REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1980 Mathematics Subject Classification (1985 Revision) can be found in the December index volumes of Mathematical Reviews.

13[45–02, 45L10, 65R20].—A. N. TIKHONOV & A. V. GONCHARSKY (Editors), *Ill-Posed Problems in the Natural Sciences*, Advances in Science and Technology in the USSR, Mathematics and Mechanics Series (translated from the Russian by M. Bloch), "Mir", Moscow, 1987, 344 pp., $21\frac{1}{2}$ cm. Price $9.95.

And now remains that we find
out the cause of this effect.
Shakespeare: HAMLET

The theory of numerical methods for integral equations of the first kind has been under vigorous development for a quarter of a century. Such equations provide the most natural mathematical expression for many inverse problems in science and technology. Although specific instances of inverse problems have been around for some time (studies of the temperature of the early earth by Fourier and Kelvin, and Bernoulli's investigation of the inhomogeneous vibrating string come to mind), an awareness of some of the common features and main difficulties of inverse problems has emerged remarkably slowly.

The reluctance to recognize the reality and significance of inverse problems is partially rooted in philosophy and history. For generations, mathematical physics was dominated by the Laplacian view that the business of natural philosophy is the precise determination of effects from causes which evolve continuously over time or space. The future-directed and outward-looking viewpoint was dominant. Thus in the "direct" problems of classical mathematical physics each cause leads to a unique effect which depends continuously on the cause. This Laplacian mindset led to Hadamard's extreme position that any realistic problem in mathematical physics must have a unique solution which depends continuously on the data of the problem. Hadamard called such problems correctly set. The term well-posed is now more common, and problems not in this class have come to be known, somewhat darkly, as ill-posed.

The demands of science and technology in recent decades have brought to the fore many problems which are inverse to the classical direct problems, that is, problems which may be interpreted as finding the cause of a given effect. Included among such problems are many questions in remote sensing or indirect measurement such as the determination of characteristics of an inaccessible region from measurements on its boundary, the determination of system parameters from input-output measurements, and the reconstruction of past events from measurements of the present state. In these problems the viewpoint is inward-looking or past-directed. Very often, in the direct problem the transition from cause to effect is accomplished by a compact integral operator and hence the associated inverse problem is ill-posed, as the inverse of a compact operator is unbounded. This ill-posedness raises serious questions about the computation and interpretation of approximate solutions, particularly since in practical circumstances the observed effects, as measured quantities, are inherently imprecise, and even small observational errors can be magnified greatly by the discontinuous solution operator. This discontinuity often becomes apparent very quickly in wild numerical instability which emerges when conventional numerical methods are used on ill-posed integral equations of the first kind.

A numerical method for ill-posed integral equations of the first kind, now known as regularization, which replaces the first-kind integral equation by an approximating well-posed equation of the second kind was first published by D. L. Phillips (1962). For the matrix case, Levenberg had earlier (1944) proposed a closely related idea, known as damped least squares, for solving a linearized version of a nonlinear least squares problem. However, it was a seminal paper of A. N. Tikhonov in 1963 that launched a rapid development of the theory of regularization for numerical solution of ill-posed problems by the Soviet school, led by Academician Tikhonov. Effective application of this method is a delicate balancing act that involves the choice of regularization parameters, smoothing norms, and approximating subspaces and the incorporation of a priori information about the solution.

The book under review is a collection of fifteen papers on the theory of regularization for ill-posed problems and the application of the method of regularization to the numerical solution of specific ill-posed inverse problems in the natural sciences. All of the papers are by Soviet mathematicians who are associated with Tikhonov's school. The first three papers are devoted to the mathematical theory of regularizing algorithms, including questions related to the solution of linear algebraic systems with approximate data, a survey of the general theory of regularizing algorithms in Hilbert spaces, and an article with a distinct topological flavor on the regularization of the computation of function values with approximately specified arguments. The remaining thirteen chapters all deal with specific inverse problems in the natural sciences. There are three papers on geophysical inverse problems, including problems in electrodynamic prospecting, seismology, gravimetry and magnetometry. Additional papers on inverse problems in plasma diagnostics, astro-physics, inverse scattering, medical tomography, and various problems in optics round out the volume. In each of these papers the physical background of the problem is examined, the main problem is cast as an integral or operator equation of the first kind, the implementation of the regularization method is discussed and the results of the numerical calculations are displayed, usually in graphical form.

In this book, leading Soviet experts provide hard-to-find information on an eclectic collection of ill-posed problems in the natural sciences. The unifying thread is the use of the regularization method to solve these problems numerically. The very low purchase price is a genuine incentive for those interested in inverse problems and numerical analysis to add this little book to their personal libraries.

C. W. GROETSCH

Department of Mathematical Sciences
University of Cincinnati
Cincinnati, Ohio 45221-0025

14[65–04, 65F05].—THOMAS F. COLEMAN & CHARLES VAN LOAN, *Handbook for Matrix Computations*, Frontiers in Applied Mathematics, Vol. 4, SIAM, Philadelphia, PA, 1988, vii+264 pp., 23 cm. Price: Softcover $24.00.

No course in numerical linear algebra is complete without laboratory assignments. There are three good reasons for this. First, the coding of efficient matrix algorithms cannot be learned in an armchair; one must actually write programs. Second, students who have not made computer runs generally believe that rounding errors are like cooties—only other people get them. Finally, this is one of the few courses where the instructor can insist on cleanly coded, well-documented programs that would be suitable for distribution in a computation center.

Unfortunately, it is not enough to give a student an assignment and a due date. Even if the student already knows a programming language, there are tricks to making it do matrix computations efficiently. Students should be encouraged to use the Basic Linear Algebra Subprograms, which, however, have complicated calling sequences with forbidding parameters. The calling sequences for the standard matrix packages, LINPACK and EISPACK, are even worse. Finally, if the instructor decides to use an interactive package, the student must learn what is essentially a new language. Helping the student master these details can eat heavily into time better spent deriving and analyzing algorithms.

The authors of this handbook have circumvented these problems by providing their students with a small book that sets out the details in concise form. It is assumed that the reader has been exposed to a high-level programming language and has had one semester of linear algebra. The book consists of four chapters. The first is an introduction to FORTRAN. The second leads the reader through the Basic Linear Algebra Subprograms (BLAS) for performing vector operations. The third is an introduction to LINPACK, a collection of subroutines for computing and applying matrix decompositions. The last chapter is an introduction to MATLAB, an interactive system for manipulating matrices. Interesting exercises are interspersed throughout the book. Let us look at each chapter in turn.

Although they are not dogmatic about it—they make a gracious nod toward the language C—the authors have chosen FORTRAN as the language of their handbook. This makes sense. Not only is most scientific computation done in FORTRAN, but FORTRAN is the language in which the best matrix packages are coded. The chapter contains a detailed description of how arrays are arranged in memory and how they appear to subroutines. This material, which is of paramount importance in matrix computations, is slighted in most treatments of FORTRAN.