REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1980 Mathematics Subject Classification (1985 Revision) can be found in the December index volumes of Mathematical Reviews.

9[73–02, 73C50].—PHILIPPE G. CIARLET, *Mathematical Elasticity*, Volume I: *Three-Dimensional Elasticity*, Studies in Mathematics and its Applications, Vol. 20, North-Holland, Amsterdam, 1988, xl + 451 pp., 23 cm. Price \$107.25/Dfl.220.00.

While many important advances in the theory of elasticity date from the first half of the previous century, the subject has been pursued with renewed vigor in the past few decades. A deep and coherent mathematical theory has been developed, although many fundamental open problems remain to challenge researchers. Several monographs—this being the latest but surely not the last—have appeared recently with the aim of introducing mathematically schooled readers to the theory, and preparing them to appreciate, or contribute to, contemporary research. For many readers, particularly those trained in partial differential equations, Ciarlet's *Mathematical Elasticity* will be among the most useful.

The book is strictly limited to static finite elasticity. (A second volume will be devoted to the more specialized topic of plate and rod theories.) Dynamic problems are not mentioned, and the linear theory appears only when required to prove something about the nonlinear model. The central problem considered is the determination of the deformations and corresponding stress fields of a three-dimensional body in equilibrium with imposed forces. The equations of equilibrium, common to all models in continuum mechanics, are three first-order PDE's in the six independent components of stress and three components of deformation. The assumption of elasticity means that the value of the stress field at a point depends only on the deformation gradient there. This constitutive assumption enables one to close the system, giving rise to a quasi-linear system of three second-order PDE's in the deformation. This system must be supplemented by boundary conditions and supplementary conditions such as injectivity or orientation-preservation of the deformation.

For a very important class of constitutive laws, these PDE's are the Euler-Lagrange equations for the minimization of a certain functional of the deformation, called the stored energy. For such *hyperelastic materials*, the boundary value problems of equilibrium may be formally viewed as problems in the calculus of variations. This alternate formulation of elasticity is dominant in much contemporary research. Ciarlet gives the boundary value problem formulation and the calculus of variations formulation equal time.

The book is divided into two parts. The first part describes the two formulations of the theory, determines the restrictions on the constitutive equations which can be derived rigorously from simple mechanical principles such as frame-indifference, and classifies the various types of constitutive laws and boundary conditions. The second part presents the two major approaches to existence. The first of these, initiated by F. Stoppelli in 1954, begins with the boundary value problem formulation and uses the implicit function theorem and the theory of the linearized elastic equations to obtain existence under the assumption of small data. The second approach, initiated by Ball in 1977, shows the existence of a minimizer of the stored energy function. Both approaches establish existence without uniqueness, which is essential, since many simple physical examples of nonuniqueness are known. Regularity of solutions remains, in large part, an open question.

Ciarlet's writing is clear and his notation consistent and carefully thought out. Complete proofs are generally given, in a style which is concise but not telegraphic. Many of the results needed from real analysis, differential geometry, functional analysis, and matrix theory are included, so that the book is surprisingly self-contained. Topics departing from, or extending the main line of the exposition, are presented in exercises, and there are extensive pointers to the literature and to open problems. In all, this book is an excellent introduction to the modern mathematical theory of elasticity.

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10[70–02, 70–08].—LESLIE GREENGARD, *The Rapid Evaluation of Potential Fields in Particle Systems*, An ACM Distinguished Dissertation, The MIT Press, Cambridge, Massachusetts, 1988, ix + 91 pp., $23\frac{1}{2}$ cm. Price \$25.00.

This book presents recent and important results obtained by V. Rokhlin and the author concerning the fast evaluation of the interactions in a system of particles governed by Coulombian forces.

In a situation where a naive computation would require an $O(N^2)$ work, it proposes an $O(N)$ algorithm based upon the remark that the logarithmic potential (in 2D) created by one particle can be replaced, away from this particle, by an inverse power expansion. For a group of particles, by simple additivity, this yields a multipole expansion valid in the external region; that is where most of the other particles are expected to live.

The author explains how to translate those multipole expansions and provide truncation estimates which indicate how many terms are needed in the expansion to reach a given accuracy. He then very clearly describes an algorithm using these tools and a hierarchy of meshes, going from the entire box containing all the particles to a desired refinement level. This hierarchy allows one to determine recursively the contribution to the potential at a given particle p_0 of all particles outside the finest box containing p_0 and its neighbors. At the end, one only needs to add the contribution of nearby particles, which is done by direct computation.