The corresponding stresses behave as $r^{\alpha-1}$ and therefore may become infinite as r goes to 0 when $\operatorname{Re}\alpha < 1$. Although this is in clear contradiction with the assumption that the strains remain small, which legitimizes the linear theory, it turns out in practice that it provides a good hint of where damage may occur. The smaller α is, the likelier is the damage.

The general theory of singular solutions is not provided in this monograph. However, assuming that the data, volume loads and surface loads, vanish in the vicinity of the corner or singular point under consideration, the authors give a very straightforward and illuminating analysis that shows the form of the singular solution there.

The analysis of the case of one single material shows that α is the solution of a rather complicated transcendental equation. The corresponding transcendental equations for multimaterials seem quite out of reach by classical analysis. This is why it is sensible to rely on numerical computation for producing approximate values of the corresponding α.

Two approaches to the actual calculation of α are described. They are based on the fact that the above u is the solution of a second-order boundary value problem for a system of two ordinary differential equations that depend quadratically on α.

In the first approach, the boundary value problem is discretized directly by the finite element method using piecewise linear functions. The approximate problem reduces to finding the zeros of a determinant $D(\alpha)$ that depends analytically on α. The method is quite cheap, yet accurate.

In the second approach, one performs a preliminary reduction of the order with respect to α, by writing the boundary value problem as a first-order system. Then one faces the more common problem of finding eigenvalues and eigenvectors to a boundary value problem for a system of ordinary differential equations. Again, this is approximated by piecewise linear elements. One is left with the problem of finding the eigenelements for a large matrix that is nonsymmetric in most cases. This method is more expensive and requires heavier equipment, but it also allows one to calculate u approximately.

The whole monograph is very clearly written. Many illustrative examples are provided. It will certainly be very helpful to engineers.

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25[49–01, 49H05].—M. J. SEWELL, *Maximum and Minimum Principles—A Unified Approach, with Applications*, Cambridge Univ. Press, Cambridge, 1987, xvi+468 pp., 23 $\frac{1}{2}$ cm. Price \$79.50 hardcover, \$34.50 paperback.

This is a text on operational variational methods. It concentrates on how to associate extremum principles with differential equations and applied problems. The book requires a minimal mathematical background and provides a large number of examples. The book has many exercises, so it could be used as a text in a mathematical methods course.

The organization is somewhat unusual. The first chapter treats mostly finite-dimensional saddle-point problems, with one small section on saddle-point problems for initial value problems. Chapter 2 is entitled "Duality and Legendre transformations" and includes some singularity theory and a number of interesting applications. Again, most of the chapter is finite-dimensional, with some infinite-dimensional examples interspersed.

The heart of the book is in Chapter 3, where the author shows how to associate dual variational problems with a saddle functional. This is done without any treatment of minimax theorems. Most of the examples involve ordinary differential equations and include variational inequalities and problems with nonstandard side conditions. Chapter 4 describes how to derive bounds on linear functionals of solutions of variational problems and develops some formal methods for finding variational principles for initial value problems.

The last chapter (Chapter 5) is 90 pages long and is entirely devoted to a theory of variational principles arising in the mechanics of solids and fluids. Much of this section reflects the author's own research and is a unique contribution to the literature.

In the first volume of Courant and Hilbert, the calculus of variations was described as being calculus on function spaces. Sewell's text provides a calculus, but not an analysis. It tries to present the theory without using the concepts and tools of real, or functional, analysis. Such an attitude was necessary in Courant's time. It is not defensible today. Most of the texts on finite-dimensional optimization theory referenced here use more analysis than this book does.

Equally surprising for a text in applied mathematics, it does not treat approximate, or numerical, methods for finding solutions. There is no discussion of second derivative conditions and no treatment of stability theory. It concentrates entirely on first derivative conditions and on setting up extremum principles. It does not study the type of critical point occurring at the solution. For the computational implementation of variational principles, it is crucial to know the type of a critical point. Algorithms for computing local minima are based on different considerations than those for finding saddle points.

This book is not directed at mathematicians. It does not define a Hilbert space, nor does it mention Banach spaces, Lebesgue or Sobolev spaces, or (metric) completeness. The author ignores many analytical details, and there is no discussion of existence questions. There is no use of the insights of convex analysis, or of minimax theory, both of which could have simplified much of the exposition. The Legendre transformation, described at length in Chapter 2, is a way to compute the convex conjugate of a given functional in convex analysis. It is the properties of this conjugate functional, not the Legendre transformation itself, that seem to be essential in much of duality theory. The author ignores the modern theory of the calculus of variations and the large volume of literature which has been written in a more sophisticated mathematical language; especially the contributions of the French, Italian and Soviet schools. Names that are missing from the bibliography include those of Brezis, Fichera, Mikhlin and Vainberg.

On the other hand, the examples and applications are of considerable interest. The book might be an appropriate introduction to these fascinating topics for someone, with an interest in applications, who is unwilling to learn elementary functional analysis. Euler would feel completely at home with this text.

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26[65-02, 65Kxx, 90Cxx].—R. FLETCHER, *Practical Methods of Optimization*, 2nd ed., Wiley, Chichester, 1987, xiv+436 pp., $23\frac{1}{2}$ cm. Price $53.95.

The development of optimization methods played a particularly vigorous role in the surge of numerical mathematics following the advent of electronic computing. Constrained optimization took center stage, first starting in the late 1940's with the development of linear and nonlinear programming by Dantzig, Charnes, Frisch, Zoutendijk, just to name a few. In the late 1950's Davidon's variable metric method, its subsequent analysis and independent work by Fletcher and Powell, yielded an entirely new perspective on the endeavor of unconstrained optimization, which at the time—hard as it is to comprehend now—was largely considered routine. Both aspects of optimization have been enriched by progress in numerical linear algebra, and their methodologies are indeed unthinkable without it.

This flourishing field, as described by one of its foremost pioneers, is the topic of this meticulously crafted book. It provides a concise, coherent, and no-nonsense description of the major practical techniques, together with evaluations and recommendations based on the author's extensive experience. His discussions touch on many alternatives and extensions to key ideas, placing them also in historical context. Experience and experiment are stressed throughout, and the text is correspondingly interspersed with illuminating numerical examples. Key theorems are stated and proved rigorously. A rich variety of exercises at the end of each chapter deepens the understanding and furnishes important additional detail. The entire material is supported by a carefully selected, yet complete, bibliography. There are only very occasional lapses into technical jargon, my favorite being "all pivots > 0 in Gaussian elimination without pivoting" (page 15). All in all, a highly recommended reading for those acquainted with the fundamentals of numerical linear algebra and multivariate calculus.

The division into two parts, on "unconstrained" and "constrained" optimization, respectively, reflects two different flavors of methodology. Each part had previously been published as separate volumes [1], [2], with the first part reprinted twice. The second edition, now combining both volumes, deepens the descriptions of key subjects in the first part and expands the subject matter of the second. Thus, in the first part, the Dennis-Moré characterization of superlinear convergence in nonlinear systems has been included and the important Chapter 2 has been substantially reorganized. In the second part, a section on "network programming", as well as a brief discussion of Karmarkar's interior point method, have been added. The reader will also find many new and stimulating questions in the problem sections. The second edition offers definitely more than the first edition, while retaining the focus and the crispness of the previous exposition.