 2	4 2 1 -246 -1
5 39	948073.98	-4 -4 4 -569 -1114	5 2 -76 -306 2036	1 -3 69 -96 1862	-1 0 14 131 -102	-1 0 -53 313 1402	0 -1 27 378 1510	2 1 10 367 -126	-1 -1 -30 357 470	-1 1 9 345 769	0 0 20 291 -350	1 0 -17 198 -170