detailed presentations of the finite element concepts found in other texts.

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23[35L65, 35L67, 65N05, 65N30, 76G15, 76H05].—S. P. SPEKREIJSE, *Multigrid Solution of the Steady Euler Equations*, CWI Tract 46, Centrum voor Wiskunde en Informatica (Centre for Mathematics and Computer Science), Amsterdam, 1988, vi+153 pp., 24 cm. Price Dfl.24.20.

The subject at hand is the numerical solution of the compressible inviscid equations of fluid dynamics, the Euler equations. Only steady state solutions are desired, though knowledge of properties of the unsteady Euler equations is required to derive suitable methods. These equations are of great interest in aerodynamics and turbomachinery applications. One goal of numerical algorithm designers in the field of computational fluid dynamics is to produce efficient, robust solvers that can be used as design tools on a routine basis. The designer must be able to vary some parameter, for example a geometrical quantity, in a specified parameter set, and study how the solution varies. The millennium is not yet upon us, though it is perhaps in sight for problems in two space dimensions.

This compact book (153 pages) presents a large amount of material in a remarkably clear and concise fashion. From a derivation of the Euler equations and general material on the Riemann problem for hyperbolic systems in Chapter 1, the author moves to a long chapter concerned with upwind discretizations of the Euler equations. The key ingredient is an approximate Riemann solver; the author prefers Osher's. All the modern apparatus of upwind schemes is clearly presented here: numerical flux functions, approximate Riemann solvers, total-variation diminishing schemes, monotonicity, and limiters. Noteworthy is the author's new, weaker, definition of monotonicity for a two-dimensional scheme, which evades the negative result of Goodman and Leveque that total-variation diminishing schemes in two space dimensions are at most first-order accurate. The new definition of monotonicity still rules out interior maxima and minima (for scalar equations) but does not preclude second-order accuracy. The third chapter gives a very brief description of the multigrid algorithm and shows some numerical solutions of a first-order accurate discretization. Practitioners in the field have found in the past few years that multigrid algorithms converge well for first-order accurate discretizations of steady fluid flow equations. The convergence is helped along by a large amount of numerical viscosity. Unfortunately, this large amount of numerical viscosity leads to unacceptable inaccuracy in the computed solutions. Multigrid algorithms applied directly to second-order discretizations (which would give acceptable accuracy) tend to converge slowly, if they converge at all. This motivates the fourth and last chapter, in which defect correction is used. This allows one to produce a second-order accurate solution, even though only problems corresponding to first-order accurate discretizations need to be solved. It seems that this is the first time defect correction has been used in the solution of any problem in computational fluid dynamics. Perhaps the use of defect correction here will spark more interest in the technique.

There are just a few, easily corrected, typographical errors in the displayed equations of this book. More numerous, though perhaps no more numerous than is the norm nowadays, are the misspelled English words. The book does not claim to be a comprehensive account of the numerical solution of the Euler equations, nor of upwind methods for the Euler equations (central differences and the competitors to Osher's scheme are not discussed). Statements like "Nowadays, finite volume schemes are almost universally used for shock capturing codes" could be misleading to a novice. There are many working codes which are based on finite difference methods and which produce very good results for problems with shocks. The expert reader who has had trouble with convergence to steady state may look at the convergence plots in Chapter 4 of this book and wonder if the algorithm would produce solutions accurate to machine zero if iterated indefinitely. One who wanted to play the devil's advocate could look at some of the plots in Chapter 3 and claim they showed a convergence rate which is not independent of grid size (the method converging slower on finer grids).

These caveats noted, this book is the best single reference for a person trying to construct a multigrid code for the steady Euler equations. In this reviewer's opinion, a novice in the field of computational fluid dynamics, armed only with this book, could produce a working code. This is a tribute to the book's completeness and clarity.

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24[65-02, 65N30].—D. LEGUILLON & E. SANCHEZ-PALENCIA, *Computation of Singular Solutions in Elliptic Problems and Elasticity*, Wiley, Chichester, 1987, 200 pp., 24 cm. Price $42.95.

The monograph is focused on the explicit computation of the singular solutions near corners and interfaces in plane elasticity theory for multimaterials.

These problems are governed by the elasticity system with piecewise constant coefficients. Indeed, the constitutive law is different in each component of the material under consideration. The jumps in the coefficients are across the interfaces between the different components. These interfaces are usually assumed to be straight, or polygonal, lines. One can also view these problems as transmission problems across these interfaces.

The general theory shows that the solutions to these problems behave as

$$r^{\alpha} u(\theta) + \text{more regular terms}$$

in polar coordinates, where r is the distance to the singular point under consideration, which is either one corner of the boundary, or a corner of an interface, or even a point where an interface meets the boundary.