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A COMPUTER SEARCH OF RANK-2 LATTICE RULES FOR MULTIDIMENSIONAL QUADRATURE

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ABSTRACT. For certain lattice rules of 'rank 2' it has been shown, in a recent paper, that a unique representation exists in a form suitable for computer evaluation. The present paper describes computer searches of such rules, reports results and identifies rules that appear promising for the numerical evaluation of practical multidimensional integrals.

1. INTRODUCTION

There is a continuing effort to find nonrandom sets of points in the multidimensional unit cube that are good abscissas for equal-weight multiple integration rules. This effort is roughly divided into two parts: the quasi-Monte Carlo method and the lattice method. Both methods aim to achieve faster convergence than the standard Monte Carlo method. Whereas the classical Monte Carlo method converges with order $1/\sqrt{N}$, where N is the number of points, the quasi-Monte Carlo method can achieve an order $(\log N)^{\gamma}/N$ for some $\gamma > 0$; a full description of the quasi-Monte Carlo method can be found in [9].

Lattice methods aim to achieve still faster rates of convergence, for integrands that are suitably well behaved. The study of lattice methods was initiated by Korobov [3] with the number-theoretic good lattice method. See also [9, 1] and references therein. Recently, a much wider class of lattice rules have been defined for the integration of smooth functions over the unit s-dimensional cube [10, 11]. In essence, a lattice rule is a rule whose abscissas are taken from a geometrical 'lattice' which includes the integer vectors as a sublattice. Subsequent work by Sloan and Lyness [12] has established a classification of lattice rules, which introduces the concept of 'rank'. The rank takes the value 1 for the number-theoretic rules of Korobov, and the value s for the s-dimensional trapezoidal rule, and more generally for any lattice rule that is an n^s copy of another rule.

The purpose of this paper is to consider in detail certain rules of rank 2. Some preliminary work has been done by Newman and Lyness [8] in a computer study of certain rank-2 and rank-3 rules in three dimensions. We now carry out computer searches in higher dimensions, and find the 'best' such rules according to a criterion introduced for number-theoretic rules by Korobov [4]. At the same time we carry out searches of rank-1 (i.e., number-theoretic) rules, so that the relative performance of the new rules can be assessed.

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In §2 we first restate the main results of [12] for lattice rules, and of [5] for the particular case of rules of rank 2. We then show how to eliminate from the computer search rules that differ only by uninteresting permutations or reflections. In §3 the full details of our computer implementation of the searching procedures are given. Section 4 contains numerical results, and §5 discusses the implications of these results. A preliminary report of this work has appeared in [13].

2. THEORY

2.1. Lattice rules for computer evaluation. Let If denote the integral of f over the unit s-dimensional cube $C^s = [0, 1]^s$,

$$If = \int_{C^s} f(\mathbf{x})\, d\mathbf{x}. \tag{1}$$

We consider only functions f which are continuous on C^s, and moreover have a continuous 1-periodic extension with respect to each variable $x^{(1)}, \dots, x^{(s)}$: that is, f is assumed to have the same values at points on opposite faces of the unit cube,

$$f(\mathbf{x})|_{x^{(i)}=0} = f(\mathbf{x})|_{x^{(i)}=1}, \qquad i = 1, \dots, s. \tag{2}$$

We may then define $\overline{f}$, the periodic extension of f, by

$$\overline{f}(\mathbf{x}) = f(\{\mathbf{x}\}), \qquad \mathbf{x} \in \mathbb{R}^s,$$

where $\{\mathbf{x}\}$ is the s-vector whose ith component is the fractional part $\{x^{(i)}\} = x^{(i)} - [x^{(i)}]$ of $x^{(i)}$. The extension to functions without the property (2) is discussed in [12]. However, for practical applications we recommend that $\overline{f}$ be at least continuous, and preferably have a continuous first derivative. A preliminary coordinate transformation is usually needed to force f to have the desired property.

The general definition of a lattice rule, as given in [10, 11], is

$$Qf = \frac{1}{N} \sum_{j=1}^{N} f(\mathbf{x}_j), \tag{3}$$

where $\mathbf{x}_1, \dots, \mathbf{x}_N$ are all the points of an infinite lattice that lie in the half-open unit cube. For our present purposes it is more convenient to begin with a representation established in [12]: there, it is shown that every lattice rule can be represented as a nonrepetitive expression of the form

$$Qf = \frac{1}{n_1 \cdots n_m} \sum_{j_1=1}^{n_1} \cdots \sum_{j_m=1}^{n_m} \overline{f}\left(j_1 \frac{\mathbf{z}_1}{n_1} + \cdots + j_m \frac{\mathbf{z}_m}{n_m}\right), \tag{4}$$

where $\mathbf{z}_1, \dots, \mathbf{z}_m$ are integer vectors, and

$$n_{i+1} \text{ divides } n_i \text{ for } i = 1, \dots, m-1, \quad n_m > 1. \tag{5}$$

Conversely, every expression of the form (4) and (5) is a lattice rule. A key feature of nonrepetitive rules of this form is that the numbers m (the rank) and $n_1, \ldots, n_m$ (the invariants) are uniquely determined. The simplest case is that of the number-theoretic rules [3],

$$Qf = \frac{1}{N}\sum_{j=1}^{N} \bar{f}\left(j\frac{\mathbf{p}}{N}\right), \tag{6}$$

for which the rank is clearly 1, and the (sole) invariant is N, provided $\mathbf{p}$ and N have no nontrivial common factor.

Though the form (4) is convenient for computer evaluation, we do not yet have a satisfactory foundation for an efficient computer search, for two reasons:

(i) A rule of this form with no restrictions on $\mathbf{z}_1, \ldots, \mathbf{z}_m$ can be repetitive, that is, there can be fewer than $n_1 \cdots n_m$ distinct abscissas.

(ii) The vectors $\mathbf{z}_1, \ldots, \mathbf{z}_m$ are far from unique, the same rule often appearing in many different guises.

In order to find an easily computable representation that leads to an efficient computer search procedure, we restrict attention to rules of rank 2.

2.2. Lattice rules of rank 2. We have established already that a lattice rule of rank 2 can be written as a nonrepetitive expression of the form

$$Qf = \frac{1}{n^2 r}\sum_{j_1=1}^{nr}\sum_{j_2=1}^{n} \bar{f}\left(j_1\frac{\mathbf{z}_1}{nr} + j_2\frac{\mathbf{z}_2}{n}\right), \tag{7}$$

where $n > 1$ and $r \geq 1$, and the two invariants are written as nr and n to satisfy (5). We shall assume throughout that n and r in (7) have no common prime factor, i.e., that their greatest common divisor (n, r) equals 1. Under this assumption the rank-2 rule can be rewritten in the form

$$Qf = \frac{1}{n^2 r}\sum_{j=1}^{r}\sum_{k_1=1}^{n}\sum_{k_2=1}^{n} \bar{f}\left(j\frac{\mathbf{z}}{r} + k_1\frac{\mathbf{y}_1}{n} + k_2\frac{\mathbf{y}_2}{n}\right), \tag{8}$$

where $\mathbf{z}, \mathbf{y}_1$ and $\mathbf{y}_2$ are integer vectors: for it may easily be verified that (8) becomes equivalent to (7) if $\mathbf{z} = \mathbf{z}_1$, $\mathbf{y}_1 = \mathbf{z}_1$ and $\mathbf{y}_2 = \mathbf{z}_2$. Conversely, it is shown in [5] that every nonrepetitive expression of the form (8) (with $n > 1$ and $(n, r) = 1$) is a rank-2 rule, and hence is expressible in the form (7): for example, we may choose in (7) $\mathbf{z}_1 = n\mathbf{z} + r\mathbf{y}_1$, $\mathbf{z}_2 = \mathbf{y}_2$.

If the components of vectors in (8) are suitably restricted, then this representation becomes unique: specifically, it is shown in [5] that the representation (8) is unique if $\mathbf{z}, \mathbf{y}_1$ and $\mathbf{y}_2$ satisfy

$$z^{(1)} = 1, \qquad \begin{bmatrix} y_1^{(1)} & y_1^{(2)} \\ y_2^{(1)} & y_2^{(2)} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \tag{9}$$

and

$$0 \leq z^{(i)} < r, \qquad 0 \leq y_j^{(i)} < n, \qquad \text{for } j \neq 1, 2,\ i = 1, \ldots, s. \tag{10}$$

Further, (9) ensures that the expression (8) is nonrepetitive.

The representation (8) is a convenient starting point for our rank-2 searches. It should be said, however, that not all rank-2 rules can be written in the form

(8). In fact, it is shown in [5] that a rank-2 rule can be written in this way if and only if its one- and two-dimensional projections onto the $x^{(1)}$ and $x^{(1)}, x^{(2)}$ subspaces have invariants nr and nr, n, respectively; in other words, if and only if the leading one- and two-dimensional projections have as many abscissas as possible (namely nr and n^2r)—or in the language of [5], if and only if the rule has 'full principal projections'.

There is still a further point to consider before we begin our computer search, namely that we would wish to eliminate from the list of rules to be searched any repetitions of rules that differ from each other only by uninteresting geometrical transformations.

2.3. The elimination of geometrically equivalent rules. We shall say that two rules are 'geometrically equivalent' if they differ only by a permutation of the variables, or by a reflection in one or more mid-planes, or by a combination of these. Since the search criterion to be described below does not distinguish between geometrically equivalent rules, it is clearly wasteful to include two or more geometrically equivalent rules in the list to be searched. We therefore seek an effective strategy for the recognition and elimination of this kind of redundancy.

The representation (8) is convenient for this purpose, as it allows the problem of geometrical equivalence to be reduced to that of the simpler rank-2 rule, with invariants n, n,

$$\tilde{Q}f = \frac{1}{n^2}\sum_{k_1=1}^{n}\sum_{k_2=1}^{n}\bar{f}\left(k_1\frac{\mathbf{y}_1}{n} + k_2\frac{\mathbf{y}_2}{n}\right). \tag{11}$$

For if $\mathbf{y}_1, \mathbf{y}_2$ and $\mathbf{y}_1', \mathbf{y}_2'$ generate two nonequivalent rules $\tilde{Q}f$ and $\tilde{Q}'f$, then the corresponding rules Qf and $Q'f$ given by (8) and (8′), where (8′) is the analogue of (8) with $\mathbf{z}, \mathbf{y}_1, \mathbf{y}_2$ replaced by $\mathbf{z}', \mathbf{y}_1', \mathbf{y}_2'$, will also be nonequivalent. On the other hand, if $\mathbf{y}_1, \mathbf{y}_2$ and $\mathbf{y}_1', \mathbf{y}_2'$ generate equivalent rules $\tilde{Q}f$ and $\tilde{Q}'f$, then it will be sufficient to include only one of these pairs in the list to be searched, provided we search over all allowed values of $\mathbf{z}$ in (8).

The strategy of using only the nonequivalent $\mathbf{y}_1, \mathbf{y}_2$ pairs from (11) and then searching over all allowed $\mathbf{z}$ vectors in (8) may not eliminate all geometrical equivalences of Qf. For example, if $n = 2$, then (11) is symmetric under reflection in every mid-plane. As a result, the replacement of $z^{(i)}$ by $r - z^{(i)}$ in the formula (8) for Qf leads to a geometrically equivalent rule. In cases such as this, where the rule (11) is symmetric in the mid-plane $x^{(i)} = \frac{1}{2}$, it is clearly sufficient to limit the component $z^{(i)}$ to the range $0 \leq z^{(i)} \leq [r/2]$. It is easy to find other situations in which the proposed strategy still leaves some geometrically equivalent pairs, but beyond some point the effort of eliminating them may not seem worthwhile.

The problem now reduces to the recognition of geometrical equivalences in the n^2-point rule (11), with the first two components of $\mathbf{y}_1, \mathbf{y}_2$ fixed by (9). In the present work a computational, rather than algebraic, method was employed for this purpose. The computational procedure is based on applying the rule (11), with the first two components of $\mathbf{y}_1$, $\mathbf{y}_2$ fixed by (9) and the other components running from 0 to $n-1$, to the function

$$f(\mathbf{x}) = \left(\sum_{i=1}^{s} h(x^{(i)})\right)^{s}, \tag{12}$$

where

$$h(x) = |x - \tfrac{1}{2}|. \tag{13}$$

As f is fully symmetric under permutations and reflections in mid-planes, two geometrically equivalent rules will clearly give the same value of $\widetilde{Q}f$; and we believe (but cannot yet prove) that two nonequivalent rules will yield different values. Thus we need retain in the list of $\mathbf{y}_1$, $\mathbf{y}_2$ pairs only those that yield a value of $\widetilde{Q}f$ not previously encountered. Table 1 gives the surviving $\mathbf{y}_1$, $\mathbf{y}_2$ pairs for some small values of n and s.

TABLE 1
Geometrically nonequivalent $\mathbf{y}_1$, $\mathbf{y}_2$ pairs for some small values of n and s.

s	n	$\mathbf{y}_1$	$\mathbf{y}_2$
3	2	100	010
		100	011
		101	011
4	2	1000	0100
		1000	0101
		1001	0101
		1000	0111
		1001	0110
		1001	0111
3	3	100	010
		100	011
		101	011
4	3	1000	0100
		1000	0101
		1001	0101
		1000	0111
		1001	0110
		1001	0111
		1011	0112

In practice, the above procedure was modified to avoid rounding error problems arising from the use of real arithmetic, by evaluating the related integer quantity

$$n^2 \widetilde{Q}F \equiv \sum_{k_1=1}^{n} \sum_{k_2=1}^{n} \overline{F}\left(k_1 \frac{\mathbf{y}_1}{n} + k_2 \frac{\mathbf{y}_2}{n}\right), \tag{14}$$

where $F(\mathbf{x}) = (\sum_{i=1}^{s} H(x^{(i)}))^s$, and $H(x) = |2nx - n|$. The possibility of integer overflow was ignored, so that the integers in (14) were actually calculated modulo some large number (on a Vax 11/750 computer), a procedure that seemed safe enough in practice. Table 2 lists the number N_y of surviving $\mathbf{y}_1, \mathbf{y}_2$ pairs for the cases $n = 2$ and $n = 3$ in 3 to 8 dimensions.

TABLE 2

The number of geometrically nonequivalent $\mathbf{y}_1, \mathbf{y}_2$ pairs.

s	N_y	
	$n = 2$	$n = 3$
3	3	3
4	6	7
5	10	12
6	16	20
7	23	30
8	32	44

2.4. The search criterion. A general discussion of possible criteria for assessing lattice and other rules is given in [6]. In the present work we use a standard criterion of Korobov and the number theorists (criterion (f') of [6]), namely: *for* α *a fixed even positive integer* (*e.g.*, $\alpha = 2$), *minimize*

$$P_\alpha = Qf_\alpha - If_\alpha = Qf_\alpha - 1, \tag{15}$$

where

$$f_\alpha(\mathbf{x}) = \phi_\alpha(x^{(1)})\phi_\alpha(x^{(2)}) \cdots \phi_\alpha(x^{(s)}), \tag{16}$$

and

$$\phi_\alpha(x) = 1 - (-1)^{\alpha/2} \frac{(2\pi)^\alpha B_\alpha(x)}{\alpha!}. \tag{17}$$

Here B_α is the Bernoulli polynomial of degree α.

The motivation lies in the form of the error expression for a lattice rule: it is shown in [11] that if the periodic extension $\bar{f}$ of the integrand in (1) has the absolutely convergent Fourier series expansion

$$\bar{f}(\mathbf{x}) = \sum_{\mathbf{m} \in \mathbb{Z}^s} a(\mathbf{m}) e^{2\pi i \mathbf{m} \cdot \mathbf{x}}, \tag{18}$$

then the lattice rule (3) corresponding to the lattice L has the error

$$Qf - If = \sum_{\mathbf{m} \in L^\perp}{}' a(\mathbf{m}), \tag{19}$$

where $L^\perp$ is the 'dual' lattice,

$$L^\perp = \{\mathbf{m} \in \mathbb{Z}^s : \mathbf{m} \cdot \mathbf{x} \in \mathbb{Z} \ \forall \mathbf{x} \in L\},$$

and the prime indicates that the term $\mathbf{m} = \mathbf{0}$ is to be omitted from the sum.

Since the particular function f_α defined by (16) has the simple Fourier series expansion

$$\overline{f}_\alpha(\mathbf{x}) = \sum_{\mathbf{m}\in\mathbb{Z}^s} \frac{1}{(\overline{m}_1\overline{m}_2\cdots\overline{m}_s)^\alpha} e^{2\pi i\mathbf{m}\cdot\mathbf{x}},$$

where

$$\overline{m} = \begin{cases} 1 & \text{if } m = 0, \\ |m| & \text{if } m \neq 0, \end{cases}$$

it follows from (15) and (19) that

$$P_\alpha = \sum_{\mathbf{m}\in L^\perp}{}' \frac{1}{(\overline{m}_1\overline{m}_2\cdots\overline{m}_s)^\alpha}. \tag{20}$$

The interesting point is that P_α is the maximum value of $|Qf - If|$ over the set of functions f whose Fourier coefficients satisfy $|a(\mathbf{m})| \leq 1/(\overline{m}_1\overline{m}_2\cdots\overline{m}_s)^\alpha$: for, with f in that set, (19) gives

$$|Qf - If| \leq \sum_{\mathbf{m}\in L^\perp}{}' |a(\mathbf{m})| \leq \sum_{\mathbf{m}\in L^\perp}{}' (\overline{m}_1\overline{m}_2\cdots\overline{m}_s)^{-\alpha} = P_\alpha,$$

with equality being achieved if $f = f_\alpha$.

In essence, the point of using P_α as the criterion is that every lattice rule finds the function f_α a difficult one to integrate, because the error expression (20) for the error in Qf_α involves no cancellation. As pointed out in [6], P_α may be a completely inappropriate measure for nonlattice rules, because many such rules find f_α trivially easy to integrate.

For the case of the rank-1 number-theoretic rules (6), it is known (see, for example, [9]) that there exist a sequence of prime numbers N and corresponding vectors $\mathbf{p}$ such that

$$P_\alpha \leq c(s, \alpha)\frac{(\log N)^{\alpha\beta(s)}}{N^\alpha}, \tag{21}$$

where $c(s, \alpha)$ and $\beta(s)$ are independent of N. Such sequences are often called 'good' lattice rules.

The following very easy result establishes the same property for a sequence of rank-2 rules having fixed smaller invariant n. Thus the judgement between rank-1 and rank-2 rules cannot be based on the notion that rank-1 rules necessarily have a better order of convergence—they do not.

Theorem. *Given $n > 1$, $s \geq 1$ and $\alpha > 1$, there exists a sequence of lattice rules with rank 2 and invariants nr, n, with $(n, r) = 1$, such that*

$$P_\alpha \leq d(n, s, \alpha)\frac{(\log N)^{\alpha\beta(s)}}{N^\alpha}, \tag{22}$$

where $N = n^2 r$, and $d(n, s, \alpha)$ and $\beta(s)$ are independent of N.

Proof. Corresponding to the rank-2 rule Qf in the form (8), there exists a corresponding r-point rule of rank 1,

$$\hat{Q}f = \frac{1}{r}\sum_{j=1}^{r} \overline{f}\left(j\frac{\mathbf{z}}{r}\right).$$

Assume r is prime and $r > n$, implying $(n, r) = 1$. By the result (21) quoted above, there exists a sequence of choices of r and $\mathbf{z}$ such that

$$\widehat{P}_\alpha \equiv \widehat{Q}f_\alpha - If_\alpha \leq c(s, \alpha)\frac{(\log r)^{\alpha\beta(s)}}{r^\alpha}.$$

Now the lattice $L(\widehat{Q})$ corresponding to the rule $\widehat{Q}$ is clearly a subset of the lattice $L(Q)$ corresponding to the rule Q, from which it follows that $L(Q)^\perp \subset L(\widehat{Q})^\perp$. Hence, for any choice of $\mathbf{y}_1, \mathbf{y}_2$ in (8),

$$\begin{aligned} P_\alpha &= \sum_{\mathbf{m}\in L(Q)^\perp}{}' \frac{1}{(\overline{m}_1 \cdots \overline{m}_s)^\alpha} \leq \sum_{\mathbf{m}\in L(\widehat{Q})^\perp}{}' \frac{1}{(\overline{m}_1 \cdots \overline{m}_s)^\alpha} = \widehat{P}_\alpha \\ &\leq c(s, \alpha)\frac{(\log r)^{\alpha\beta(s)}}{r^\alpha} \leq d(n, s, \alpha)\frac{(\log N)^{\alpha\beta(s)}}{N^\alpha}, \end{aligned}$$

where $d(n, s, \alpha) = n^{2\alpha}c(s, \alpha)$. □

It may be remarked that the proof yields a poor value for the constant in (22), compared to that in (21). However, the numerical calculations later in the paper strongly suggest that this is merely an artifact of the proof, at least for small values of n.

3. THE COMPUTER SEARCH

The object is to search over geometrically nonequivalent rank-2 rules with invariants nr, n and $(n, r) = 1$, which have full principal projections. This is achieved in practice by searching over all $\mathbf{y}_1, \mathbf{y}_2$ pairs from the restricted list as described in subsection 2.3, and a set of allowed $\mathbf{z}$ vectors satisfying (9), (10). In each case we applied the criterion in subsection 2.4 to obtain the rules giving the smallest P_α values. In practice we minimized P_2 and P_6 separately, and also performed comparable searches on rank-1 rules of the form (6), so that the relative performances of rank-1 and rank-2 rules could be quantified.

The search procedure described in this work differs from previous searches [1, 2, 7] in two main respects: first, that rank-2 rules are being searched for the first time, and second (due to the ever increasing power and improving architecture of computers) that search programs have been written specifically to take advantage of the Cyber 205 vector capabilities. A full statement of the search procedure now follows.

For the rank-2 quadrature rule in the form (8), the ingredients to be specified are the dimension s, the positive integers n and r (with $(n, r) = 1$), the pair of vectors $\mathbf{y}_1, \mathbf{y}_2$, and the vector $\mathbf{z}$. Given s and n, the pairs $\mathbf{y}_1, \mathbf{y}_2$ are chosen as in subsection 2.3 (some examples, for small s, being given in Table 1). Recall that there are N_y such pairs, with N_y given in Table 2. In a particular search we compute, for fixed s, n and r, the values of P_2 and P_6 for all $\mathbf{y}_1, \mathbf{y}_2$ pairs, and for a set of $\mathbf{z}$ vectors as specified below. We retain from the search only the rules which minimize P_2 or P_6 (or both).

Since it is impossible in practice to search over all possible $\mathbf{z}$ vectors, we set up, for given s and r, two separate search sets of $\mathbf{z}$ vectors, one with components of $\mathbf{z}$ chosen systematically, as below, and another (of equal size) with components chosen at random. The first set consists of vectors of the one-parameter (Korobov [4]) form

$$\mathbf{z} = (1, a, a^2, \dots, a^{s-1}) \quad (\mathrm{mod}\ r), \tag{23}$$

with $1 \le a < r$, or, for the special case $n = 2$, $1 \le a < r/2$. The second set consists of vectors of the form

$$\mathbf{z} = (R^{(1)}, \dots, R^{(s)}),$$

with $R^{(i)}$ a random variable in the range 1 to r. Then, for $n > 2$, the total number of $\mathbf{z}$ vectors in each of the search sets is $(r - 1)$, and for $n = 2$ it is $(r - 1)/2$. The two sets of $\mathbf{z}$ vectors (which we shall call 'Korobov' and 'random', respectively) were searched separately.

For comparison we also carried out searches of rank-1 rules with the same total number of points (i.e., $N = n^2 r$), and with $\mathbf{p}$ in (6) given either by

$$\mathbf{p} = (1, a, a^2, \dots, a^{s-1}) \quad (\mathrm{mod}\ N) \tag{24}$$

with $1 \le a \le N/2$, or by

$$\mathbf{p} = (R^{(1)}, \dots, R^{(s)}),$$

with $R^{(i)}$ a random variable in the range 1 to N. The number of $\mathbf{p}$ vectors of the first ('Korobov') type is $[N/2]$. The number of $\mathbf{p}$ vectors of the second ('random') type was taken to be $N_y(r - 1)$ if $n > 2$, or $N_y(r - 1)/2$ if $n = 2$, to give the same number of 'random' rules searched as in the rank-2 case.

Searches were restricted to 3, 4, 5, 6, 7 and 8 dimensions, with $n = 2$ or $n = 3$, and to selected values of $N < 131\,070$, with r such that $N = n^2 r$ and $(n, r) = 1$. Higher values of n were excluded after preliminary studies showed that the results with $n = 5$ were usually worse, for a given total number of points N, than those with $n = 3$ and (especially) $n = 2$. The restriction to $N < 131\,070$ originates in the architecture of the Cyber 205 computer and the consequent structure of the programs written to do the searching: the programs contain vectors of length equal to the number of abscissas in a rule—or more precisely to half that number, because of symmetry in the functions ϕ_α defined in (17). (The maximum vector length in the Cyber 205 is 65 535 $(= 2^{16} - 1)$.) Details of the vectorization of the searches are given in the Appendix.

Rules falling in three 'windows' of N values were considered, as follows:

(i) 1000-point window with a full search (described in subsection 3.1);
(ii) 10 000-point window with both full and 'reduced' searches (described in subsection 3.2);
(iii) 100 000-point window with a further reduced search (described in subsection 3.3).

The searches were done in the above order. At each stage the results provided suggestions for reducing the search procedure for the next window, so as to keep the work required within acceptable limits. The ways in which the searches were reduced are stated in subsections 3.1, 3.2 and 3.3.

3.1. 1000-point window. Rules were searched for a number of N values in the range 948 to 1052, including all the values of N that, for a given n, satisfy $(n, r) = 1$ and $N = n^2r$ for some r. With $n = 2$ these are $N = 948, 956, 964, \ldots, 1052$ (i.e., 14 values); with $n = 3$ these are $954, 963, 981, 990, 1008, 1017, 1035$, and 1044 (i.e., 8 values). At each N, rank-1 and rank-2 rules were searched (in the latter case with $n = 2$ or $n = 3$, as appropriate), for both 'Korobov' and 'random' vectors $\mathbf{p}$ and $\mathbf{z}$, and (in the rank-2 case) for all of the N_y nonequivalent $\mathbf{y}_1, \mathbf{y}_2$ pairs. The numbers of rules searched by this 'full search' procedure are as set out in Table 3.

TABLE 3
Numbers of rules searched in the 'full' search used for the 1000=point window.

Type of Rule	Number of Rules Searched	
	$n = 2$	$n = 3$
(1) Rank 2, 'Korobov' $\mathbf{z}$ vectors	$\frac{(r-1)}{2}N_y$	$(r-1)N_y$
(2) Rank 2, 'random' $\mathbf{z}$ vectors	$\frac{(r-1)}{2}N_y$	$(r-1)N_y$
	(for comparison with the rank-2 searches with $n = 2$)	(for comparison with the rank-2 searches with $n = 3$)
(3) Rank 1, 'Korobov' $\mathbf{p}$ vectors	$\frac{N}{2}$	$\frac{N}{2}$
(4) Rank 1, 'random' $\mathbf{p}$ vectors	$\frac{(r-1)}{2}N_y$	$(r-1)N_y$

Note that there is a discrepancy between the numbers of rank-1 and rank-2 rules searched, arising from the different numbers of 'Korobov' vectors $\mathbf{z}$ and $\mathbf{p}$ when the parameter a is allowed its full range in (23) or (24). Consequently, it is not possible to search exactly the same number of rules.

As we shall see in §4, a striking conclusion from the searches in the 1000-point window was the overwhelming predominance among the 'best' rank-2 rules of rules with a particular $\mathbf{y}_1, \mathbf{y}_2$ pair, namely $\mathbf{y}_1 = \mathbf{Y}_1$, $\mathbf{y}_2 = \mathbf{Y}_2$, where

$$\mathbf{Y}_1 := (1, 0, 0, 0, \ldots, 0), \qquad \mathbf{Y}_2 := (0, 1, 0, 0, \ldots, 0), \tag{25}$$

that is, with all components zero except for those fixed by (9). Whatever may be the origin of this phenomenon, it immediately suggests an empirical way of dramatically reducing the time for the rank-2 searches: namely, that instead of considering all of the N_y nonequivalent $\mathbf{y}_1, \mathbf{y}_2$ pairs, one could impose from the start $\mathbf{y}_1 = \mathbf{Y}_1$ and $\mathbf{y}_2 = \mathbf{Y}_2$. The resulting concept of a 'reduced search' (in which the number of rules searched is exactly as in Table 3, but with N_y replaced by 1) was central to our consideration of rules in the 10 000-point window, to keep computer costs within acceptable limits.

3.2. 10 000-point window. To test the validity of the reduced search strategy, we first considered in detail $N = 9972$, carrying out the following searches:

(i) rank-2 $n = 2$ full search;
(ii) rank-2 $n = 2$ reduced search,

where a full search is as in Table 3, and a reduced search, as above, refers to the inclusion of only the pair $\mathbf{y}_1 = \mathbf{Y}_1$, $\mathbf{y}_2 = \mathbf{Y}_2$. Thus, in a reduced search, N_y in Table 3 is replaced by 1. The results (to be discussed in §4) convinced us to rely solely on the reduced search strategy in the future.

We then investigated 8 values of N between 9972 and 10 764 chosen such that both $n = 2$ and $n = 3$ rank-2 rules exist (requiring N to be a multiple of 36, and $N/36$ a multiple of neither 2 nor 3). The N values satisfying these conditions are 9972, 10 116, 10 188, 10 332, 10 404, 10 548, 10 620, 10 764. For these we carried out:

(i) rank-2 $n = 2$ reduced search;
(ii) comparison rank-1 search;
(iii) rank-2 $n = 3$ reduced search;
(iv) comparison rank-1 search.

3.3. 100 000-point window. For $N \approx 100\,000$ the search procedure was very expensive and required the full memory resources of Cyber 205, so we further reduced the searches by choosing even the parameter a in the 'Korobov' $\mathbf{z}$ vector at random (following the example of Haber [2] for the rank-1 case). For $N = 100\,044$ we considered:

(i) $s = 3$ to 5, rank-2 $n = 2$ reduced search with 'random' and 'Korobov-random' vectors $\mathbf{z}$, with the number of rules of each type searched equal to $r/20$;
(ii) $s = 3$ to 8, rank-2 $n = 2$ reduced search with 'random' and 'Korobov-random' vectors $\mathbf{z}$, with the number of rules of each type searched equal to $r/200$.

And for $N = 131\,004$ we considered:

(iii) $s = 7$ and 8, rank-2 $n = 2$ reduced search with 'random' and 'Korobov-random' vectors $\mathbf{z}$, with the number of rules of each type searched equal to $r/200$.

4. RESULTS OF THE COMPUTER SEARCHES

For each of the searches described in §3, the 'best' rules, as judged by P_2 and P_6 separately, are recorded in the microfiche supplement at the end of this issue.

Here we attempt to give an overall view of the results so obtained, and at the end select (in Tables 4, 5 and 6) some rules that seem to be particularly promising.

A first useful observation is that a rule that minimizes P_2 usually gives a reasonably small value of P_6 and vice versa—a fact that is easily understood from the error expression (20). (For a more complete discussion, see [6].) In the following we therefore concentrate on the results for P_2.

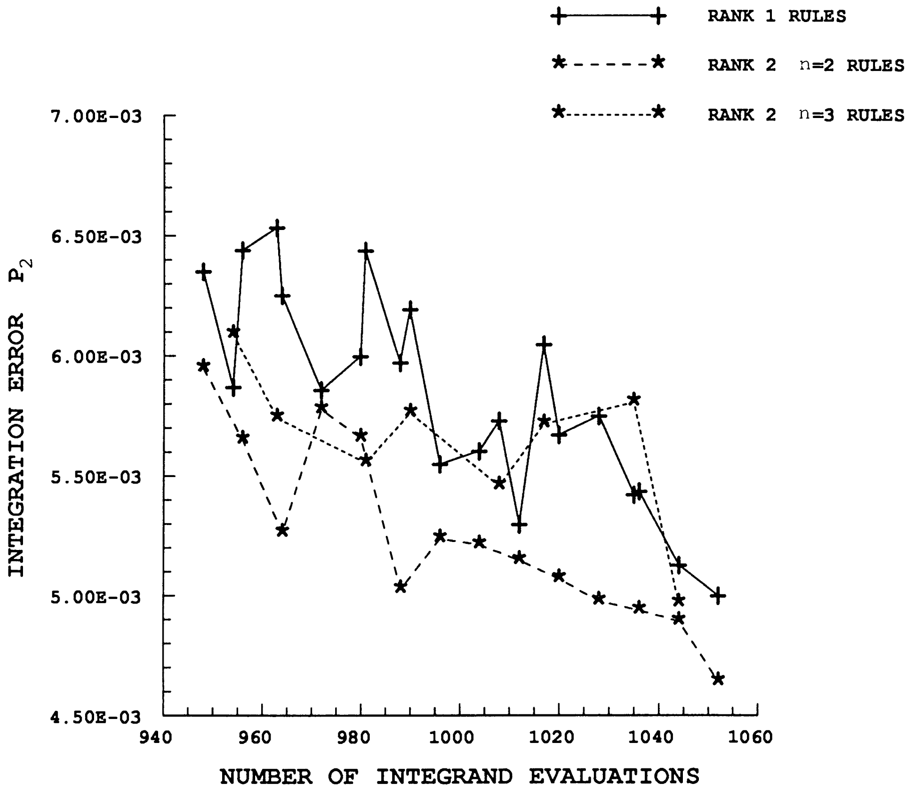

FIGURE 1

Integration error P_2 (as defined in (15)) as a function of the number of integrand evaluations (N or n^2r) in 3 dimensions, for the 1000-point window. Each point gives the smallest P_2 value obtained from searches at fixed N.

The full results also record, in each case, separate 'best' results for 'Korobov' and 'random' vectors **z** or **p**. (See subsection 3.1.) In the following, however, we shall generally quote just the minimum of the two values so obtained.

4.1. The 1000-point window. Figures 1 to 3 show the minimum values of P_2 obtained, for dimensions 3, 5 and 8, for all values of N in the 1000-point window. The three separate graphs in each case are for the rank-1 case (solid line), the rank-2 $n = 3$ case (short dashed line) and the rank-2 $n = 2$ case (long dashed line). (The lines are included merely to guide the eye between the plotted points, and have no other significance.)

For the three-dimensional results in Figure 1 one sees the rank-2 $n = 3$ rules performing, on the whole, as well as the rank-1 results, and the rank-2 $n = 2$ rules performing rather better than either of the others.

Similar conclusions hold in all dimensions; but the apparent superiority of the rank-2 $n = 2$ rules becomes much more marked as the dimensionality

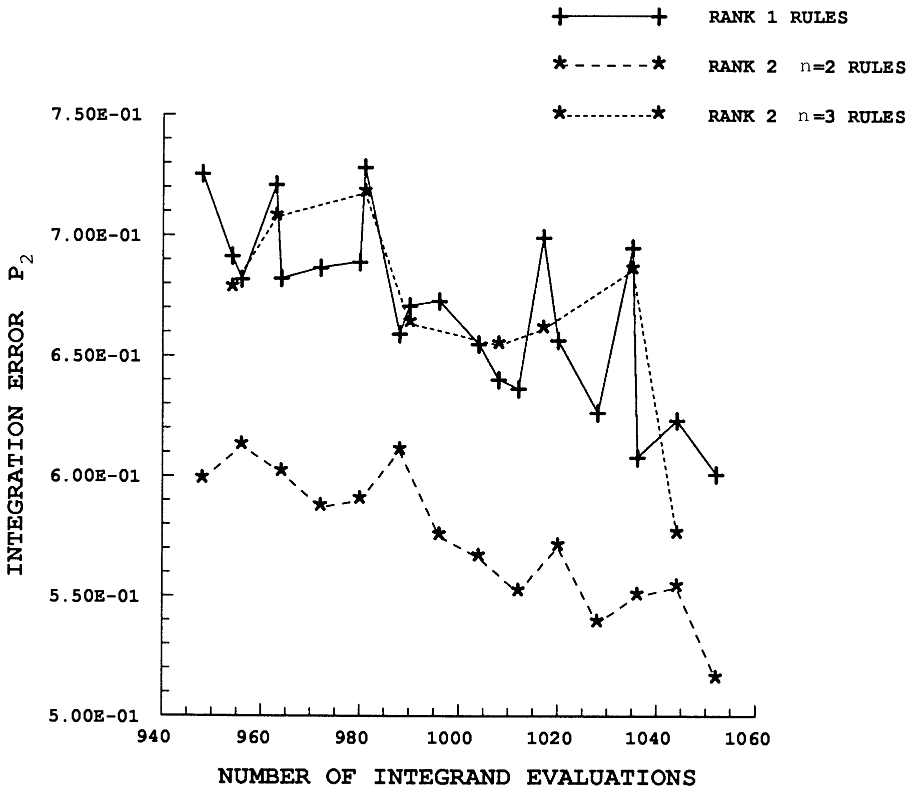

FIGURE 2

Integration error P_2 (as defined in (15)) as a function of the number of integrand evaluations (N or n^2r) in 5 dimensions, for the 1000-point window. Each point gives the smallest P_2 value obtained from searches at fixed N.

increases: for, in the 5-dimensional and 8-dimensional results of Figures 2 and 3, the graph of the best rank-2 $n = 2$ rules is very well separated from both of the others. In *no* case, in any of the figures, have we found a rank-1 rule that competes with the best of the rank-2 $n = 2$ rules.

Of the rules in Figures 1 to 3, some have $\mathbf{z}$ or $\mathbf{p}$ vectors of 'Korobov' type, and some have 'random' $\mathbf{z}$ or $\mathbf{p}$ vectors. The honors are about even, with the proportion of 'Korobov' vectors rising slowly (to about 60%) as the dimension s increases.

A striking (and to us totally unexpected) observation from the complete tabulation of the 'best' rules in the 1000-point window is that most of them (for example, 65% of them when $s = 3$, and 86% when $s = 6$) have the particular $\mathbf{y}_1, \mathbf{y}_2$ pair given by (25). The bias is particularly striking for the larger values of s, since the total number N_y of possible $\mathbf{y}_1, \mathbf{y}_2$ pairs rises rapidly with s (see Table 2).

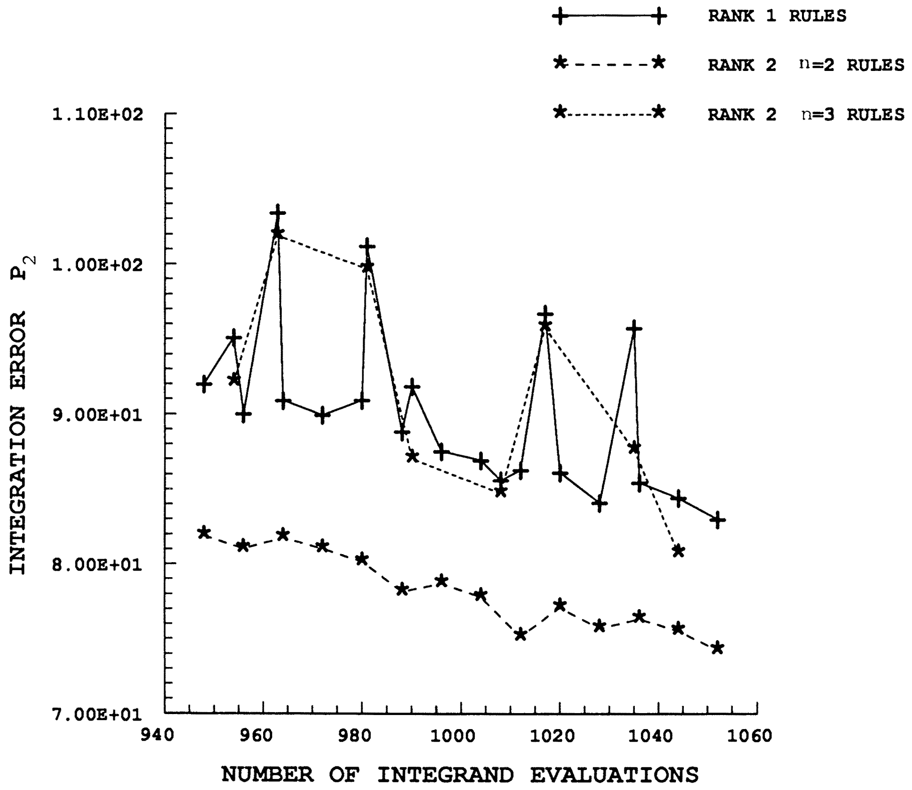

FIGURE 3

Integration error P_2 (as defined in (15)) as a function of the number of integrand evaluations (N or n^2r) in 8 dimensions, for the 1000-point window. Each point gives the smallest P_2 value obtained from searches at fixed N.

A bias towards the special $\mathbf{y}_1$, $\mathbf{y}_2$ pair given by (25) also persists, though in less marked form, in the rank-2 $n = 3$ tabulated results for the 1000-point window. In this case, about one third of the 'best' rules have the special $\mathbf{y}_1$, $\mathbf{y}_2$ pair for all dimensions from 3 to 8, compared to a much smaller expected proportion (see Table 2) if the distribution were statistical.

In summary, the salient features of the results for the 1000-point window appear to be that the 'best' of the rank-2 $n = 2$ rules consistently perform better than both the rank-1 and rank-2 $n = 3$ rules; and that these best rules have a very strong bias towards the particular $\mathbf{y}_1$, $\mathbf{y}_2$ pair given by (25).

In Table 4 we have selected, rather subjectively, a subset of rules from the complete tabulation of rules for the 1000-point window, that seem to us particularly good as judged by the performance for both P_2 and P_6. All, of course, are rank-2 $n = 2$ rules, and (with just one exception) all have the special vector pair $\mathbf{y}_1$, $\mathbf{y}_2$ given by (25). Many of these rules minimize *both* P_2 and P_6. If

TABLE 4

A selection of rank-2 rules from the computer searches, in the 1000-point window, for 3 to 8 dimensions. All have $n = 2$ and, except for the one case shown otherwise, the special vector pairs $\mathbf{y}_1 = \mathbf{Y}_1$, $\mathbf{y}_2 = \mathbf{Y}_2$ given by (25).

s	$\mathbf{y}_1$	$\mathbf{y}_2$	$\mathbf{z}$	r	N	No. of rules searched	P_2	P_6
3			85 173 215	253	1012	504	0.48779E − 02	0.35318E − 10
3			24 69 235	257	1028	512	0.49103E − 02	0.55095E − 10
3			1 118 197	259	1036	516	0.49369E − 02	0.88647E − 10
3			1 125 108	263	1052	524	0.46389E − 02	(0.62585E − 10)
3			1 77 143	263	1052	524	(0.47765E − 02)	0.57511E − 10
4			59 13 151 219	251	1004	750	0.68468E − 01	0.56933E − 07
4			134 181 44 175	255	1020	762	0.68871E − 01	0.52421E − 07
4	1000	0111	1 50 169 162	259	1036	774	0.68817E − 01	0.28850E − 07
4			1 50 133 75	263	1052	786	0.64661E − 01	0.50868E − 07
5			165 155 50 116 31	243	972	968	0.57793E + 00	(0.22054E − 04)
5			125 29 142 76 12	243	972	968	(0.59426E + 00)	0.16535E − 04
5			34 210 142 39 155	261	1044	1040	0.55351E + 00	0.17458E − 04
5			1 37 84 24 117	257	1028	1024	0.53875E + 00	0.12919E − 04
5			1 90 210 227 179	263	1052	1048	0.51525E + 00	(0.10321E − 04)
5			1 103 89 225 31	263	1052	1048	(0.55621E + 00)	0.96425E − 05
6			231 85 107 143 5 147	243	972	1210	0.35331E + 01	0.12668E − 02
6			61 16 65 76 73 226	251	1004	1250	0.34273E + 01	(0.17762E − 02)
6			65 46 116 239 179 57	251	1004	1250	(0.34313E + 01)	0.89440E − 03
6			250 92 116 155 5 159	263	1052	1310	0.32123E + 01	0.77498E − 03
6			1 98 136 178 86 12	263	1052	1310	0.31434E + 01	0.73189E − 03
7			99 38 138 228 106 201 99	243	972	1452	0.16982E + 02	(0.41247E − 01)
7			6 24 173 229 97 206 77	243	972	1452	(0.17953E + 02)	0.15417E − 01
7			120 178 189 203 141 199 25	257	1028	1536	0.16441E + 02	(0.13001E + 00)
7			147 6 120 170 173 35 179	257	1028	1536	(0.16750E + 02)	0.10966E − 01
7			1 32 44 183 221 212 169	245	980	1464	0.17332E + 02	0.90423E − 02
7			1 82 42 103 222 214 72	257	1028	1536	0.16245E + 02	(0.90464E − 02)
7			1 77 18 101 67 19 178	257	1028	1536	(0.16486E + 02)	0.80419E − 02
8			198 132 75 93 201 181 159 174	255	1020	1778	0.75847E + 02	(0.45566E + 00)
8			92 3 144 60 185 141 109 37	255	1020	1778	(0.76042E + 02)	0.22930E + 00
8			90 39 112 244 12 78 118 156	261	1044	1820	0.72828E + 02	0.25408E + 00
8			1 62 49 2 124 98 4 248	253	1012	1764	0.75147E + 02	0.24833E + 00
8			1 60 181 77 149 261 143 164	263	1052	1834	0.74247E + 02	(0.26772E + 00)
8			1 68 153 147 2 136 43 31	263	1052	1834	(0.74308E + 02)	0.23635E + 00

this is not the case, then we show two different rules for the same values of s and N, one minimizing P_2, and one minimizing P_6. (The quantity *not* minimized by the particular rule is enclosed in parentheses.)

The rules in Table 4 in which the $\mathbf{z}$ vectors are of the 'Korobov' form are distinguished by the fact that $z^{(1)} = 1$; in all other cases, the $\mathbf{z}$-vectors are 'random'.

Finally, Table 4 allows us to make a useful point about the use of P_2 as a search criterion. If we recall that P_2 is the error in the integration of a function (namely f_2) whose exact integral is 1, it may seem bizarre in the extreme to persist with P_2 when its value is comparable to or greater than 1. Yet, we see in Table 4 that rules that minimize P_2 often also minimize P_6, which is an integration error of much more sensible size. Thus, the use of P_2 as a criterion may still be defensible even when its value is large.

4.2. The 10 000-point window. We recall from subsection 3.2 that for the 10 000-point window the first step was to carry out both a 'full' search and a 'reduced' search (i.e., one restricted to the special pair $\mathbf{y}_1 = \mathbf{Y}_1$, $\mathbf{y}_2 = \mathbf{Y}_2$) for the rank-2 $n = 2$ case, the purpose being to assess the effectiveness of the reduced search strategy.

The full search required in total 2940 cpu seconds on a Cyber 205, compared with 185 cpu seconds for the reduced search. Yet, in 9 cases out of 12 (counting separately the P_2 and P_6 minimizations, and dimensions from 3 to 8), the two searches yielded exactly the same result. In the other three cases, the full search yielded marginally better results, but hardly enough to justify fifteen times the expense. We therefore resolved to concentrate exclusively on reduced searches.

Adopting the reduced search, rules for a range of N-values between 9972 and 10 764 were searched as described in §3, and the results compared graphically in a manner similar to the 1000-point window. Again, a comparison of rank-1 and rank-2 rules was made, and it was found that the rank-2 $n = 2$ rules consistently performed better than the others in four to eight dimensions, and the performance of the rank-2 $n = 3$ and rank-1 rules were about equal. This can be seen for the cases of three, five and eight dimensions in Figures 4, 5 and 6. The case of three dimensions, in Figure 4, has all three types of rules giving roughly equal performance. The superiority of the rank-2 $n = 2$ rules thus

TABLE 5

A selection of rank-2 rules from the computer searches, in the 10 000 - point window, for 3 to 8 dimensions. All have $n = 2$ and the special vector pairs $\mathbf{y}_1 = \mathbf{Y}_1$, $\mathbf{y}_2 = \mathbf{Y}_2$ given by (25).

s	$\mathbf{z}$	r	N	No. of rules searched	P_2	P_6
3	1 1186 2056	2601	10404	1300	0.92970E − 04	
3	1026 1597 2113	2637	10548	1318	0.96583E − 04	
4	1 425 2398 1448	2583	10332	1291	0.21414E − 02	0.17337E − 11
4	1 337 178 1972	2637	10548	1318	0.21490E − 02	(0.15916E − 11)
4	1 473 376 242	2691	10764	1345	0.21399E − 02	(0.22879E − 11)
4	1 638 703 1808	2691	10764	1345	(0.21573E − 02)	0.15490E − 11
5	1 721 253 1576 334	2547	10188	1273	0.26107E − 01	0.17376E − 08
5	1 961 571 235 1690	2637	10548	1318	0.24746E − 01	0.13403E − 08
5	1 988 1759 1522 1006	2655	10620	1327	0.24233E − 01	0.78143E − 09
6	1 1159 1012 1288 250 1939	2547	10188	1273	0.20869E + 00	0.18181E − 06
6	1554 968 1704 1618 989 1163	2583	10332	1291	0.21016E + 00	(0.44878E − 06)
6	1747 213 901 552 1690 699	2583	10332	1291	(0.22026E + 00)	0.44234E − 06
6	1 422 199 1673 2431 1052	2655	10620	1327	0.19951E + 00	(0.51608E − 06)
6	1 253 289 1432 1216 2323	2655	10620	1327	(0.20859E + 00)	0.27331E − 06
7	1 758 1138 2465 961 32 1009	2583	10332	1291	0.12749E + 01	(0.23946E − 04)
7	1 514 730 685 802 1531 1702	2583	10332	1291	(0.12752E + 01)	0.15347E − 04
7	1887 1679 713 121 257 252 2415	2601	10404	1300	0.12471E + 01	0.20018E − 04
7	1 1211 2617 1880 94 812 1117	2691	10764	1345	0.11802E + 01	0.20949E − 04
8	1 1151 361 350 424 1547 244 674	2547	10188	1273	0.67017E + 01	(0.16587E − 02)
8	1 641 814 2186 376 1598 424 1802	2547	10188	1273	(0.67083E + 01)	0.33740E − 03
8	1 895 2014 1459 490 808 622 283	2637	10548	1318	0.62780E + 01	(0.87553E − 03)
8	1 620 2035 1214 1135 2258 2350 1376	2637	10548	1318	(0.63798E + 01)	0.33268E − 03
8	1 83 1507 1295 2536 590 532 1100	2691	10764	1345	0.63424E + 01	(0.30119E − 02)
8	1 1199 607 1223 2473 2336 2224 2486	2691	10764	1345	(0.64452E + 01)	0.44928E − 03

becomes much more marked as the dimensionality increases; and only in three dimensions, at a high level of convergence, have we found a rank-1 rule that competes with the best rank-2 rules. Moreover, if the best rank-1 rules have **p** vectors of Korobov type, then they have been found by searches of about four times as many rules as the best rank-2 $n = 2$ rules, and about nine times as many as the best rank-2 $n = 3$ rules. Similar results also hold when the criterion is P_6, except that the P_6 errors obtained with the best rank-2 $n = 3$ rules are consistently greater than in the rank-1 case. Table 5 contains a selection of some of our best rank-2 $n = 2$ rules which appear useful for practical applications.

TABLE 6

A selection of rank-2 rules from the computer searches, in the 100 000-point window, for 3 to 8 dimensions. All have $n = 2$ and the special vector pairs $\mathbf{y}_1 = \mathbf{Y}_1$, $\mathbf{y}_2 = \mathbf{Y}_2$ given by (25).

s	$\mathbf{z}$	r	N	No. of rules searched	P_2	P_6
3	15348 10173 9373	25011	100044	125	0.22978E − 05	
3	7319 23489 7010	25011	100044	1250	0.21899E − 05	
3	1 2962 19594	25011	100044	125&1250	0.23751E − 05	
4	8619 4353 6230 7317	25011	100044	125	0.81738E − 04	
4	7317 17161 15361 20189	25011	100044	1250	0.70548E − 04	
4	1 5416 20164 10198	25011	100044	125	0.98855E − 04	
4	1 3112 5287 20917	25011	100044	1250	0.61731E − 04	
5	5477 3083 3651 6657 22019	25011	100044	125	0.11765E − 02	0.36948E − 12
5	21049 2618 14782 19415 18425	25011	100044	1250	0.11338E − 02	(0.35527E − 12)
5	191 21046 5773 23486 18635	25011	100044	1250	(0.12302E − 02)	0.11369E − 12
5	1 4292 13168 17207 19972	25011	100044	125	0.10872E − 02	0.36948E − 12
5	1 6968 6673 2015 9349	25011	100044	1250	0.10633E − 02	0.11369E − 12
6	13619 13979 18315 2471 8357 20188	25011	100044	125	0.11447E − 01	0.92854E − 10
6	1 3476 2263 12734 18925 4370	25011	100044	125	0.10697E − 01	0.13260E − 09
7	21555 6000 20758 18301 23971 19852 12294	25011	100044	125	0.92860E − 01	(0.12859E − 06)
7	9552 24926 22900 19119 21046 1864 22864	25011	100044	125	(0.93370E − 01)	0.33851E − 07
7	1 12245 24091 14561 21037 9776 4474	25011	100044	125	0.97294E − 01	(0.16702E − 06)
7	1 9734 9088 23696 5422 4538 3466	25011	100044	125	(0.99940E − 01)	0.64325E − 07
8	14747 13874 19956 21322 20288 5245 19183 10465	25011	100044	125	0.56734E + 00	(0.91846E − 05)
8	3212 5555 17120 4251 18381 3147 4888 9868	25011	100044	125	(0.57631E + 00)	0.37302E − 05
8	1 1342 172 5725 4573 9271 11215 18919	25011	100044	125	0.54047E + 00	(0.93545E − 06)
8	1 5066 3070 20789 20764 19169 17452 22958	25011	100044	125	(0.54125E + 00)	0.72957E − 06
7	21684 1655 10615 20557 15184 15457 22297	32751	131004	163	0.65006E − 01	0.15978E − 07
7	1 16000 18184 16867 3760 29164 20503	32751	131004	163	0.67529E − 01	0.15569E − 07
8	18133 7323 20067 4975 16402 32438 690 25405	32751	131004	163	0.41951E + 00	0.78090E − 06
8	1 8243 21475 32021 8794 10979 8884 32327	32751	131004	163	0.41074E + 00	0.86272E − 06

4.3. The 100 000-point window. The full details of these searches are given at the end of §3, and *all* the rules found in this search are listed in Table 6. This table contains two rules in each dimension for $N = 100\,044$ when the number of rules searched is 125; and six rules in 3 to 5 dimensions, with the same N, but with the number of rules searched being 1250. The larger search could not be continued in 6, 7 and 8 dimensions due to insufficient computer resources. Table 6 also contains two rules in each of 7 and 8 dimensions for $N = 131\,004$. (The $N = 100\,044$ point rules were considered to give adequate convergence, in up to 6 dimensions, for most practical applications, so the further effort of searching $N = 131\,004$ was restricted to 7 and 8 dimensions.) The computer

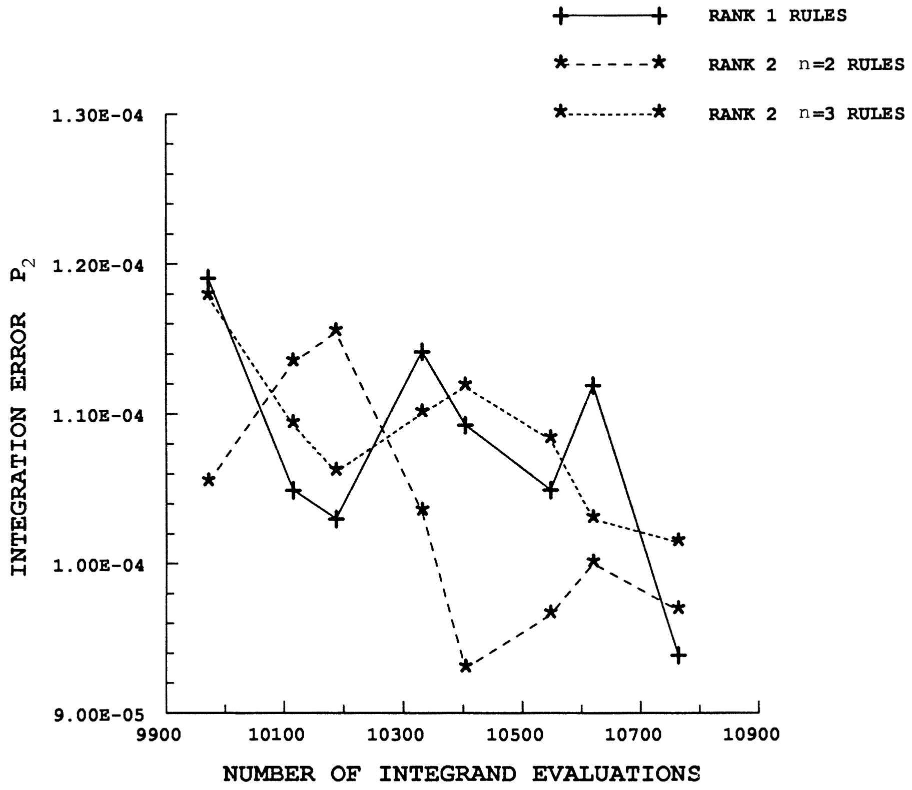

FIGURE 4

Integration error P_2 (as defined in (15)) as a function of the number of integrand evaluations (N or n^2r) in 3 dimensions, for the 10 000-point window. Each point gives the smallest P_2 value obtained from searches at fixed N.

cpu time taken for all the $N = 100\,044$ results with 125 searches in Table 6 was 1031 seconds; the $N = 131\,004$ results for $s = 7$ and 8 alone took 979 seconds and required the full memory resources of the Cyber 205 for their attainment.

The quality of the rules in Table 6 may be assessed by comparison with rank-1 rules previously published by Maisonneuve [7] and Haber [2]. The quantity P_2 in this work is the same as $|R_N F_s^2|$ of Haber (where $s =$ dimension) and $P^{(2)}(g)$ of Maisonneuve; this was checked computationally by using some of Haber's and Maisonneuve's rules in the programs used to compute P_2. A comparison between the rank-1 $N = 100\,063$ rules in Maisonneuve and our rank-2 $n = 2$, $N = 100\,044$ results from searching 125 rules shows that the latter gives P_2 values which are between 47% and 74% of Maisonneuve's $P^{(2)}(g)$ values. A comparison between the rank-1 $N = 98\,304$ rules in Haber and our rank-2 $n = 2$, $N = 100\,044$, with 125 searched, rules shows that the latter give P_2 values

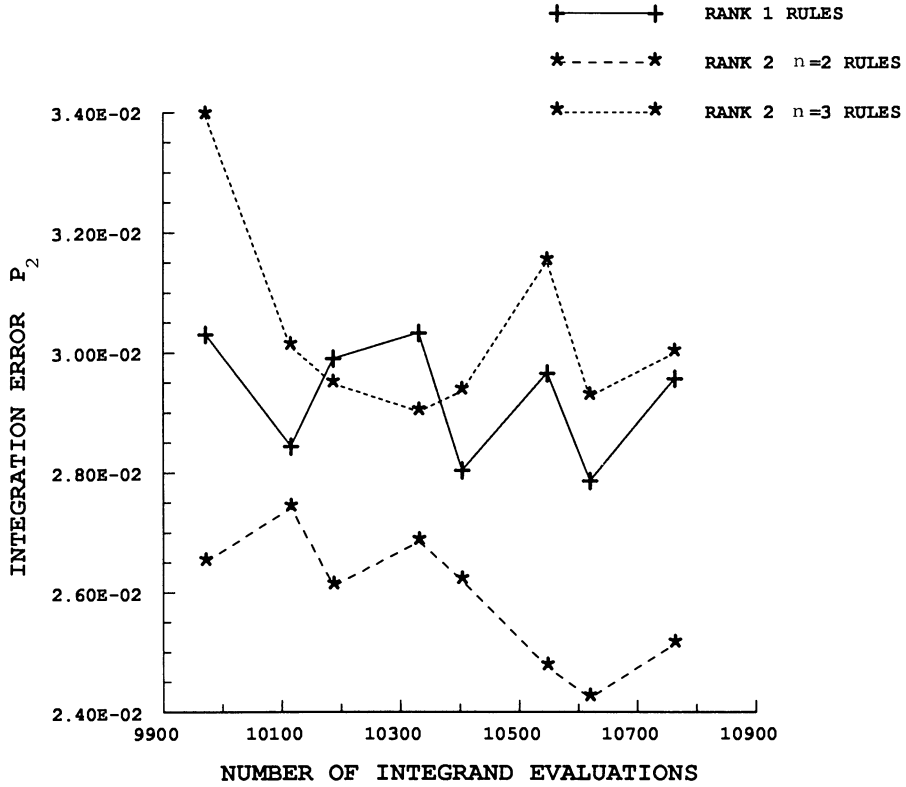

FIGURE 5

Integration error P_2 (as defined in (15)) as a function of the number of integrand evaluations (N or n^2r) in 5 dimensions, for the 10 000-point window. Each point gives the smallest P_2 value obtained from searches at fixed N.

which are between 54% and 78% of Haber's $|R_N F_s^2|$ rules. These comparisons illustrate that the rules given in Table 6 offer a considerable advantage over the previously published rank-1 rules, according to the standard criterion.

5. DISCUSSION

Two empirical conclusions emerge clearly from the computer searches reported in this paper. The first is that, by the standard criterion and for dimensions greater than 3, the 'best' of our rank-2 $n = 2$ rules (i.e., lattice rules with invariants $2r$, 2 and r odd) consistently outperform the best rank-1 rules obtained with a comparable search effort. This is true whether we look at rules with around 1000, or 10 000 or 100 000 abscissas.

The second conclusion is that for the best of the rank-2 $n = 2$ rules there is a marked tendency for the vectors $\mathbf{y}_1$, $\mathbf{y}_2$ in (8) to have the special values $\mathbf{y}_1 = \mathbf{Y}_1$, $\mathbf{y}_2 = \mathbf{Y}_2$ given by (25).

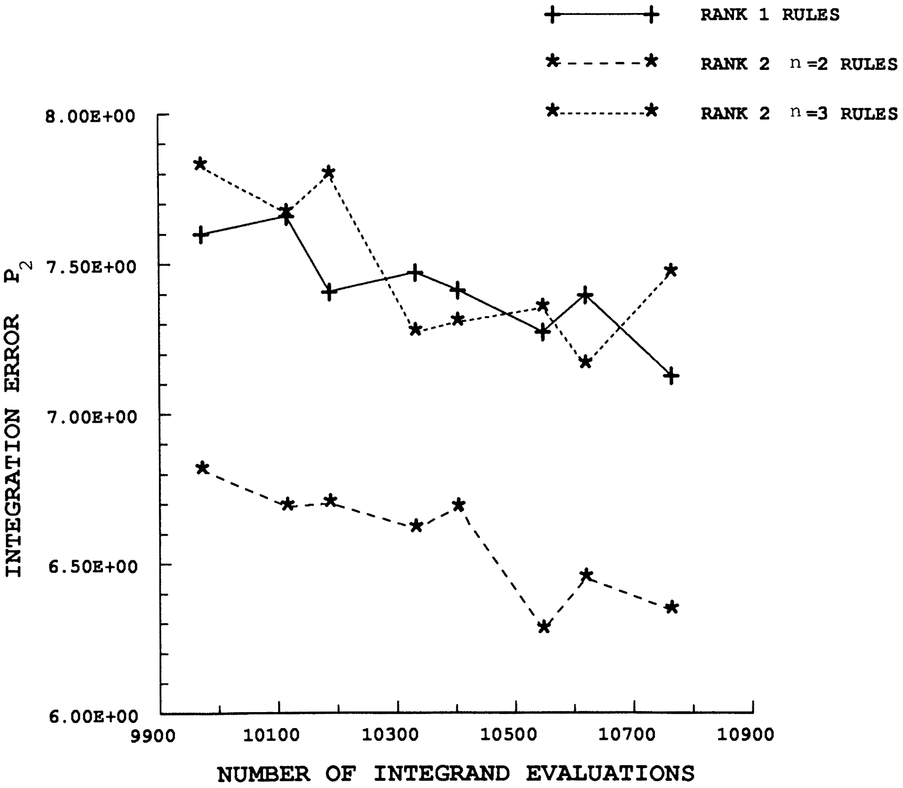

FIGURE 6

Integration error P_2 (as defined in (15)) as a function of the number of integrand evaluations (N or n^2r) in 8 dimensions, for the 10 000-point window. Each point gives the smallest P_2 value obtained from searches at fixed N.

At the present time, no convincing explanation has emerged for either phenomenon.

APPENDIX. PROGRAMMING CONSIDERATIONS

Here we consider only the case of rank-2 rules. The situation for rank-1 rules is similar, and even easier.

Using (15) and (8), the object is to compute, for α a positive even integer,

$$P_\alpha = \frac{1}{n^2 r} \sum_{j=1}^{r} \sum_{k_1=1}^{n} \sum_{k_2=1}^{n} \overline{f}_\alpha \left(j\frac{\mathbf{z}}{r} + k_1 \frac{\mathbf{y}_1}{n} + k_2 \frac{\mathbf{y}_2}{n} \right) - 1, \tag{A1}$$

with $f_\alpha(\mathbf{x}) = \prod \phi_\alpha(x^{(i)})$, and ϕ_α given by (17).

The function ϕ_α has the symmetry property

$$\phi_\alpha(x) = \phi_\alpha(1-x), \tag{A2}$$

leading to

$$f_\alpha(\mathbf{x}) = f_\alpha((1, 1, \dots, 1) - \mathbf{x}).$$

Since the lattice rule (8), like every lattice rule, is symmetric about the center of the cube, this symmetry may be used to reduce the number of terms needed in the sum: it can easily be seen that (A1) can be replaced by

$$P_\alpha = \frac{2}{n^2 r} \sum_{j=0}^{[r/2]}{}^{*} \sum_{k_1=1}^{n} \sum_{k_2=1}^{n} \overline{f}_\alpha\left(j\frac{\mathbf{z}}{r} + k_1\frac{\mathbf{y}_1}{n} + k_2\frac{\mathbf{y}_2}{n}\right) - 1, \tag{A3}$$

where the asterisk indicates that the term with $j = 0$ is to be halved, and if r is even, also that with $j = r/2$.

In our searches we evaluated P_α for many different rules, all having the same values of n and r. Now the values of f_α needed for all rules with given n and r are products of a quite small set of ϕ_α values, namely the set

$$\{\phi_\alpha(l/nr) : l = 0, 1, \dots, nr - 1\}. \tag{A4}$$

Thus the first step in a search, following an idea of [8], was to calculate an array of these nr values of ϕ_α. (The number of ϕ_α values can be further reduced by a factor of close to two, by exploiting the symmetry (A2); but this is of no great consequence.)

For a given rule, our object was to construct a vector, of length $n^2([r/2]+1)$, of the values of f_α occurring in (A3), from which it is a trivial matter to deduce P_α. Since

$$\begin{aligned} \overline{f}_\alpha\left(j\frac{\mathbf{z}}{r} + k_1\frac{\mathbf{y}_1}{n} + k_2\frac{\mathbf{y}_2}{n}\right) &= \prod_{i=1}^{s} \phi_\alpha\left(\left\{j\frac{z^{(i)}}{r} + k_1\frac{y_1^{(i)}}{n} + k_2\frac{y_2^{(i)}}{n}\right\}\right) \\ &= \prod_{i=1}^{s} \phi_\alpha\left(\left\{\frac{njz^{(i)} + rk_1y_1^{(i)} + rk_2y_2^{(i)}}{nr}\right\}\right), \end{aligned} \tag{A5}$$

an intermediate step was to create, for each component $x^{(i)}$, a vector of length $n^2([r/2]+1)$, labelled by j, k_1, k_2, of the ϕ_α values occurring in (A5), after which it is an efficient operation on the Cyber 205 to form the vector of f_α values by taking component-by-component products of the s individual vectors.

A final comment concerns the method of formation of the vectors of ϕ_α values needed in (A5). For the ith component, the values needed are of the form $\phi_\alpha(l/nr)$, where

$$l = njz^{(i)} + rk_1y_1^{(i)} + rk_2y_2^{(i)} \pmod{nr} \tag{A6}$$

for $0 \le j \le [r/2]$, $1 \le k_1 \le n$, $1 \le k_2 \le n$. Thus for each component one first forms an integer vector of length $n^2([r/2]+1)$ by means of (A6), and then forms the vector of ϕ_α values from the values already stored (see (A4) above). The process of forming a vector from a vector of indices of a stored array can be efficiently handled on the Cyber 205 by means of a utility, the 'vector data motion' macro Q8VGATHR.

ACKNOWLEDGMENTS

We are grateful to Dr. J. N. Lyness for valuable discussions, to the Australian Research Council for generous financial support and to CSIRO for a merit award for use of the CSIRONET Cyber 205 computer.

BIBLIOGRAPHY

1. S. Haber, *Experiments on optimal coefficients*, Applications of Number Theory to Numerical Analysis (S. K. Zaremba, ed.), Academic Press, New York, 1972, pp. 11-37.
2. ——, *Parameters for integrating periodic functions of several variables*, Math. Comp. **41** (1983), 115-129.
3. N. M. Korobov, *Properties and calculation of optimal coefficients*, Dokl. Akad. Nauk SSSR **132** (1960), 1009-1012; English transl., Soviet Math. Dokl. **2** (1961), 1288-1291.
4. ——, *Number-theoretic methods of approximate analysis*, Fizmatgiz, Moscow, 1963. (Russian)
5. J. N. Lyness and I. H. Sloan, *Some properties of rank-*2 *lattice rules*, Math. Comp. **53** (1989), 627-637.
6. J. N. Lyness, *Some comments on quadrature rule construction criteria*, Numerical Integration (H. Brass and G. Hämmerlin, eds.), ISNM 85, Birkhäuser, 1988, pp. 117-129.
7. D. Maisonneuve, *Recherche et utilisation des 'Bons Treillis'. Programmation et résultats numériques*, Applications of Number Theory to Numerical Analysis (S. K. Zaremba, ed.), Academic Press, New York, 1972, pp. 121-201.
8. B. Newman and J. N. Lyness, *Good lattice rules for the integration of periodic functions of several variables*, unpublished, 1986.
9. H. Niederreiter, *Quasi-Monte Carlo methods and pseudo-random numbers*, Bull. Amer. Math. Soc. **84** (1978), 957-1041.
10. I. H. Sloan, *Lattice methods for multiple integration*, J. Comput. Appl. Math. **12** & **13** (1985), 131-143.
11. I. H. Sloan and P. J. Kachoyan, *Lattice methods for multiple integration: theory, error analysis and examples*, SIAM J. Numer. Anal. **24** (1987), 116-128.
12. I. H. Sloan and J. N. Lyness, *The representation of lattice quadrature rules as multiple sums*, Math. Comp. **52** (1989), 81-94.
13. I. H. Sloan and L. Walsh, *Lattice rules—classification and searches*, Numerical Integration (H. Brass and G. Hämmerlin, eds.), ISNM 85, Birkhäuser, 1988, pp. 251-260.

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SUPPORTING TABLES FOR: I.H. SLOAN AND J. WALSH-
" A COMPUTER SEARCH OF RANK 2 LATTICE RULES
FOR MULTIDIMENSIONAL QUADRATURE", MATH. COMP., 1989.

NOTE: If P2 is minimised by a particular rule but P6 is not,
then the value of P6 is shown in parenthesis and vice-versa.

CONTENTS:

Korobov-type rank 1 rules for comparison with rank 2 rules having n=2.

Rank 2 rules having n=2.

Korobov-type rank 1 rules for comparison with rank 2 rules having n=3.

Rank 2 rules having n=3.

KOROBOV-TYPE RANK 1 RULES FOR COMPARISON WITH RANK 2

RULES HAVING n = 2.

1,000 POINT DATA

---

s = 3

P2	P6	p			N	NO. RULES SEARCHED.
0.654478E-02	0.216868D-09	514	504	597	940	334
0.642908D-02	0.276088E-09	788	353	726	906	337

```
  0.641108E-02    0.249798E-09   844  535   54     964    360

  0.585638E-02    0.860920E-10   982  368  673     972    363

  0.599708E-02 ( 0.235198E-09)   785  946  846     980    366
( 0.615928E-02)   0.172018E-09   738  391  874     980    366

  0.597058E-02 ( 0.307268E-09)   150  248  365     988    369
( 0.598108E-02)   0.134388E-09   595  362  930     988    369

  0.554048E-02    0.200508E-09   729  992  440     996    372

  0.560378E-02    0.965258E-10    85  101  549    1004    375

  0.623658E-02 ( 0.434378E-09)   400  165  348    1012    378
( 0.665768E-02)   0.344078E-09    85  422  573    1012    378

  0.567038E-02 ( 0.264638E-09)   282   36  441    1020    381
( 0.600948E-02)   0.132098E-09   690  553  921    1020    381

  0.590348E-02 ( 0.628898E-09)   406   46  835    1028    384
( 0.619508E-02)   0.150268E-09   927  653  565    1028    384

  0.563458E-02 ( 0.306728E-09)   208   35  850    1036    387
( 0.599758E-02)   0.233108E-09   132  754   45    1036    387

  0.512808E-02 ( 0.117418E-08)   852  802   81    1044    390
( 0.556068E-02)   0.111838E-09   495  636  309    1044    390

  0.589108E-02    0.822228E-10   430    6  373    1052    393

  0.635180D-02 ( 0.361320D-09)     1  307  196     948    948
( 0.608630D-02)   0.250820D-09     1  106  388     948    948

  0.660940D-02 ( 0.561340D-09)     1  290  852     956    956
( 0.688090D-02)   0.931330D-10     1  105  909     956    956

  0.620180D-02 ( 0.208130D-09)     1  254  882     964    964
( 0.640120D-02)   0.171420D-09     1  293   53     964    964

  0.617980D-02 ( 0.213180D-09)     1  134  440     972    972
( 0.621080D-02)   0.191270D-09     1  142  724     972    972

  0.636710D-02 ( 0.393730D-09)     1  202  684     980    980
( 0.640830D-02)   0.300040D-09     1  114  256     980    980

  0.601270D-02    0.961100D-10     1  426  672     988    988

  0.576140D-02 ( 0.198470D-09)     1  106  280     996    996
( 0.611040D-02)   0.190090D-09     1  270  448     996    996

 •0.588880D-02 ( 0.125700D-09)     1  307  877    1004   1004
( 0.588880D-02)   0.125700D-09     1  131  585    1004   1004

  0.559000D-02    0.914640D-10     1  306  532    1012   1012

  0.576700D-02 ( 0.522000D-09)     1  286  196    1020   1020
( 0.588800D-02)   0.841200D-10     1  352  484    1020   1020

  0.574800D-02 ( 0.228630D-09)     1  322  804    1028   1028
( 0.608280D-02)   0.216300D-09     1  159  600    1028   1028

  0.588700D-02    0.135390D-09     1  150  100    1036   1036

  0.582190D-02    0.110830D-09     1  288  760    1044   1044
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    0.49904E-02   0.12580E-09     1   322   500    1052     1052


                        N = 4
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        P2              P6                  C                 N     NO. RULES SEARCHED.
                                   -----------------
     0.86352E-01 ( 0.25610E-06)  490   710   174   945       948       708
   ( 0.96040E-01)  0.22041E-06   744   515   432   233       948       708

     0.90623E-01 ( 0.37126E-06)  148   178   336   467       956       714
   ( 0.10317E+00)  0.34305E-06   979   367    83   149       956       714

     0.90283E-01   0.14036E-06   503   502   133   100       964       720

     0.87012E-01   0.10397E-06   940   177   894   841       972       726

     0.79963E-01   0.10730E-06   132   385   816   206       980       732

     0.88024E-01 ( 0.74227E-06)  112   634   640   667       988       738
   ( 0.87659E-01)  0.16161E-06   123   307   766   924       988       738

     0.73000E-01   0.80085E-07   872   181   918   862       996       744

     0.82719E-01   0.16050E-06   136   265     6    16      1004       750

     0.79039E-01   0.14126E-06   482   956   830   879      1012       756

     0.77029E-01   0.11308E-06   175   362   784    40      1020       762

     0.76201E-01 ( 0.21004E-06)  116   398   176   313      1028       768
   ( 0.80000E-01)  0.10219E-06   888  1003   476   392      1028       768

     0.80090E-01 ( 0.20250E-05)   14   190   355   256      1036       774
   ( 0.80000E-01)  0.11747E-06   727   323   245   675      1036       774

     0.75719E-01 ( 0.22381E-06)  774   462  1040   313      1044       780
   ( 0.78881E-01)  0.11799E-06   376   929   943   500      1044       780

     0.75640E-01 ( 0.10417E-06)  200   120   230   785      1052       786
   ( 0.80000E-01)  0.12087E-06    68   167   493   720      1052       786

     0.80000E-01 ( 0.19130E-06)    1   322   382   532       948       948
   ( 0.80000E-01)  0.91577E-07     1   275   733   999       948       948

     0.80000E-01   0.28141E-06     1   374   300   340       956       956

     0.80023E-01 ( 0.39041E-06)    1   334   696   140       964       964
   ( 0.87000E-01)  0.28774E-06     1   194   500   682       964       964

     0.80000E-01 ( 0.34023E-06)    1   348   160   928       972       972
   ( 0.10000E+00)  0.33030E-06     1   223   197    19       972       972

     0.80012E-01 ( 0.81500E-06)    1   222   284   320       980       980
   ( 0.80000E-01)  0.28773E-06     1   333   149   617       980       980

     0.81700E-01   0.12865E-06     1   142   404    64       988       988

     0.80112E-01 ( 0.16330E-06)    1   326   780   110       996       996
   ( 0.87000E-01)  0.15000E-06     1   439   490   295       996       996

     0.77043E-01 ( 0.22204E-06)    1   146   232   740      1004      1004
```

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( 0.876648E-01)  0.133338E-06     :  465  237   45     1004     1004

  0.778978E-01   0.211506E-06     :  218  972  368     1012     1012

  0.768348E-01   0.164788E-06     :   46   76  436     1020     1020

  0.749908E-01   0.779648E-07     :  426  548   97     1028     1028

  0.786908E-01   0.127938E-06     :  166  620  356     1036     1036

  0.770448E-01 ( 0.120668E-06)    :  358  796 1000     1044     1044
( 0.802398E-01)  0.111638E-06     :  191  905  215     1044     1044

  0.731408E-01   0.906048E-07     :  459  414  116     1052     1052


                    s = 5
                  ---------


      P2             P6                    P                    N      NO. RULES SEARCHED.
                               --------------------------
  0.760058E+00 ( 0.107068E-03)  204  630  716  752  915      948     1180
( 0.779798E+00)  0.341008E-04   714  849  888  433    9      948     1180

  0.724728E+00 ( 0.788678E-04)  298   84  228  765  344      956     1190
( 0.785168E+00)  0.283228E-04   667  546  807  527  120      956     1190

  0.723858E+00 ( 0.464758E-04)  686  103  100  122  648      964     1200
( 0.733988E+00)  0.375008E-04   127  728  402  924  694      964     1200

  0.688768E+00   0.276908E-04   696  334  112    7  372      972     1210

  0.689808E+00 ( 0.688178E-04)  109  484  166  132  356      980     1220
( 0.788228E+00)  0.335488E-04   849  781  276  413  970      980     1220

  0.697288E+00 ( 0.898988E-04)  492  359  776  330  360      988     1230
( 0.698398E+00)  0.271618E-04   732  944  214  742  201      988     1230

  0.672808E+00 ( 0.713338E-04)  380  944  960  386  827      996     1240
( 0.688808E+00)  0.311398E-04    66  665  176  470  560      996     1240

  0.661068E+00 ( 0.447238E-04)  636  196  855  312  262     1004     1250
( 0.738068E+00)  0.373158E-04   962  321  173  225  585     1004     1250

  0.636408E+00   0.246348E-04   904  602  294  601  988     1012     1260

  0.688758E+00 ( 0.613708E-04)  690  676  696   60   59     1020     1270
( 0.768348E+00)  0.356768E-04   750  301  701  763  734     1020     1270

  0.671608E+00 ( 0.105808E-03)  764  508  481  352 1016     1028     1280
( 0.766858E+00)  0.356398E-04   717  587  954  545  129     1028     1280

  0.687948E+00   0.137638E-04   978  320  262  776  535     1036     1290

  0.682808E+00 ( 0.300008E-04)  105  896  766   22   42     1044     1300
( 0.698088E+00)  0.252258E-04  1033  214  228  234   58     1044     1300

  0.688888E+00   0.168998E-04   910  987  138  754   94     1052     1310

  0.725868E+00 ( 0.683908E-04)    1   26  676  512   40      948      948
( 0.730768E+00)  0.309638E-04     1  362  220    8   52      948      948

  0.681088E+00 ( 0.288038E-04)    1   14  696  836  685      956      956
```

```
( 0.77241E+00)  0.27095E-04    1   77  193  521  921     956     956

  0.68221E+00 ( 0.36774E-04)   1  122  744  650  476     964     964
( 0.75462E+00)  0.20634E-04    1  457  625  701  205     964     964

  0.69753E+00   0.37196E-04    1  398  940  872   12     972     972

  0.68019E+00 ( 0.91343E-04)   1  214  716  344  116     980     980
( 0.76241E+00)  0.21052E-04    1   67  569  803  361     980     980

  0.65037E+00   0.22254E-04    1  446  784  772  120     988     988

  0.68808E+00 ( 0.60864E-04)   1   86  424  608  496     996     996
( 0.77708E+00)  0.34190E-04    1  343  121  647  697     996     996

  0.65504E+00 ( 0.22122E-04)   1  178  560  284  352    1004    1004
( 0.65704E+00)  0.20245E-04    1  302  848  876  500    1004    1004

  0.63604E+00   0.19031E-04    1  310  872  756  568    1012    1012

  0.60633E+00   0.95741E-04    1   26  676  236   16    1020    1020

  0.62844E+00 ( 0.29629E-04)   1  414  748  244  272    1028    1028
( 0.73650E+00)  0.25205E-04    1   33   61  885  637    1028    1028

  0.64447E+00   0.22095E-04    1  358  738  344  904    1036    1036

  0.63878E+00 ( 0.50430E-04)   1   94  404  604  400    1044    1044
( 0.69808E+00)  0.36209E-04    1  106  796  856  952    1044    1044

  0.62201E+00   0.29988E-04    1  162  996  396 1032    1052    1052


                         N = 6
                       ---------


      P2            P6                    C                    N      NO. RULES SEARCHED.
                                 ---------------------------
  0.41090E+01   0.19030E-02   522  280  736  512   24  343     948     1888

  0.41898E+01 ( 0.29767E-02)   48  625  510  542  288  720     956     1904
( 0.49891E+01)  0.20045E-02   188  273  806  434  419  106     956     1904

  0.41588E+01 ( 0.26811E-02)  710  120  727   78  766  918     964     1920
( 0.42883E+01)  0.29031E-02   288   82  250  355  146  158     964     1920

  0.40124E+01 ( 0.27481E-02)  556  982  291  984  892   20     972     1936
( 0.47818E+01)  0.25718E-02   628  178  738  204  367  569     972     1936

  0.40884E+01 ( 0.28850E-02)  232  150  110  874   11  196     980     1952
( 0.48788E+01)  0.10691E-02   927  882  143  122  320  165     980     1952

  0.38873E+01 ( 0.40898E-02)  734  496  832  725  488  888     988     1968
( 0.41142E+01)  0.28388E-02   788  512  344  293  198   22     988     1968

  0.38888E+01 ( 0.41277E-02)  768  588  908  731  492  490     996     1984
( 0.38219E+01)  0.19428E-02   372  912  196  170  320  983     996     1984

  0.38211E+01 ( 0.10825E-02)  912  388  718  148   41  632    1004     2000
( 0.39487E+01)  0.13088E-02   181   96  172  114  978  150    1004     2000

  0.38881E+01 ( 0.48859E-02)  924  288  168  527  856  498    1012     2016
( 0.46727E+01)  0.24348E-02    11  212  768  396  843  688    1012     2016
```

```
  0.[illegible]D+01 ( 0.[illegible]D-02)  142  960  276  010  010  504    1020    2032
( 0.[illegible]D+01)  0.1[illegible]D-02  646   64  682  674  342  931    1020    2032

  0.[illegible]D+01   0.1[illegible]D-02  930  560   62  277  992  114    1020    2048

  0.370[illegible]E+01   0.131[illegible]E-02  910  976  100  543  440  334    1036    2064

  0.370[illegible]D+01 ( 0.220170D-02)  012  072  230   26  [illegible]   20    1044    2080
( 0.4[illegible]D+01)  0.109[illegible]D-02  649  152  630  936  222  847    1044    2080

  0.377[illegible]D+01 ( 0.[illegible]D-02)  760  [illegible]  600  564  637  400    1052    2096
( 0.377[illegible]D+01)  0.170[illegible]D-02  [illegible]  644  320  620  066   73    1052    2096

  0.60[illegible]D+01 ( 0.[illegible]D-02)    1  142  296  320  134  544     960     960
( 0.621[illegible]D+01)  0.10[illegible]D-02    1  200  076  752   40  224     960     960

  0.600777E+01   0.103[illegible]D-02    1  120  500  424  004  220     956     956

  0.[illegible]D+01 ( 0.17[illegible]D-02)    1  110  430  376   24  904     964     964
( 0.[illegible]D+01)  0.133[illegible]D-02    1  322  536   16   24   16     964     964

  0.[illegible]D+01 ( 0.[illegible]D-02)    1  270  496  836  100  304     972     972
( 0.[illegible]D+01)  0.218[illegible]D-02    1  434   52  200  700  952     972     972

  0.[illegible]D+01 ( 0.1[illegible]D-02)    1  [illegible]   64  460  176  352     980     980
( 0.4[illegible]D+01)  0.101[illegible]D-02    1  326  436   36  936   16     980     980

  0.[illegible]D+01   0.12[illegible]D-02    1   20  900  306  820  140     [illegible]     [illegible]

  0.[illegible]D+01 ( 0.[illegible]D-02)    1   10  100    0   60  [illegible]     996     996
( 0.[illegible]D+01)  0.[illegible]D-02    1  104  [illegible]  [illegible]  [illegible]  [illegible]     996     996

  0.[illegible]D+01 ( 0.1[illegible]D-02)    1  270  286  976  [illegible]  [illegible]    1004    1004
( 0.[illegible]D+01)  0.13[illegible]D-02    1  204  112  [illegible]  [illegible]    0    1004    1004

  0.377[illegible]D+01   0.10[illegible]D-02    1  270  100  720  [illegible]  [illegible]    1012    1012

  0.[illegible]D+01 ( 0.4[illegible]D-02)    1  404   76  044  676  904    1020    1020
( 0.[illegible]D+01)  0.213[illegible]D-02    1  437  209  113  421  377    1020    1020

  0.[illegible]D+01 ( 0.[illegible]D-02)    1  166  [illegible]  724  [illegible]  140    [illegible]    [illegible]
( 0.[illegible]D+01)  0.1[illegible]D-02    1  342  [illegible]  152  [illegible]  296    [illegible]    [illegible]

  0.[illegible]D+01 ( 0.[illegible]D-02)    1  206  [illegible]  [illegible]  120  726    1036    1036
( 0.[illegible]D+01)  0.7[illegible]D-03    1  346  376  204  206  310    1036    1036

  0.377[illegible]D+01 ( 0.221[illegible]D-02)    1  420   14  206  204  560    1044    1044
( 0.[illegible]D+01)  0.113[illegible]D-02    1  490  329  [illegible]   49  439    1044    1044

  0.[illegible]D+01   0.[illegible]D-02    1   60  136  704  612   12    1052    1052
```

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                     [illegible] = 7
                     -----------

      F2             F6                          D                         S   NO. [illegible] [illegible].
                                  ------------------------------------------
  0.[illegible]D+02 ( 0.102170D-02)  277  [illegible]  [illegible]  [illegible]  434  [illegible]  [illegible]    940   2714
( 0.211970D-02)  0.4[illegible]D-01  420  [illegible]  476  707   50  [illegible]  370    940   2714

  0.[illegible]D+02 ( 0.13[illegible]D-02)  614  [illegible]  274  [illegible]  704   20  904    [illegible]   2737
( 0.[illegible]D-02)  0.379[illegible]D-01  [illegible]  306  041  [illegible]  [illegible]  [illegible]  [illegible]    936   2737
```

```
  0.200272E+02 ( 0.104106E+00)  576  150  503  632  491   76  436     964    2763
( 0.215008E+02)  0.580087E-01   882  464   11  419  888  280   90     964    2763

  0.201378E+02 ( 0.797608E-01)  496  959    8  196  502  464  920     972    2783
( 0.200400E+02)  0.754308E-01   312  597  234   40  944  912  720     972    2783

  0.202064E+02 ( 0.771758E-01)  619  812  600  500  744  922  102     980    2806
( 0.200618E+02)  0.490664E-01    28  300  768   39  224   76  436     980    2806

  0.198568E+02 ( 0.738088E-01)  846 [illegible] 776  368  700  960  475     988    2829
( 0.200398E+02)  0.838008E-01   912  830  144  434  474  450  169     988    2829

  0.198432E+02    0.417368E-01   410  221   92   76  176  532  304     996    2852

  0.199988E+02 ( 0.106108E+00)   16   24  502  802  559  476  634    1004    2875
( 0.206208E+02)  0.543728E-01   924  656  155  766   90  364  404    1004    2875

  0.190298E+02    0.413768E-01    72  632  702  664  377  396  960    1012    2898

  0.190628E+02 ( 0.132218E+00)  464   56  390  121  456  760  776    1020    2921
( 0.195238E+02)  0.487918E-01   804  308   40  853  494  374  832    1020    2921

  0.198108E+02    0.428088E-01   452  948  432  194  530  123  716    1028    2944

  0.190080D+02 ( 0.100808E+00)  600  254  829 1032  210  620  552    1036    2967
( 0.228200D+02)  0.371828D-01   883  604  242  170  608  373  270    1036    2967

  0.180808E+02 ( 0.756178E-01)  760  228  242  320   37  612   88    1044    2990
( 0.180478E+02)  0.462818D-01   560  290  360  396  908  170  276    1044    2990

  0.170638E+02 ( 0.183088E+00)  526  900  808  304 1005  644   28    1052    3013
( 0.198348E+02)  0.477088D-01    30  322  868   42  210   80  408    1052    3013

  0.200308E+02 ( 0.118018D-01)    1  334  840  460   64  520  196     948     948
( 0.200808E+02)  0.190388D-01    1   32  404  220  100  304   52     948     948

  0.208278D+02 ( 0.488388D-01)    1  302  304  292  232  276  100     956     956
( 0.200888D+02)  0.295088D-01    1  106  100   25  852  732  600     956     956

  0.190808D+02 ( 0.103088D+00)    1  400   60    0  760  480   64     964     964
( 0.201238D+02)  0.657178D-01    1  126  452   76  900  612  904     964     964

  0.208888D+02 ( 0.107288D+00)    1  290  612  476  616  740  180     972     972
( 0.200618D+02)  0.498888D-01    1   34  184  424  888  256  920     972     972

  0.190708D+02    0.424118D-01    1  282  304  308  296   52  764     980     980

  0.188088D+02    0.111618D-01    1   22  404  760  100  224  976     988     988

  0.198378D+02 ( 0.136288D+00)    1   10  108    4   40  480   16     996     996
( 0.197708D+02)  0.321788D-01    1   26  676  644  800   92  408     996     996

  0.207868D+02    0.124308D-01    1  314  980  604  124   80   84    1004    1004

  0.188798D+02    0.130888D-01    1  294  416  804    4  164  652    1012    1012

  0.188888D+02    0.127828D-01    1  206  376   56  616  856   76    1020    1020

  0.181948D+02    0.107348D-01    1  210  924  776  586  900  796    1028    1028

  0.188088D+02    0.130888D-01    1  146  596 1020  904  612   88    1036    1036

  0.180228D+02    0.427298D-01    1  110  392  820  712  496   64    1044    1044

  0.188768D+02    0.127488D-01    1  330  872  724  840  980  288    1052    1052
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                      N = 8
                    ---------

      P2           P6                               P                            K    NO. RULES SEARCHED.
                                 ---------------------------------------
  0.9[illegible]D+02 ( 0.245382E+01)  868   16  490  380  819  796   14  904     948     3776
( 0.97204E+02)  0.401008E+00  924  524  676  124  898  204  511  930     948     3776

  0.932108E+02 ( 0.543827E+00)  828  343  132  596  428  332  390  344     956     3808
( 0.90801E+02)  0.40640E+00   314  101  408  522  214  164  240  510     956     3808

  0.934758E+02 ( 0.480098E+00)  588  282  788  616   72  123  170  260     964     3840
( 0.10[illegible]D+03)  0.479028E+00  882  144   10  294  835    4  335  100     964     3840

  0.9[illegible]D+02 ( 0.235038E+01)  424  456  476  656  719  540   30   40     972     3872
( 0.9[illegible]D+02)  0.4[illegible]E+00  796  490   84  160   52  456  213  100     972     3872

  0.[illegible]E+02 ( 0.237208E+01)  406  772  504  960  170  944  756  400     980     3904
( 0.9[illegible]D+02)  0.437468E+00  349   18  474   12  950  474  636  200     980     3904

  0.[illegible]D+02   0.341[illegible]D+00  912  522  609   56  904  172  722  814     988     3934

  0.911248D+02 ( 0.6[illegible]E+00)  980  100  116  212  540  900  201   18     996     3968
( 0.9[illegible]D+02)  0.481100D+00  366  090  102  600   20  174  373  960     996     3968

  0.9[illegible]D+02 ( 0.2[illegible]D+01)  426  452  364  260  500  000   00  367    1004     4000
( 0.9[illegible]D+02)  0.4[illegible]D+00  304  170  374  700  190  420  762  907    1004     4000

  0.[illegible]D+02   0.417[illegible]D+00  331  152  436  940   40  304  410  000    1012     4032

  0.[illegible]D+02 ( 0.2[illegible]D+01)  300  620  228  360   60   81  228  304    1020     4064
( 0.9[illegible]D+02)  0.437[illegible]E+00  644  344  317  292  646  332  942  276    1020     4064

  0.[illegible]D+02 ( 0.2[illegible]D+01)  336  320  304  224  070  360  279  520    1028     4096
( 0.101[illegible]D+03)  0.4[illegible]D+00   43  421  335  893  914   93  909  106    1028     4096

  0.[illegible]D+02 ( 0.2[illegible]D+01)  131  110  700  910  610  720  254  734    1036     4128
( 0.[illegible]D+02)  0.4[illegible]D+00  400  702  520  604  820  496  370  959    1036     4128

  0.[illegible]D+02 ( 0.230[illegible]D+01)  090  302   24  929  300  714  000  274    1044     4160
( 0.[illegible]D+02)  0.4[illegible]D+00  600  040  216  106  414  525  406 1004    1044     4160

  0.[illegible]D+02 ( 0.479[illegible]E+00) 1000   12  104  981  036  000  972 1032    1052     4192
( 0.[illegible]D+02)  0.375[illegible]D+00  600  325  510  071  042  339  526  164    1052     4192

  0.9[illegible]D+02   0.332718D+00    1  314    4  300   16  204   64  100     940      940

  0.9[illegible]D+02   0.321[illegible]D+00    1  300  000  670    4  396    9  790     956      956

  0.[illegible]D+02 ( 0.070[illegible]D+00)    1  490   60    6  760  680   64  844     964      964
( 0.[illegible]D+02)  0.256[illegible]D+00    1  164  564  116  900  036  024  100     964      964

  0.[illegible]D+02 ( 0.419[illegible]D+00)    1  306  320    0  664  600   64  452     972      972
( 0.[illegible]D+02)  0.377[illegible]D+00    1  250  210  100  700   40  200   14     972      972

  0.[illegible]D+02 ( 0.238350D+01)    1  300  704   92  596  700  624  932     980      980
( 0.10[illegible]D+03)  0.342[illegible]D+00    1   07  709  983  921  747  309  423     980      980

  0.[illegible]D+02 ( 0.22[illegible]D+01)    1  146  500  904  336  204  104  276     988      988
( 0.[illegible]D+02)  0.370[illegible]D+00    1  106  340  476   40  202  324  752     988      988
```

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  0.87559E+02 ( 0.33603E+00)    1   50  500  500  100   20    4  200      996      996
( 0.88974E+02)  0.31410E+00     1  492  256  884  796  212  592  488      996      996

  0.88959E+02   0.33680E+00     1   50  492  504  100  904    4  200     1004     1004

  0.86200E+02   0.31773E+00     1   82  800  500  124  604    4  200     1012     1012

  0.86153E+02   0.30636E+00     1  338    4  332   16  319   64  212     1020     1020

  0.86112E+02   0.32274E+00     1  454  516  900    4  700    8  540     1020     1020

  0.85479E+02   0.41302E+00     1  442  596  200  904  700   64  316     1036     1036

  0.84435E+02 ( 0.41036E+00)    1  346  700 1036  364  664   64  220     1044     1044
( 0.85761E+02)  0.39147E+00     1  150  952   80  112  992  136  808     1044     1044

  0.83014E+02   0.32164E+00     1  110  520  220    4  440    8  880     1052     1052


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10,000 POINT DATA
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FULL SEARCH


                         n = 3
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     P2            P6              p           N    NO. RULES SEARCHED.

                                ------------

  0.11000D-03 (-0.14211E-13) 8630 9376 2727    9972     4984
( 0.17700D-03)  0.00000E+00  8165 4152 9632    9972     4984

  0.11000D-03 (-0.14211E-13)    1 1033 9297    9972     9972
( 0.10690D-03)  0.00000E+00     1  254 4484    9972     9972

                         n = 4
                       ---------

     P2            P6              p                N    NO. RULES SEARCHED.

                                ----------------

  0.29810D-02 ( 0.76397E-11) 9534 2044 4077   55    9972     7476
( 0.29000D-02)  0.45191E-11  1621 4500 4165 8792    9972     7476

  0.28000D-02 ( 0.54143D-11)    1  480 4700 2276    9972     9972
( 0.27000D-02)  0.47740D-11     1  200 2223 9316    9972     9972

                         n = 5
                       ---------

     P2            P6                  p                N    NO. RULES SEARCHED.
                                ---------------------
  0.33170D-01 ( 0.14030D-07)  683 5166 1612 5775 3300    9972     9900
( 0.35470D-01)  0.49997E-08  9852 9874 1716 8230 3895    9972     9900

  0.30000D-01 ( 0.20715D-08)    1 1990 1210 6416 2000    9972     9972
( 0.31200D-01)  0.25914E-08     1 4222 5320 4096 1064    9972     9972
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```
                    N = 6
                  ---------

      P2            P6                     P                N    NO. RULES SEARCHED.

                              -------------------------
  0.247876E+00 ( 0.170268E-05)  232 4788 3240 9707 4774 5640    9972   12460
( 0.283166E+00)  0.695161E-04  3241 8366 3788 2911 2837 0953    9972   12460

  0.266146E+00 ( 0.955018E-06)    1 2426 1996 5074 5100 1424    9972    9972
( 0.260771E+00)  0.406038E-04    1 6754 3964 7740 7396 9304    9972    9972

                    N = 7
                  ---------

      P2            P6                        P                  N    NO. RULES SEARCHED.

                              --------------------------------
  0.162528E+01 ( 0.135388E-03) 1109 4638 0954 1342 2548 4434 1000    9972   14952
( 0.170518E+01)  0.415408E-04  0190 4433 9375 1506 0217  957   21    9972   14952

  0.150038E+01 ( 0.391808E-04)    1  886 9856 2276 3484 5228 6708    9972    9972
( 0.153718E+01)  0.318888E-04    1 6798 5420 6653 5096 0416 3368    9972    9972

                    N = 8
                  ---------

      P2            P6                           P                    N    NO. RULES SEARCHED.

                              -------------------------------------
  0.795218E+01 ( 0.256108E-02) 0858 9768 5132  868  103 4396 6009 9896    9972   17644
( 0.887268E+01)  0.108658E-02  5176 3536 3087 1750 2264 1036 3020 1156    9972   17644

  0.768088E+01 ( 0.192578E-02)    1 3050 8888 3268 0006  364 8108 8260    9972    9972
( 0.788878E+01)  0.167158E-02    1 3178 8828 9108 1000 6904 2512 5536    9972    9972

SECOND SEARCH


                    N = 3
                  ---------

      P2            P6             P           N    NO. RULES SEARCHED.

                              --------------
  0.130008E-03 (-0.142118E-13) 2828 9877 5085    9972     1246
( 0.177638E-03)  0.000000E+00  6168 4152 9632    9972     1246

  0.119088E-03 (-0.142118E-13)    1 1883 9897    9972     4986
( 0.188888E-03)  0.000000E+00    1  394 6884    9972     4986

                    N = 4
                  ---------

      P2            P6             P                N    NO. RULES SEARCHED.

                              -------------------
  0.290188E-08 ( 0.768878E-11) 9534 2064 6877   35    9972     1246
( 0.288848E-08)  0.848818E-11  9151 6687 2637 7376    9972     1246

  0.268888E-08 ( 0.548488E-11)    1  694 4768 2276    9972     4986
( 0.279888E-08)  0.677688E-11    1  396 2838 9016    9972     4986

                    N = 5
```

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     P2              P6                    P                    N     NO. RULES SEARCHED.

                                  ----------------------
  0.347388E-01 ( 0.900618E-00) 4107 2210 8159 1058 6044      9972     1246
( 0.378366E-01)  0.743898E-00  7758 6424 3215 5010 1634      9972     1246

  0.303108E-01 ( 0.207158E-00)    1 1993 1216 6610 2022      9972     4906
( 0.313338E-01)  0.759148E-00     1 4222 5120 4096 1044      9972     4906

                 N = 6
               ---------

     P2              P6                       P                    N     NO. RULES SEARCHED.

                                  ---------------------------
  0.301838E+00 ( 0.156908E-05) 4738 2261 3270 3450 1150 3772      9972     1246
( 0.301678E+00)  0.100078E-05  3011 6250 4797 3021 9690 2570      9972     1246

  0.300148E+00 ( 0.955018E-06)    1 2426 1996 5076 5100 1424      9972     4906
( 0.300778E+00)  0.000003E-06     1 4754 3964 7760 7396 9204      9972     4906

                 N = 7
               ---------

     P2              P6                          P                    N     NO. RULES SEARCHED.

                                  --------------------------------
  0.162908E+01 ( 0.110778E-03) 1919 9734  582 3556 2506 5460 5256      9972     1246
( 0.198388E+01)  0.112078E-03  9912  639 2396 0010 1943 5622 0945      9972     1246

  0.180000E+01 ( 0.391008E-04)    1  926 9056 2276 3404 5220 4700      9972     4906
( 0.153718E+01)  0.310000E-04     1 4790 9420 0052 5090 0416 3360      9972     4906

                 N = 8
               ---------

     P2              P6                             P                    N     NO. RULES SEARCHED.

                                  -------------------------------------
  0.828678E+01 ( 0.338358E-01) 3064  112 3316 7916 2232 6436 3696 1677      9972     1246
( 0.820000E+01)  0.275418E-02  8110 7610 1316 5005 0550 7052 5034 9014      9972     1246

  0.700000E+01 ( 0.198878E-02)    1 3650 9000 3240 0064  344 9100 0240      9972     4906
( 0.790878E+01)  0.167158E-02     1 3170 0020 9100 1000 6904 2512 5536      9972     4906


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                 N = 3
               ---------

       P2            P6              P            N       NO. RULES SEARCHED.

                                 -------------
  0.128138E-03 ( 0.000000E+00) 4107 0602 0314    10116     1264
( 0.100000E-03)  0.000000E+00  6944 4320 1309    10116     1264

  0.104908E-03 (-0.142118-130     1 1051 6093    10116     5050
( 0.301908E-03)  0.000000E+00     1  237 5509    10116     5050

  0.119858E-03 (-0.142118-13) 3532 2094 6295    10100     1273
( 0.198288E-03)  0.000000E+00  7403 9529  401    10100     1273
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```
  0.103018E-03 (-0.142118E-13)     1 2302 1404   10100     5094
( 0.104978E-03)  0.00000E+00       1  231 2421   10100     5094

  0.123418E-03 (-0.142118E-13) 4942  901 6220   10332     1291
( 0.224990E-03)  0.00000E+00   7500 9443  407   10332     1291

  0.116108E-03 (-0.142118E-13)     1 2433 9585   10332     5164
( 0.227008E-03)  0.00000E+00       1  339 5441   10332     5164

  0.115348E-03 (-0.142118E-13) 5420 5014  739   10404     1300
( 0.180738E-03)  0.00000E+00   3377 10204 4167  10404     1300

  0.100000E-03 (-0.142118E-13)     1 2162 7100   10404     5202
( 0.100838E-03)  0.00000E+00       1  300 9572   10404     5202

  0.100300E-03 (-0.142118E-13) 3009 8034 6764   10540     1310
( 0.154638E-03)  0.00000E+00   7927 2037 8555   10540     1310

  0.100900E-03 (-0.142118E-13)     1 4749 1377   10540     5274
( 0.167070E-03)  0.00000E+00       1  230 3904   10540     5274

  0.121030E-03 ( 0.00000E+00)  7310 6462 6591   10620     1327
( 0.190280E-03)  0.00000E+00   9004 1365 4719   10620     1327

  0.111000E-03 ( 0.00000E+00)      1 4790 7264   10620     5310
( 0.227200E-03)  0.00000E+00       1  271 9721   10620     5310

  0.000040E-04 ( 0.00000E+00)   150 5905 1072   10764     1345
( 0.192100E-03)  0.00000E+00   1000  339 3476   10764     1345

  0.101300E-03 ( 0.00000E+00)      1 2970 9712   10764     5382
( 0.167900E-03)  0.00000E+00       1  267 6765   10764     5382


                   n = 4
                 ---------

        P2           P6              P              N      NO. NULLS SEARCHED.

                            -----------------

  0.200418E-02 ( 0.100000E-10) 2600 5005 3344 2206   10110     1264
( 0.290040E-02)  0.10000E-10    390 0402 0097 4099   10116     1264

  0.236108E-02   0.375178E-11     1  525 2000 3061   10116     5058

  0.200000E-02 ( 0.200000E-10) 2042  776 0009 0820   10100     1273
( 0.317700E-02)  0.125300E-10  3079 5236 3703 2005   10100     1273

  0.200670E-02 ( 0.660000E-11)    1 2146  340 6292   10180     5094
( 0.277020E-02)  0.300050E-11     1 1031 340010007   10180     5094

  0.290020E-02   0.670000E-11  7006 0904 4900 2021   10332     1291

  0.200000E-02 ( 0.500320E-11)    1 7200 0404 0900   10332     5164
( 0.900000E-02)  0.370000E-11     1 4300 10096 2456  10332     5164

  0.200000E-02 ( 0.313090E-10) 1930 3009 5796 2520   10404     1300
( 0.300000E-02)  0.372700E-11  9012 9040 1001 0523   10404     1300

  0.220000E-02 ( 0.305118E-11)    1 3703 0420 7204   10404     5202
( 0.220040E-02)  0.312642E-11     1 3906 4012 2052   10404     5202

  0.242700E-02 ( 0.602990E-11) 0030 1097 2970 4016   10540     1310
( 0.247020E-02)  0.341060E-11  0735 0670 9500 3704   10540     1310
```

```
  0.245000E-02 ( 0.750338E-11)    1  525 1377 5661   10548    5274
( 0.247908E-02)  0.406012E-11     1 1617 9333 1021   10548    5274

  0.243708E-02   0.531498E-11  9600 9335 1636 8702   10620    1327

  0.241398E-02 ( 0.997018E-11)    1 270310200 7047   10620    5310
( 0.[illegible]-02)  0.433043E-11     1 4010 1924 9092   10620    5310

  0.2[illegible]E-02 ( 0.[illegible]E-11) 5154  759 3590 2463   10764    1345
( 0.27[illegible]-02)  0.412118E-11  5048 9915 [illegible] [illegible]   10764    1345

  0.220778E-02 ( 0.630968E-11)    1  748 7553 4220   10764    5382
( 0.220318E-02)  0.381378E-11     1  710 0956 0009   10764    5382

                    n = 5
                  ---------

      P2          P6                 P                   N      NO. RULES SEARCHED.

                         -----------------------
  0.[illegible]-01   0.[illegible]-04  7610 2060 2767 7370 9608   10116    1264

  0.[illegible]-01 ( 0.[illegible]-00)    1  770 6172 [illegible] [illegible]   10116    5058
( 0.[illegible]-01)  0.[illegible]-00     1 3742 2020 2100 3432   10116    5058

  0.[illegible]-01 ( 0.157[illegible]-07) 1511 [illegible] [illegible] 1400 [illegible]   10100    1273
( 0.[illegible]-01)  0.967100-00  3132 [illegible] [illegible] [illegible] 5137   10100    1273

  0.[illegible]-01 ( 0.[illegible]-00)    1 1070 [illegible] [illegible] [illegible]   10100    5094
( 0.[illegible]-01)  0.210400-00     1 [illegible]  [illegible] [illegible] [illegible]   10100    5094

  0.[illegible]-01   0.[illegible]-00  1032 1347 7142  372 [illegible]   10032    1281

  0.[illegible]-01   0.[illegible]-00     1  759 7021 5571 2601   10032    5106

  0.[illegible]-01 ( 0.[illegible]-00) [illegible] [illegible] [illegible] 7094 7900   10004    1300
( 0.[illegible]-01)  0.[illegible]-00  2307 0972 9917 5167 3926   10004    1300

  0.[illegible]-01   0.119[illegible]-00     1 1014 1030 [illegible] 1900   10004    5202

  0.[illegible]-01 ( 0.[illegible]-00) 3470 [illegible] 1394 [illegible]   10040    1310
( 0.[illegible]-01)  0.[illegible]-00  [illegible]  500 [illegible] 7267 [illegible]   10040    1310

  0.[illegible]-01 ( 0.[illegible]-00)    1 1002 [illegible] 7904 [illegible]   10540    5270
( 0.[illegible]-01)  0.[illegible]-00     1 9115 [illegible] [illegible] [illegible]   10540    5270

  0.[illegible]-01   0.[illegible]-00  3334  [illegible] 246 2092   10620    1327

  0.[illegible]-01 ( 0.[illegible]-00)    1 [illegible] [illegible] [illegible] [illegible]   10620    5310
( 0.[illegible]-01)  0.[illegible]-00     1 [illegible] 0776 9076   10620    5310

  0.[illegible]-01 ( 0.[illegible]-00) [illegible] [illegible] [illegible] [illegible]   10764    1345
( 0.[illegible]-01)  0.[illegible]-00  [illegible] [illegible] 2237 3344 [illegible]   10764    1345

  0.[illegible]-01 ( 0.122[illegible]-07)    1 [illegible] [illegible] [illegible] [illegible]   10764    5382
( 0.[illegible]-01)  0.[illegible]-00     1 2190 [illegible] [illegible] 0776   10764    5382

                    n = 6
                  ---------

      P2          P6                 P                   N      NO. RULES SEARCHED.

                         ---------------------------------
  0.[illegible]-00 ( 0.[illegible]-00)  234  [illegible] 2200 [illegible] [illegible] [illegible]   10116    1264
( 0.[illegible]-00)  0.130[illegible]-03  [illegible] [illegible] 6730 7719 [illegible] [illegible]   10116    1264
```

```
0.63030E-02 0.21406E-03  556 324 167      956   313
0.46575E-02              706 879 730      963   310
            0.23330E-03   47 643 947      963   310
0.65033E-02 0.10113E-03  323 956 730      981   334
0.46570E-02 0.29484E-03  526 640 643      990   337
0.65025E-02              778 163 233     1008   333
            0.23169E-03  649 437 473     1000   333
0.63331E-02              703 600 920     1017   334
            0.29534E-03  610 417 734     1017   336
0.84307E-02 0.93974E-10  251 879  75     1033   343
0.33338E-02               42 614 629     1044   343
            0.10933E-03  493 434 339     1044   343

0.58404E-02                1 600 444      954   954
            0.23734E-03    1  91 649      954   374
0.65331E-02                1 444 444      943   943
            0.21008E-03    1 333 616      943   943
0.64308E-02                1 447 444      981   981
            0.19433E-03    1 417 252      981   981
0.41803E-02                1 179 001      990   990
            0.16302E-03    1 921 002      990   990
0.57200E-02                1 330 444     1008  1008
            0.20043E-03    1 779 434     1008  1008
0.60440E-02                1 310 344     1017  1017
            0.14022E-03    1 104 295     1017  1017
0.54215E-02                1 818 331     1033  1033
            0.12133E-03    1 219 331     1033  1033
0.54215E-02 0.11606E-03    1 646 780     1044  1044
......................................................................

                 p = 4
              ---------

    P3          P6              P              N    NO. NOTES SEARCHED.
                        ------------------
0.89857E-01             344 303 338 732      954   733
            0.23431E-04  13 109 890 302      954   733
0.90012E-01 0.22330E-04 337 301 003 170      963   742
0.53109E-01             300 733 932 721      981   754
            0.11209E-04 372 113 319 220      981   766
0.81370E-01             434 900 908 944      990   763
            0.14404E-04 375 114 332 222      990   763
0.90337E-01             143  13 131 743     1000   777
            0.90790E-01 331 409 137 321     1000   777
0.94073E-01             997 103 900 653     1017   781
            0.17370E-04 532  73 303 349     1017   784
0.90043E-01 0.14151E-04 920 375 009 100     1033   799
0.73714E-01             774 443 1040 513    1044   803
            0.11730E-04 374 339 943 339     1044   803

0.93533E-01               1 776 202 704      904   954
            0.27733E-04   1 170 302 430      904   954
0.96301E-01               1 979 117 333      943   963
            0.34734E-04   1 906 117 430      943   963
0.90730E-01               1 900 632 910      981   981
            0.31300E-04   1 433 341 433      981   981
0.90000E-01               1 330 394 512      990   990
            0.31497E-04   1 910 874 131      990   990
0.90401E-01               1 914 310 344     1000  1000
            0.34977E-04   1 104 394 490     1000  1000
0.90303E-01               1 900  98  55     1017  1017
            0.13900E-04   1 900 374 764     1017  1017
0.70030E-01               1 301 241  81     1033  1033
            0.93320E-07   1 903 479 340     1033  1033
0.77944E-01               1 400 736  44     1044  1044
            0.11143E-04   1 131 949 215     1044  1044
```

```
    PS           PE                  P                    N    NO. BITES SEARCHED.
```

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    PS           PE              P                     N    NO. BITES SEARCHED.
```

```
0.109094E+03                443 340 446 325  35 422 462 333    943   4444
             0.410322E+00    24 433 342 622 833 340  55 232    443   4444
0.105992E+03                133 140 130 422 478 643 124 656    901   4125
             0.431196E+00   266 449 262  43 343 225 863 806    901   4125
0.910446E+03                423 524  49 524 446 990   4 423    992   4129
             0.503916E+00   416 503 942 503 443 325 303 134    992   4129
0.988008E+03                137 980 940 986 888  88 331 986   1009   4444
             0.411308E+00   341 396 379 416 684 359 003 369   1009   4444
0.103066E+03                396 273 406 070   0 326 464 153   1213   4606
             0.329813E+00   030 349 036 346 046 483 605 032   1213   4058
0.100300E+03                644 480 849 884 213 444 409 032   1033   2018
             0.513895E+03   416 453 240 458 243 206 322 443   1044   2215
0.964439E+03                493 335 224 634 344 214 944 314   1044   2093
             0.405048E+00   600 040 316 106 424 354 404 1004  1044   2040

0.931108E+03                  1 316 446 346 314 804  46 163    494    494
             0.430533E+00     1 610 401 403 409 204 341 403    494    494
0.103439E+03                  1 313  53 403 156 944 433 449    943    943
             0.393493E+00     1 263 140  13 563 113 341  63    463    463
0.101318E+03 0.333448E+00     1 301 494 394 334 336 410 355    801    801
0.053138E+03                  1 356 444 803 344 456  44 305    401    401
             0.539915E+00     1 451 134 814 334  14 344 354    443    443
0.836124E+03                  1 334 416 1000 393 946  44 306  1008   1008
             0.413308E+00     1 136 436 1000 443 344  44 304  1008   1008
0.041396E+03                  1 303 144 414 336 336 646 433   1013   1013
             0.316381E+00     1  01 433 304 113   4 383 666   1013   1013
0.831008E+03                  1  31 104 883  34 1013  4 313   1033   1033
             0.330304E+00     1 316 334 341 436 138 416 343   1033   1033
0.044330E+03                  1 346 300 1036 344 444 44 330   1044   1044
             0.301416E+00     1 130 443  03 113 983 134 908   1044   1044
```


10,000 POINT DATA

REDUCED SEARCH

```
                   a = 3
                 ---------

   P2           P4          b              n     NO. RULES SEARCHED.

( 0.180000-03)  0.000000E+00   1 234 4004    9972    4996
  0.110000-03 (-0.14211E-13)   1 1033 8587   9972    4996

( 0.177003-03)  0.000000E+00  6183 4133 8632   9972     334
  0.130000-03 (-0.14211E-13)  2632 9977 3903   9972     334
                             -------------

   P2           P4          b                    n     NO. RULES SEARCHED.

                   a = 4
                 ---------

  0.230000-03 ( 0.74097E-13)  9336 2866 4077  55    9972     334
( 0.200000-03)  0.74740E-13   2000 2000 2000 4173   9972     334

  0.300000-03 ( 0.84143E-13)   1 496 4308 2276      9972    4996
```

```
[illegible]
[illegible]

[illegible]
[illegible]

[illegible]
[illegible]

[illegible]
[illegible]
                                  ---------------
      P3          P4              P              N     NO. WATER REACHED.

                       ---------
                        N = [illegible]

.................................................................................

[illegible]
[illegible]

[illegible]
[illegible]
                                  ------------------------------------
      P3          P4                           P                      N     NO. WATER REACHED.

                       ---------
                        N = [illegible]

[illegible]
[illegible]

[illegible]
[illegible]
                                  ------------------------------
      P3          P4                         P                   N     NO. WATER REACHED.

                       ---------
                        N = [illegible]

[illegible]
[illegible]

[illegible]
                                  --------------------------
      P3          P4                       P                N     NO. WATER REACHED.

                       ---------
                        N = [illegible]

[illegible]
[illegible]

[illegible]
[illegible]
                                   ---------------------
      P3          P4                     P              N     NO. WATER REACHED.

                       ---------
                        N = [illegible]

[illegible]
```

```
0.133612E-03 (-0.163122E-13)  6943 961  0520  10332   516
( 0.234688E-03)  0.000000E+00  7509 9463 637  10332   516

0.114108E-03 (-0.163211E-13)  1 2433 9303     10332   5144
( 0.233008E-03)  0.000000E+00  1 233 3401     10332   5144

0.122318E-03 ( 0.000000E+00)  9939 0729 4210  10404   376
( 0.100738E-03)  0.000000E+00  4468101777 4167  10404   376

0.100008E-03 (-0.163211E-13)  1 2102 7100     10404   5202
( 0.164936E-03)  0.000000E+00  1 280 5572     10404   5202

0.123372E-03 ( 0.000000E+00)  6106 470 9907   10560   396
( 0.156036E-03)  0.000000E+00  7927 3037 9333  10560   396

0.100000E-03 (-0.163211E-13)  1 4760 1377     10560   5276
( 0.169276E-03)  0.000000E+00  1 230 3904     10560   5276

0.137036E-03 ( 0.000000E+00)  7310 9402 6591  10620   390
( 0.136236E-03)  0.000000E+00  9906 1393 4719  10620   390

0.111000E-03 ( 0.000000E+00)  1 4790 7204     10620   3310
( 0.187000E-03)  0.000000E+00  1 271 9721     10620   3310

0.800000E-04 ( 0.000000E+00)  130 3905 1972   10704   590
( 0.183100E-03)  0.000000E+00  1003 339 9476  10704   590

0.101000E-03 ( 0.000000E+00)  1 2970 9712     10704   3302
( 0.167000E-03)  0.000000E+00  1 747 6705     10704   3302

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                                    a - 4
                                  ---------

    P3           P4           P                  N     NO. HOLES SEARCHED.
                        ------------------

0.300610E-02 ( 0.100000E-10)  2810 3003 3344 2306  10116   343
( 0.300000E-02)  0.100000E-10  380 5003 0007 0000  10116   343

0.300000E-08  0.373170E-11  1 323 3000 3001     10116   3000

0.300000E-08  0.300000E-10  3002 776 0000 4420  10300   300

0.300070E-02 ( 0.030000E-11)  1 2106 300 0300    10100   3004
( 0.377220E-02)  0.300000E-11  1 1001 30003000   10100   3004

0.300000E-02  0.133103E-10  4906 3377 3000 100  10300   376

0.300000E-02 ( 0.300033E-11)  1 2303 0400 0300   10332   3106
( 0.300000E-02)  0.370300E-11  1 43001000 3030   10332   3106

0.300000E-02  0.372700E-11  0013 3103 3001 0303  10400   376

0.300000E-02 ( 0.300333E-11)  1 3703 0300 7004   10400   5302
( 0.330000E-02)  0.313000E-11  1 3000 0013 3003   10400   5302

0.300700E-03 ( 0.000000E-11)  0300 1007 3743 0010  10300   300
( 0.307000E-02)  0.341000E-11  0733 0030 0700 3740  10300   300

0.300000E-02 ( 0.700000E-11)  1 323 1377 1001    10000   3274
( 0.307000E-02)  0.000000E-11  1 1037 0333 7031   10300   3274

0.303700E-02  0.341000E-11  0000 0330 1031 0700  10300   300
```

```
( 0.1[illegible]-01)  0.1[illegible]-06   1 0 0 0  0 1 1 1   144 209   0 319 245   2    900   735

  0.1[illegible]-01   0.7[illegible]-07   1 0 0 0  0 1 0 0     7 140  33 170 247   2    900   730

  0.1[illegible]-01 ( 0.1[illegible]-06)  1 0 0 1  0 1 1 1   245 190  61 169 260   2    944   744
( 0.1[illegible]-01)  0.1[illegible]-06   1 0 0 0  0 1 1 1     1 245 133  90 260   2    940   744

  0.1[illegible]-01 ( 0.6[illegible]-06)  1 0 0 1  0 1 1 0   103  63 101  72 251   2   1004   730
( 0.1[illegible]-01)  0.3[illegible]-00   1 0 0 0  0 1 0 0   110  67 170   1 231   2   1004   730

  0.1[illegible]-01   0.1[illegible]-06   1 0 0 0  0 1 1 1   136 100  69 133 233   2   1012   736

  0.1[illegible]-01 ( 0.7[illegible]-06)  1 0 0 0  0 1 0 0   106  29 101 211 233   2   1020   742
( 0.1[illegible]-01)  0.1[illegible]-06   1 0 0 0  0 1 0 0    40  64 229 247 233   2   1020   742

  0.1[illegible]-01 ( 0.3[illegible]-00)  1 0 0 1  0 1 1 0   205 170 100  01 237   2   1030   740
( 0.1[illegible]-01)  0.3[illegible]-00   1 0 0 1  0 1 1 0   162  63 144 103 237   2   1030   740

  0.1[illegible]-01   0.004[illegible]-07 1 0 0 0  0 1 1 1   100 306  30 100 239   2   1034   774

  0.1[illegible]-01 ( 0.2[illegible]-06)  1 0 0 1  0 1 1 1    63 164 133  94 301   2   1004   700
( 0.1[illegible]-01)  0.9[illegible]-07   1 0 0 0  0 1 0 0   133 220  79 230 301   2   1004   700

  0.1[illegible]-01   0.0[illegible]-07   1 0 0 0  0 1 0 0    63 213 134 170 303   2   1032   706

  0.1[illegible]-01 ( 0.1[illegible]-06)  1 0 0 0  0 1 0 0     1  20 202  30 233   2    940   708
( 0.1[illegible]-01)  0.1[illegible]-06   1 0 0 0  0 1 0 1     1  60  63 103 237   2    900   708

  0.1[illegible]-01 ( 0.1[illegible]-06)  1 0 0 0  0 1 0 0     1  69  11  01 230   2    900   714
( 0.1[illegible]-01)  0.1[illegible]-06   1 0 0 1  0 1 1 0     1  00  36 150 230   2    926   714

  0.1[illegible]-01   0.0[illegible]-07   1 0 0 0  0 1 0 0     1  39  73  33 261   2    944   720

  0.1[illegible]-01 ( 0.2[illegible]-06)  1 0 0 0  0 1 0 0     1  37 134 109 243   2    972   736
( 0.1[illegible]-01)  0.2[illegible]-06   1 0 0 0  0 1 0 0     1  17  46  33 243   2    972   730

  0.[illegible]-01 ( 0.1[illegible]-06)   1 0 0 0  0 1 1 1     1  66 170 223 245   2    900   735
( 0.[illegible]-01)  0.1[illegible]-06    1 0 0 0  0 1 1 1     1  10  79 107 243   2    900   735

  0.1[illegible]-01   0.0[illegible]-07   1 0 0 0  0 1 0 0     1  36  61 220 247   2    900   730

  0.1[illegible]-01   0.[illegible]-07    1 0 0 0  0 1 1 1     1  70  40 230 300   2    900   744

  0.1[illegible]-01   0.0[illegible]-07   1 0 0 0  0 1 0 0     1  06 100 213 301   2   1000   730

  0.[illegible]-01   0.[illegible]-07     1 0 0 0  0 1 0 0     1  63 177 120 213   2   1052   730

  0.1[illegible]-01   0.[illegible]-06    1 0 0 0  0 1 1 1     1 130  70 300 300   2   1030   745

  0.[illegible]-01 ( 0.1[illegible]-06)   1 0 0 0  0 1 1 1     1  00  69  90 297   2   1000   700
( 0.[illegible]-01)  0.[illegible]-07     1 0 0 0  0 1 0 0     1 110  93 130 297   2   1030   700

  0.[illegible]-01   0.[illegible]-07     1 0 0 0  0 1 1 1     1  00 100 103 300   2   1030   774

  0.[illegible]-01 ( 0.1[illegible]-06)   1 0 0 0  0 1 0 0     1  07  13 217 300   2   1000   700
( 0.[illegible]-01)  0.1[illegible]-06    1 0 0 0  0 1 0 0     1  30 131 242 300   2   1000   700

  0.[illegible]-01 ( 0.[illegible]-07)    1 0 0 0  0 1 0 0     1  00 102  77 302   2   1075   700
( 0.[illegible]-01)  0.[illegible]-07     1 0 0 0  0 1 0 0     1  30 130  73 302   2   1075   700

              0 - 3
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   N8        N9        A7       A5          B           A    N    N    NO. WATER SAMPLED.
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                                        ---------   ---------   ------------------

0.10730E+00 ( 0.73006E-04) 1 0 0 0 0  0 1 0 0 1  171 103 173  70 173 237   2   900  1100
( 0.11300E+00)  0.31700E-04 1 0 0 0 0  0 1 0 0 0  100  34 160  20  00 237   2   900  1100

0.10030E+00 ( 0.74520E-04) 1 0 0 0 0  0 1 0 1 1  130 203  40 137 150 234   2   940  1140
( 0.11300E+00)  0.20140E-04 1 0 0 0 0  0 1 0 0 1  105  40 233 149 133 234   2   940  1140

0.10730E+00 ( 0.66470E-04) 1 0 0 0 0  0 1 0 0 1  205  70 103 171  47 241   2   940  1200
( 0.11370E+00)  0.20147E-04 1 0 0 0 0  0 1 0 0 0   00   4  23 170  37 241   2   940  1200

0.11180E+00 ( 0.11520E-03) 1 0 0 0 1  0 1 1 1 0  241 105 131 141 105 243   2   972  1210
( 0.11300E+00)  0.20847E-04 1 0 0 0 0  0 1 0 0 0  107  19  21 100  03 243   2   972  1210

0.10000E+00    0.21073E-04 1 0 0 0 0  0 1 0 0 0    1 230 100 101  44 245   2   900  1220

0.10380E+00    0.29470E-04 1 0 0 0 0  0 1 1 1 1  243 164  50 133  13 245   2   900  1230

0.10830E+00 ( 0.37480E-04) 1 0 0 0 0  0 1 1 1 1   74  34  18 130 133 249   2   940  1240
( 0.10830E+00)  0.10020E-04 1 0 0 0 0  0 1 0 0 0   74  34  10 170 133 249   2   940  1240

0.90420E-01    0.11832E-04 1 0 0 0 0  0 1 0 0 0   19 102  73 151 203 231   2  1004  1250

0.90790E-01 ( 0.60000E-04) 1 0 0 0 0  1 0 1 0 0  237 100 107  14 220 253   2  1012  1260
( 0.10250E+00)  0.13770E-04 1 0 0 0 0  0 1 0 0 0  174   1 210 215 237 253   2  1012  1260

0.80000E-01 ( 0.78131E-04) 1 0 0 0 1  0 1 0 1 1  172 104 300 157 123 253   2  1030  1270
( 0.10970E+00)  0.23000E-04 1 0 0 0 0  0 1 0 0 0  103   1 217 216 230 253   2  1030  1270

0.80000E-01 ( 0.80021E-04) 1 0 0 0 1  0 1 0 1 0  104 133 233  90 102 257   2  1030  1280
( 0.10000E+00)  0.20000E-04 1 0 0 0 0  0 1 0 0 0   22 120 300  33 112 257   2  1030  1280

0.90020E-01 ( 0.27123E-04) 1 0 0 0 0  0 1 0 0 0   24 300 101  90 124 249   2  1034  1290
( 0.10113E+00)  0.30000E-04 1 0 0 0 0  0 1 0 0 0   72 237 170 130  30 249   2  1034  1290

0.90000E-01    0.17400E-04 1 0 0 0 0  0 1 0 0 0   34 230 103  30 153 301   2  1004  1300

0.90300E-01 ( 0.30470E-04) 1 0 0 0 0  0 1 0 1 1  300  99 170  13  97 303   2  1032  1310
( 0.07700E-03)  0.13300E-04 1 0 0 0 0  0 1 0 0 0   00   9  33 109  14 300   2  1032  1310

0.10000E+00 ( 0.30070E-04) 1 0 0 0 0  0 1 0 1 1    1 107  73 337 113 337   2   900  1100
( 0.11000E+00)  0.20700E-04 1 0 0 0 0  0 1 0 0 0    1 113 190  44  79 337   2   900  1100

0.30000E+00 ( 0.00300E-00) 1 0 0 0 0  0 1 0 0 0    1  03 103  02 103 300   2   900  1100
( 0.30000E+00)  0.30070E-04 1 0 0 0 0  0 1 0 0 0    1  52  73  76 120 309   2   900  1100

0.10000E+00 ( 0.30000E-00) 1 0 0 0 0  0 1 1 1 1    1 110 107 133  24 301   2   904  1300
( 0.11130E+00)  0.30000E-04 1 0 0 0 0  0 1 0 0 0    1  59 107  47 122 301   2   904  1300

0.10080E+00 ( 0.00700E-00) 1 0 0 0 0  0 1 0 0 1    1  03 303  20 127 303   2   972  1310
( 0.10000E+00)  0.10000E-04 1 0 0 0 0  0 1 0 0 0    1  93  34  71 104 303   2   972  1310

0.30700E+00 ( 0.10000E-00) 1 0 0 0 1  0 1 0 0 1    1  30 100 301  31 309   2   900  1320
( 0.11000E+00)  0.30000E-04 1 0 0 0 0  0 1 0 0 0    1 110  11  39 171 309   2   900  1320

0.30000E+00 ( 0.00700E-00) 1 0 0 0 0  0 1 0 0 0    1  30  00 300 157 307   2   900  1330
( 0.30700E+00)  0.30100E-04 1 0 0 0 0  0 1 0 0 0    1  59  23 123  33 307   2   900  1330

0.30000E+00 ( 0.30100E-00) 1 0 0 0 0  0 1 0 0 0    1  00 300 123 300 300   2   900  1340
( 0.10700E+00)  0.30000E-04 1 0 0 0 0  0 1 0 0 0    1  00  23 300 127 300   2   900  1340

0.10700E+00 ( 0.30130E-00) 1 0 0 0 0  0 1 0 0 0    1  00 300 313  71 302   2  3004  1350
( 0.10700E+00)  0.12700E-04 1 0 0 0 0  0 1 0 0 0    1  00 300  03 307 302   2  3004  1350

0.30700E-01 ( 0.30000E-00) 1 0 0 0 0  0 1 0 0 0    1  20 140  01  69 379   2  3012  1300
( 0.30000E+00)  0.10300E-04 1 0 0 0 0  0 1 0 0 0    1  30 170 110  90 379   2  3012  1300
```

```
 0.000000-01 ( 0.100000-04) 1 0 0 0 0  0 1 0 0 0   1  43  13 130 104 255   2   1030   1330
( 0.131300+00)  0.134000-04 1 0 0 0 1  0 1 1 1 0   1  31 164  81 171 255   2   1030   1330

 0.003000-01 ( 0.133000-04) 1 0 0 0 0  0 1 0 0 0   1  49  33 134 107 253   2   1030   1303
( 0.000210-01)  0.129100-04 1 0 0 0 0  0 1 0 0 0   1  37  64  34 113 253   2   1030   1303

 0.067730-01 ( 0.217000-04) 1 0 0 0 0  0 1 0 0 0   1 134  43 133 316 344   2   1034   1320
( 0.103000+00)  0.134000-04 1 0 0 0 0  0 1 0 0 0   1  43 313  34 311 344   2   1034   1320

 0.000310-01    0.213300-04 1 0 0 0 0  0 1 0 0 1   1  33 134 110 330 301   2   1044   1300

 0.003130-01 ( 0.313130-04) 1 0 0 0 0  0 1 0 0 0   1  33 143 330 144 303   2   1033   1310
( 0.000000-01)  0.044320-05 1 0 0 0 0  0 1 0 0 0   1 101  00 333  31 303   2   1033   1310


                                   0 - 0
                                ---------

      P3         P4          Y1            Y2                  X                T    A     N    NO.MOLES SEARCHED.
                          -----------   -----------   ----------------------

 0.000000+00 ( 0.100000-03) 1 0 0 0 0 0  0 1 0 0 0 0   50 129  17 161  15 162 237   2   940   1000
( 0.000000+00)  0.100000-03 1 0 0 0 0 0  0 1 0 0 0 0   70 133 100 300  41  34 237   2   940   1000

 0.301000+00 ( 0.303000-03) 1 0 0 0 0 1  0 1 0 0 1 0   49  90 117 100 130 140 230   2   956   1004
( 0.100000+00)  0.330000-03 1 0 0 0 0 0  0 1 0 0 0 0   70 130 100 300  41  34 230   2   956   1004

 0.000000+00    0.031000-03 1 0 0 0 0 0  0 1 0 0 0 0  130 103 130 233 216 161 241   2   940   1020

 0.000000+00 ( 0.331000-03) 1 0 0 0 0 0  0 1 1 1 1 1   00 130 101  30  31  17 243   2   972   1004
( 0.000000+00)  0.130000-03 1 0 0 0 0 0  0 1 0 0 0 0   00  60 110  23 176 103 243   2   972   1004

 0.000130+00    0.100000-03 1 0 0 0 0 0  0 1 0 0 0 1  143 213 300 210 143 141 243   2   900   1003

 0.000000+00 ( 0.103000-03) 1 0 0 0 0 0  0 1 0 0 0 0   33 130  10 160  16 160 241   2   900   1000
( 0.000000+00)  0.103330-03 1 0 0 0 0 0  0 1 0 0 0 1   01  10 130 100 110 300 241   2   900   1000

 0.000000+00 ( 0.103030-03) 1 0 0 0 0 0  0 1 0 0 0 0  101 100  10 230 233 100 240   2   900   1004
( 0.000000+00)  0.233000-03 1 0 0 0 0 0  0 1 0 0 1 1  130  33 134  00 110 303 240   2   900   1004

 0.000000+00 ( 0.100000-03) 1 0 0 0 0 0  0 1 0 0 0 0   43 234  10   1  24 134 231   2  1004   2000
( 0.000000+00)  0.103030-03 1 0 0 0 0 0  0 1 0 0 0 0  100 140  20  43  43  24 231   2  1004   2000

 0.000000+00    0.130000-03 1 0 0 0 0 0  0 1 0 0 0 0  130 113 130  03  33  30 233   2  1013   2030

 0.000000+00 ( 0.300000-03) 1 0 0 0 0 0  0 1 0 0 1 1  100  00 100 100   3 133 200   2  1000   2033
( 0.000000+00)  0.100000-03 1 0 0 0 0 0  0 1 0 0 0 0   00  30 111  33 110 103 200   2  1030   2033

 0.000000+00 ( 0.300000-03) 1 0 0 0 0 0  0 1 0 0 1 1   33 130  30 113 103 230 203   2  1030   2000
( 0.000000+00)  0.130000-03 1 0 0 0 0 0  0 1 0 0 0 0   00 230  10   1  24 130 203   2  1030   2000

 0.000000+00 ( 0.303100-03) 1 0 0 0 1 1  0 1 1 1 0 1   03  13  00  30  13 233 230   2  1030   2000
( 0.000000+00)  0.100000-03 1 0 0 0 0 0  0 1 1 1 1 1  103  37 113   3 110 130 230   2  1030   2000

 0.000000+00    0.110000-03 1 0 0 0 0 0  0 1 0 0 0 0   10 210  03 200  13 133 203   2  2000   2000

 0.000000+00 ( 0.310000-03) 1 0 0 0 0 0  0 1 0 1 1 1   00 103 300 130 130  03 203   2  1003   2000
( 0.000000+00)  0.100000-03 1 0 0 0 0 0  0 1 1 1 1 1  103  13 130 200 203 100 203   2  1003   2000

 0.000000+00 ( 0.131100-03) 1 0 0 0 0 0  0 1 0 1 1 1    1  03 200  03  00 230 233   2   940   1000
( 0.000000+00)  0.000000-00 1 0 0 0 0 0  0 1 0 0 0 0    1  33 133 230  03  03 233   2   940   1000

 0.000300+00 ( 0.300100-03) 1 0 0 0 0 0  0 1 0 0 0 1    1  00  00 103 103  13 230   2   940   1000
( 0.000030+00)  0.100130-03 1 0 0 0 0 0  0 1 1 1 1 1    1 113 103  30 133  11 230   2   940   1000

 0.000000+00 ( 0.300000-03) 1 0 0 0 0 0  0 1 0 0 1 1    1  10  00  03 130 232 241   2   940   1000
```

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( 0.59155E+00)  0.12107E-02  1 0 0 0 0 0  0 1 0 0 0 0  1  30 177   0 269 211 261  2   944  1000

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( [illegible])   0.10081E-02  1 0 0 0 0 0  0 1 0 0 0 0  1  30 166 161  51 101 263  2   990  1043

  [illegible]  ( 0.13000E-02) 1 0 0 0 0 0  0 1 0 0 0 0  1  30 199  77  07 100 267  2   988  1088
( [illegible])   [illegible]  1 0 0 0 0 0  0 1 0 0 0 0  1  67  63 164 129 136 267  2   988  1088

  [illegible]    [illegible]  1 0 0 0 0 0  0 1 0 0 0 0  1 122 100 100 108 120 269  2   940  1088

  [illegible]    [illegible]  1 0 0 0 0 0  0 1 0 0 0 0  1 108 110 184 119  31 231  2  1084  2000

  [illegible]    [illegible]  1 0 0 0 0 0  0 1 0 0 0 0  1  03  30   7  73 133 233  2  1013  2018

  [illegible]    [illegible]  1 0 0 0 0 0  0 1 0 0 0 0  1 113  19 107 104 300 230  2  1030  2032

  [illegible]  ( [illegible]) 1 0 0 0 0 0  0 1 0 0 0 0  1  90 231 163  11  20 237  2  1050  2000
( [illegible])   [illegible]  1 0 0 0 0 0  0 1 0 0 0 0  1  91  37  47 109 109 237  2  1050  2000

  [illegible]  ( [illegible]) 1 0 0 0 1 1  0 1 1 1 0 1  1  10 100 223 120  26 249  2  1030  2004
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  [illegible]    [illegible]  1 0 0 0 0 0  0 1 0 0 0 0  1  90 126 170  00  13 200  2  1003  2000


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                                  ---------

   P2           P6            Y1:            Y2:                    t                  z   n    m   NO.RULES SEARCHED.

                           -------------  -------------  ---------------------------

  0.10000E+01 ( 0.10640E-03)  1 0 0 0 0 0 0  0 1 0 0 0 0 0   31   6 134 161 131  30 210 237  2  940  2714
( 0.30000E+01)  0.44700E-01   1 0 0 0 0 0 0  0 1 0 0 0 1 1   94 139  79  64 100 203 100 237  2  940  2714

  0.10425E+01 ( 0.01016E-01)  1 0 0 0 0 0 0  0 1 1 1 1 1 1  103  13 137  94  99 231 170 239  2  930  2737
( 0.10031E+01)  0.44400E-01   1 0 0 0 0 0 0  0 1 0 0 0 0 0  230  05 100 107 131 231 217 230  2  960  2737

  0.10470E+01 ( 0.44736E-01)  1 0 0 0 0 0 0  0 1 0 0 0 0 0   11  10 137 133  31 107 203 241  2  940  2700
( 0.10470E+01)  0.25807E-01   1 0 0 0 0 0 0  0 1 0 0 1 1 1    0 330  00 173 107 100 233 241  2  940  2700

  0.10335E+01 ( 0.32035E-01)  1 0 0 0 0 0 0  0 1 0 0 1 1 1   30 100 101 147 133 117  99 203  2  972  2703
( 0.10007E+01)  0.47116E-01   1 0 0 0 0 0 0  0 1 1 1 1 1 1  103 107 170  40 130  63 103 203  2  972  2703

  0.10000E+01   0.43106E-01   1 0 0 0 0 0 0  0 1 0 0 0 0 0  130  23 100 183 107 171  83 203  2  940  2004

  0.17030E+01 ( 0.77070E-01)  1 0 0 0 0 0 0  0 1 0 0 0 1 1  103  00 100 140 130  10 100 203  2  988  2054
( 0.30010E+01)  0.42137E-01   1 0 0 0 0 0 0  0 1 0 0 0 0 1  133 330  40 131 210  14  94 203  2  988  2054

  0.17700E+01 ( 0.02540E-01)  1 0 0 0 0 0 0  0 1 0 1 1 1 1   31 135 130 220 110   5 173 209  2  944  2035
( 0.10070E+01)  0.42000E-01   1 0 0 0 0 0 0  0 1 0 0 0 0 0  100 173 109 113 109  00  23 209  2  944  2035

  0.17070E+01 ( 0.14334E+00)  1 0 0 0 0 1 1  0 1 0 1 1 0 1  233 230 110 203  30  19  30 231  2 1004  2073
( 0.10352E+01)  0.44335E-01   1 0 0 0 0 0 0  0 1 0 0 0 0 0  300  63 120 134 122 203 022 231  2 1004  2073

  0.17600E+01 ( 0.30107E-01)  1 0 0 0 0 0 0  0 1 0 0 0 0 0   69 210   1 243  92 230  94 233  2 1012  2000
( 0.10003E+01)  0.44372E-01   1 0 0 0 0 0 0  0 1 0 0 0 0 0  103  13  92 128 117 110 172 233  2 1012  2000

  0.17307E+01 ( 0.43133E-01)  1 0 0 0 0 0 0  0 1 0 0 0 0 0  111  63 127 203 120   0 210 249  2 1050  2321
( 0.17737E+01)  0.21100E-01   1 0 0 0 0 0 0  0 1 0 1 1 1 1   43 229 130  07  23  43 232 233  2 1050  2321
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θ = 0

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  [illegible] [illegible] 1 0 0 0 0 0  0 1 0 0 0 0  [illegible]  3  [illegible]  [illegible]

( [illegible] [illegible] 1 0 0 0 0 0  0 1 0 0 0 0  [illegible]  3  [illegible]  [illegible]
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( [illegible] [illegible] 1 0 0 0 0 0  0 1 0 0 0 0  [illegible]  3  [illegible]  [illegible]
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( [illegible] [illegible] 1 0 0 0 0 0  0 1 0 0 1 1  [illegible]  3  [illegible]  [illegible]
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( [illegible] [illegible] 1 0 0 0 0 0  0 1 0 0 0 1  [illegible]  3  [illegible]  [illegible]
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( [illegible] [illegible] 1 0 0 0 0 0  0 1 0 0 0 1  [illegible]  3  [illegible]  [illegible]
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  [illegible] [illegible] 1 0 0 0 0 0  0 1 0 0 0 0  [illegible]  3  [illegible]  [illegible]

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( [illegible] [illegible] 1 0 0 0 0 0  0 1 0 0 0 0  [illegible]  3  [illegible]  [illegible]
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                          -----------  -----------  ---------------------
  [illegible]  [illegible]  [illegible]  [illegible]  [illegible]  [illegible] [illegible] [illegible] [illegible]

                  ---------
                  [illegible]

( [illegible] [illegible] 1 0 0 0 0  0 1 0 0 0  1 [illegible]  3  [illegible]  [illegible]
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( [illegible] [illegible] 1 0 0 0 0  0 1 0 0 1  1 [illegible]  3  [illegible]  [illegible]
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( [illegible] [illegible] 1 0 0 0 0  0 1 0 0 0  1 [illegible]  3  [illegible]  [illegible]
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  [illegible] [illegible] 1 0 0 0 0  0 1 0 0 0  1 [illegible]  3  [illegible]  [illegible]

( [illegible] [illegible] 1 0 0 0 1  0 1 1 1 0  1 [illegible]  3  [illegible]  [illegible]
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( [illegible] [illegible] 1 0 0 0 0  0 1 0 0 0  1 [illegible]  3  [illegible]  [illegible]
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( [illegible] [illegible] 1 0 0 0 0  0 1 0 0 0  1 [illegible]  3  [illegible]  [illegible]
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( [illegible] [illegible] 1 0 0 0 0  0 1 0 0 0  1 [illegible]  3  [illegible]  [illegible]
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( [illegible] [illegible] 1 0 0 0 0  0 1 0 0 0  1 [illegible]  3  [illegible]  [illegible]
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[illegible]

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( 0.1000E+03)  [illegible]   1 0 0 0 0 0 0  0 1 0 0 0 0 0  [illegible]  3  [illegible]  [illegible]
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( 0.1000E+03)  [illegible]   1 0 0 0 0 0 0  0 1 0 0 0 0 0  [illegible]  3  [illegible]  [illegible]
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  [illegible]   [illegible]  1 0 0 0 0 0 0  0 1 0 0 0 0 0  [illegible]  3  [illegible]  [illegible]

( [illegible])  [illegible]  1 0 0 0 0 0 0  0 1 0 0 0 0 0  [illegible]  3  [illegible]  [illegible]
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  [illegible]   [illegible]  1 0 0 0 0 0 0  0 1 0 0 0 0 0  [illegible]  3  [illegible]  [illegible]

( [illegible])  [illegible]  1 0 0 0 0 0 0  0 1 0 0 0 0 0  [illegible]  3  [illegible]  [illegible]
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( [illegible])  [illegible]  1 0 0 0 0 0 0  0 1 0 1 1 1 1  [illegible]  3  [illegible]  [illegible]
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( [illegible])  [illegible]  1 0 0 0 0 0 0  0 1 0 1 1 1 1  [illegible]  3  [illegible]  [illegible]
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( [illegible])  [illegible]  1 0 0 0 0 0 0  0 1 0 0 0 0 0  [illegible]  3  [illegible]  [illegible]
  [illegible] ( [illegible]) 1 0 0 0 0 0 0  0 1 0 0 0 0 0  [illegible]  3  [illegible]  [illegible]

  [illegible]   [illegible]  1 0 0 0 0 0 0  0 1 0 0 0 0 0  [illegible]  3  [illegible]  [illegible]

( [illegible])  [illegible]  1 0 0 0 0 0 0  0 1 0 0 0 0 0  [illegible]  3  [illegible]  [illegible]
  [illegible] ( [illegible]) 1 0 0 0 0 0 0  0 1 0 0 0 0 0  [illegible]  3  [illegible]  [illegible]
```

```
0.7603[illegible]E+02   0.3000[illegible]E+00  1 0 0 0 0 0 0 0  0 1 0 0 0 0 0 0   :  92 176 136 133  13  65  50 239   2   1036   4130

0.737000E+02 ( 0.316013E+00) 1 0 0 0 0 0 0 0  0 1 0 0 0 0 0 0   :  10 138  62   1   5 190 173 261   2   1044   4160
( 0.7637[illegible]E+02) 0.3000[illegible]E+00  1 0 0 0 0 0 0 0  0 1 0 0 0 0 0 0   :  40  34  55 112  43 136 137 261   2   1044   4160

0.7624[illegible]E+02 ( 0.3677[illegible]E+00) 1 0 0 0 0 0 0 0  0 1 0 0 0 0 0 0   :   1  41 101  77 149 241 143 146 264   2   1052   4180
( 0.7630[illegible]E+02) 0.2363[illegible]E+00  1 0 0 0 0 0 0 0  0 1 0 0 0 0 0 0   :  40 133 147   2 130  43  31 264   2   1052   4180
```

NOTE: The following rules were found with a different PHI2 function,
modified as suggested in section 6 of SLOAN(1985), Lattice Methods
for Multiple Integration.

PHI2=1+q*2*pi**2(1/6-u-u**2)   (this should only change the P2 errors.)

WITH q = 2

a = 3

```
      P2            P6          y1      y2         s          r    n     N    NO.RULES SEARCHED.
                               -----   -----  ------------

0.3[illegible]D-01 ( 0.[illegible]D-00) 1 0 0   0 1 0   200 129 140   237   2   900   334
( 0.[illegible]D-01) 0.[illegible]D-09  1 0 0   0 1 1   100  53 164   237   2   900   334

0.3[illegible]D-01 ( 0.1[illegible]D-00) 1 0 0   0 1 0   206 144  70   239   2   936   337
( 0.3[illegible]D-01) 0.1[illegible]D-09  1 0 0   0 1 0   170 133  93   239   2   946   337

0.3[illegible]D-01 ( 0.2[illegible]D-00) 1 0 0   0 1 0   133 102  44   261   2   944   340
( 0.3[illegible]D-01) 0.[illegible]D-10  1 0 0   0 1 0    74 140 150   261   2   944   340

0.3[illegible]D-01 ( 0.[illegible]D-00) 1 0 0   0 1 0   219 201 133   263   2   972   343
( 0.3[illegible]D-01) 0.1[illegible]D-00  1 0 0   0 1 1   236  33 213   263   2   972   343

0.3[illegible]D-01 ( 0.[illegible]D-00) 1 0 0   0 1 0   165 180   2   264   2   990   346
( 0.3[illegible]D-01) 0.[illegible]D-00  1 0 0   0 1 0   166 130  76   263   2   990   346

0.3[illegible]D-01 ( 0.[illegible]D-00) 1 0 0   0 1 0   130 210 217   267   2   900   349
( 0.[illegible]D-01) 0.1[illegible]D-00  1 0 1   0 1 1    72 180  97   267   2   900   349

0.3[illegible]D-01 ( 0.[illegible]D-00) 1 0 0   0 1 0    13 200 125   200   2   946   372
( 0.[illegible]D-01) 0.1[illegible]D-00  1 0 1   0 1 1   130 210   1   200   2   946   372

0.3[illegible]D-01 ( 0.[illegible]D-00) 1 0 0   0 1 0    64 140  63   281   2  1004   375
( 0.[illegible]D-01) 0.[illegible]D-00  1 0 0   0 1 1    64 180 134   281   2  1004   375

0.3[illegible]D-01 ( 0.[illegible]D-00) 1 0 0   0 1 0    74 129 247   283   2  1012   378
( 0.[illegible]D-01) 0.1[illegible]D-00  1 0 0   0 1 1   164   0  37   283   2  1012   378

0.3[illegible]D-01 ( 0.111[illegible]D-00) 1 0 0   0 1 0   147 213   2   236   2  1020   381
( 0.[illegible]D-01) 0.[illegible]D-10  1 0 1   0 1 1   130 101  21   236   2  1020   381

0.3[illegible]D-01   0.[illegible]D-10  1 0 0   0 1 0   216 157 160   237   2  1020   384

0.3[illegible]D-01 ( 0.[illegible]D-00) 1 0 0   0 1 0    14 230  36   290   2  1020   387
( 0.3[illegible]D-01) 0.3[illegible]D-00  1 0 0   0 1 1   134 140 110   239   2  1020   387

0.3[illegible]D-01   0.1[illegible]D-09  1 0 0   0 1 0    69 130  60   301   2  1000   388

0.3[illegible]D-01 ( 0.[illegible]D-00) 1 0 0   0 1 0   101 104 130   263   2  1008   390
( 0.3[illegible]D-01) 0.11[illegible]D-09  1 0 0   0 1 0    77 134 237   300   2  1008   390

0.3[illegible]D-01 ( 0.[illegible]D-00) 1 0 0   0 1 0     1  76  60   237   2   900   304
```

[illegible]

P2 P4 Y1 Y2 S T M N NO.RULES SEARCHED.

K = 3

[illegible]

```
( 0.[illegible]) 0.[illegible] 1 0 0 0 0 0 1 0 0 0 [illegible] 2 1012 1300
  0.[illegible] ( 0.[illegible]) 1 0 0 0 0 0 1 0 0 0 [illegible] 2 1012 1300

( 0.[illegible]) 0.[illegible] 1 0 0 0 0 0 1 0 0 0 [illegible] 2 1004 1320
  0.[illegible] ( 0.[illegible]) 1 0 0 0 0 0 1 0 0 0 [illegible] 2 1004 1320

  0.[illegible]   0.[illegible] 1 0 0 0 0 0 1 0 0 0 [illegible] 2 800 1300

( 0.[illegible]) 0.[illegible] 1 0 0 0 0 0 1 0 0 0 [illegible] 2 800 1330
  0.[illegible] ( 0.[illegible]) 1 0 0 0 0 0 1 0 0 0 [illegible] 2 800 1330

( 0.[illegible]) 0.[illegible] 1 0 0 0 0 0 1 0 0 0 [illegible] 2 900 1330
  0.[illegible] ( 0.[illegible]) 1 0 0 0 0 0 1 0 0 0 [illegible] 2 900 1330

( 0.[illegible]) 0.[illegible] 1 0 0 0 0 0 1 0 0 0 [illegible] 2 972 1310
  0.[illegible] ( 0.[illegible]) 1 0 0 0 0 0 1 0 0 0 [illegible] 2 972 1310

( 0.[illegible]) 0.[illegible] 1 0 0 0 0 0 1 0 0 0 [illegible] 2 944 1300
  0.[illegible] ( 0.[illegible]) 1 0 0 0 0 0 1 0 0 0 [illegible] 2 944 1300

( 0.[illegible]) 0.[illegible] 1 0 0 0 0 0 1 0 0 0 [illegible] 2 904 1280
  0.[illegible] ( 0.[illegible]) 1 0 0 0 0 0 1 0 0 0 [illegible] 2 904 1280

  0.[illegible]   0.[illegible] 1 0 0 0 0 0 1 0 0 0 [illegible] 2 800 1100

( 0.[illegible]) 0.[illegible] 1 0 0 0 0 0 1 0 0 0 [illegible] 2 1003 1310
  0.[illegible] ( 0.[illegible]) 1 0 0 0 0 0 1 0 0 0 [illegible] 2 1003 1310

( 0.[illegible]) 0.[illegible] 1 0 0 0 0 0 1 0 0 0 [illegible] 2 1004 1300
  0.[illegible] ( 0.[illegible]) 1 0 0 0 0 0 1 0 0 0 [illegible] 2 1004 1300

( 0.[illegible]) 0.[illegible] 1 0 0 0 0 0 1 0 0 0 [illegible] 2 1030 1300
  0.[illegible] ( 0.[illegible]) 1 0 0 0 0 0 1 0 0 0 [illegible] 2 1030 1300

( 0.[illegible]) 0.[illegible] 1 0 0 0 0 0 1 0 0 0 [illegible] 2 1030 1300
  0.[illegible] ( 0.[illegible]) 1 0 0 0 0 0 1 0 0 0 [illegible] 2 1030 1300

( 0.[illegible]) 0.[illegible] 1 0 0 0 0 0 1 0 0 0 [illegible] 2 1030 1310
  0.[illegible] ( 0.[illegible]) 1 0 0 0 0 0 1 0 0 0 [illegible] 2 1030 1310

( 0.[illegible]) 0.[illegible] 1 0 0 0 0 0 1 0 0 0 [illegible] 2 1012 1300
  0.[illegible] ( 0.[illegible]) 1 0 0 0 0 0 1 0 0 0 [illegible] 2 1012 1300

( 0.[illegible]) 0.[illegible] 1 0 0 0 0 0 1 0 0 0 [illegible] 2 1004 1290
  0.[illegible] ( 0.[illegible]) 1 0 0 0 0 0 1 0 0 0 [illegible] 2 1004 1290

( 0.[illegible]) 0.[illegible] 1 0 0 0 0 0 1 0 0 0 [illegible] 2 900 1290
  0.[illegible] ( 0.[illegible]) 1 0 0 0 0 0 1 0 0 0 [illegible] 2 900 [illegible]

( 0.[illegible]) 0.[illegible] 1 0 0 0 0 0 1 1 1 1 [illegible] 2 800 1330
  0.[illegible] ( 0.[illegible]) 1 0 0 0 0 0 1 0 0 0 [illegible] 2 800 1330

( 0.[illegible]) 0.[illegible] 1 0 0 0 0 0 1 0 0 0 [illegible] 2 900 1330
  0.[illegible] ( 0.[illegible]) 1 0 0 0 0 0 1 0 0 0 [illegible] 2 900 1330

( 0.[illegible]) 0.[illegible] 1 0 0 0 0 0 1 0 0 0 [illegible] 2 972 1310
  0.[illegible] ( 0.[illegible]) 1 0 0 0 0 0 1 0 0 0 [illegible] 2 972 1310

( 0.[illegible]) 0.[illegible] 1 0 0 0 0 0 1 0 0 0 [illegible] 2 944 1300
  0.[illegible] ( 0.[illegible]) 1 0 0 0 0 0 1 0 0 0 [illegible] 2 944 1300

( 0.[illegible]) 0.[illegible] 1 0 0 0 0 0 1 0 0 1 [illegible] 2 970 1100
  0.[illegible] ( 0.[illegible]) 1 0 0 0 0 0 1 0 0 0 [illegible] 2 970 1100

( 0.[illegible]) 0.[illegible] 1 0 0 0 0 0 1 0 0 0 [illegible] 2 940 1103
```

```
  0.230200D+00 ( 0.900000E-06)    1 1606 1672 3700 0072 9124   10100    5094
( 0.200000E+00)  0.090000D-06     1 2129 9109  593 9373 7013   10100    5094

  0.274140E+00 ( 0.430330E-05) 6550  056 6910 6030 3665 9430   10332     574
( 0.200000E+00)  0.102000E-05  7325 9923 4197 2000    1 2074   10332     574

  0.242271E+00   0.612020E-06     1 1202 0656  100 9004 5204   10337    5166

  0.243060E+00 ( 0.102390E-05) 2194 299910200 1300 2442 7206   10404     570
( 0.247900E+00)  0.707900E-06  4172 6979 3900 4000 9040 7030   10404     570

  0.230310E+00 ( 0.540030E-06)    1 4210 0000 5420 0694 2560   10404    5202
( 0.200700D+00)  0.000050E-06     1 1002 0016  720 7104 9960   10404    5202

  0.232120E+00 ( 0.100670E-05) 4026 3495 2962  602 3064 3606   10500     506
( 0.303012E+00)  0.700060E-06  7090 2520 4057 3141 2277 5500   10500     506

  0.227700E+00 ( 0.001570E-06)    1 3274 2300 4034  124 5152   10540    5274
( 0.227430E+00)  0.232900E-06     1 2074 0400 5420 2956 2356   10540    5274

  0.230750D+00 ( 0.170000D-05) 0000 0006 1346 3406 3420 5323   10620     590
( 0.206100E+00)  0.120170E-05  1050 4353 7910 5670 0006 0530   10620     590

  0.201300D+00 ( 0.122300D-05)    1 4050  300 3972 7456 2640   10620    5310
( 0.200000E+00)  0.440900E-06     1 3434 4156 9044 4216 2404   10620    5310

  0.200000D+00 ( 0.790000D-06) 2200 0276 0070 1294 0573 5034   10764     590
( 0.270200D+00)  0.021000D-06   236 5134 2770  955 4025 7060   10764     590

  0.200000D+00 ( 0.115390D-05)    1 4304 1012 4270 6700 5036   10764    5302
( 0.300000D+00)  0.044200D-06     1 2343 0000  063 9517 4259   10764    5302

.................................................................................

                          Q = 7
                        ---------

        PE           PG                   P                  N    NO. RULES SEARCHED.
                               -------------------------------

  0.107100D+01 ( 0.100000D-03) 3130 2416 0004 1072 7454 610010000  10116     562
( 0.100170D+01)  0.100010D-03  0040 2001  370 2300 7970  915  100  10116     562

  0.100070D+01 ( 0.300000D-04)    1  974 7000 0000 7104 0064 5702  10116    5000
( 0.100400D+01)  0.200000D-04     1 2139 3009 0011  021 3123 3037  10116    5050

  0.197000D+01 ( 0.120000D-03) 2006 0004 3200 3302 7007 2012 2300  10100     506
( 0.107730D+01)  0.112300D-03  4300 0519 3290 1003 0002 7072 9701  10100     506

  0.140100D+01 ( 0.300000D-04)    1 2002 6216  020 5036 261010100  10100    5094
( 0.100010D+01)  0.370710D-04     1 0006 2512 7200 377210000  434  10100    5094

  0.100000D+01   0.120070D-03  7190 3000 7100 0000 1900 4304 1000  10332     576

  0.140000D+01 ( 0.421000D-04)    1  310 3006 3000 9000 9000 0000  10032    3166
( 0.140010D+01)  0.300000D-04     1  730 0004 9000 3504   32 3502  10032    3166

  0.100000D+01 ( 0.130000D-03) 3624 1272 0000 0010 7236 1901 4176  10404     570
( 0.170700D+01)  0.320720D-04  7400 0025 3000 1000 1406 2011 6220  10404     570

  0.140700D+01 ( 0.200000D-04)    1 0002 9006 0000 0002 1304 2000  10404    3002
( 0.140900D+01)  0.200900D-04     1 3206 0040 5700 7000 7732  704  10404    3002

  0.140000D+01   0.577000D-04   306 5000 2077 3204 2000 3303 0036  10040     506

  0.140000D+01 ( 0.720010D-04)    1 2206 9000 7532 0026 4900 3000  10040    5274
```

```
( 0.14600E+01)  0.30503E-04      1 1570 7216  540 5720 6064 6104   10540    5274

  0.15375E+01 ( 0.53246E-03) 1457 3924 1040 1240 0640 6176 4672   10620     590
( 0.15701E+01)  0.79000E-04  406610160 9042 2424 2771 4310 5734   10620     590

  0.16012E+01 ( 0.45679E-04)     1 1522 1324 7040  670 9352 2944   10620    5310
( 0.14204E+01)  0.22050E-04      1 1530 7004 1952 7356 4320 0144   10620    5310

  0.14602E+01 ( 0.07621E-04) 0510 0627 5054 1106 3552  070 9500   10764     590
( 0.15097E+01)  0.78973E-04  7300  010 3000 0900 6760 9290 1427   10764     590

  0.13560E+01 ( 0.37090E-04)     1 3802 5300 1000 5476  012 3000   10764    5302
( 0.14171E+01)  0.27400E-04      1  422 5060 7964 2462 7100 3020   10764    5302

....................................................................................

                    S = 8
                   -------

      P2            P6                        D                             N      NO. RULES SEARCHED.
                                ---------------------------------------
  0.85850E+01 ( 0.33732E-01) 1002 0096 1966 4630 2314 7065 0490 1202   10116      562
( 0.96570E+01)  0.25044E-02  7262 6640 8616 3027 6444  300 5109 3316   10116      562

  0.78800D+01 ( 0.22320D-02)    1 1014 2096 3140  032 0272 4616 3040   10116     5050
( 0.78000E+01)  0.11009E-02     1 3566  544 7700 2572 6656 3160 9452   10116     5050

  0.80042E+01   0.21000E-02  059210096101 77 5306 1726 2602 9434 5256   10180      564

  0.76990E+01 ( 0.10973E-02)    1 2310 3016 7020 2116 3212 3412 6200   10180     5094
( 0.77303E+01)  0.13720E-02     1 2306 4704 7316  204 0776 6202 2204   10180     5094

  0.80543E+01   0.39000E-02   110 1210 0072 0317 5274 5004 2134  302   10332      574

  0.76727E+01 ( 0.27950D-02)    1 2734 478010144 2600 1192 4340 5032   10332     5164
( 0.78000D+01)  0.15702D-02     1 4526 6652 9836 7000 6040 0300 9040   10332     5164

  0.78000E+01 ( 0.42620D-02) 7000 7294 3440 0906 4500 0352 7200 7796   10404      570
( 0.81160D+01)  0.24233E-02  1162 0000 7075 4000 0010 1017 1264 4336   10404      570

  0.76240D+01   0.11320D-02     1 0910 7020 3304 0620 9772 3630 5000   10404     5302

  0.78500D+01 ( 0.33420D-02) 3030 3056 6029 7634 0062 1162 0006 6110   10540      906
( 0.82179D+01)  0.30062D-02  4227 6905  962 990010016 7319 9115 4003   10540      900

  0.78761D+01 ( 0.30611D-02)    1 3042 3304 0506 9704 1016 7264 2000   10540     5274
( 0.72072D+01)  0.11110E-02     1 3270 7620 9620 6300 2004 479610060   10540     5274

  0.76060D+01 ( 0.60812D-02) 613610564 3006 1036 0002 0048 0036 7276   10620      590
( 0.80007D+01)  0.30100D-02  5520 6200 6950 6404 5237 5733 3052 6037   10620      590

  0.72070D+01 ( 0.24381D-02)    1 3610 482610412 7006 1900  704 2082   10620     5310
( 0.76014D+01)  0.14000D-02     1  742 0944 9500 5296  232 2294 4180   10620     5310

  0.76520D+01 ( 0.43070D-02) 0036 0074 120010130 0900 2404  764 2497   10764      590
( 0.80007D+01)  0.31047D-02  220710089 3304 9197 6270 3267 4220 1204   10764      590

  0.71000D+01 ( 0.21300D-02)    1 2900  600 4600 5004 3304 0030 1424   10764     5302
( 0.76001E+01)  0.12004E-02     1 1322 306010232 7120 0904 3100 1000   10764     5302
```

```
          RANK 2 RULES HAVING n = 3.
          --------------------------

-----------------------------------------------------------------------------------------------

-1000 POINT DATA
----------------

------------------------------------------------------------------------------------------------
----------------------------------------------------------------------------------------------

                      n = 3
                    ---------

       P2           P6          Y1     Y2       x          r   n       N    NO.RULES SEARCHED.
                             -----  -----   ----------

0.62200e-02                  1 0 0  0 1 0   36  24  31 106  3      994       315
            0.73749e-09      1 0 0  0 1 0   63  25  40 106  3      994       315

0.57003e-02 0.93476e-10      1 0 0  0 1 0   70  99  23 107  3      963       310

0.63816e-02                  1 0 0  0 1 0   70 101  25 109  3      981       324
            0.36300e-09      1 7 0  0 1 1   30  67  56 109  3      981       324
```

```
0.370070e-02 0.167530e-09  1 0 0  0 1 1   94  82  23 110  3     990     327

0.545670e-02 0.154460e-09  1 0 0  0 1 0   20  34  50 112  3    1000     333

0.571400e-02 0.164400e-09  1 0 0  0 1 0   34  99   4 113  3    1017     336

0.[illegible]e-02 0.173020e-09  1 0 0  0 1 0   44   5  10 115  3    1035     342

0.496720e-02 0.310950e-10  1 0 0  0 1 0  111  43  21 116  3    1044     345


0.[illegible]E-02 0.134650E-09  1 0 1  0 1 1    1  90  44 106  3     954     315

0.[illegible]E-02               1 0 0  0 1 0    1  20  79 107  3     963     318
             0.250100E-09  1 0 0  0 1 0    1  87  79 107  3     963     318

0.555410E-02               1 0 0  0 1 0    1  74  26 109  3     981     324
             0.517130E-10  1 0 0  0 1 0    1  35  26 109  3     981     324

0.[illegible]E-02               1 0 1  0 1 1    1  16  36 110  3     990     327
             0.[illegible]E-09  1 0 1  0 1 1    1   6  36 110  3     990     327

0.[illegible]E-02               1 0 0  0 1 0    1  19  25 112  3    1000     333
             0.[illegible]E-09  1 0 1  0 1 1    1 106  36 112  3    1000     333

0.[illegible]E-02               1 0 0  0 1 0    1  63  14 113  3    1017     336
             0.[illegible]E-09  1 0 1  0 1 1    1   6  36 113  3    1017     336

0.[illegible]E-02               1 0 0  0 1 1    1  52  50 115  3    1035     342
             0.124570E-09  1 0 0  0 1 1    1  53  49 115  3    1035     342

0.[illegible]E-02               1 0 0  0 1 0    1  10  13 116  3    1044     345
             0.[illegible]E-10  1 0 0  0 1 0    1  97  13 116  3    1044     345
.........................................................................................

                        n = 4
                      ---------

       P2          P6        y1        y2             c            r   n      m    NO.RULES SEARCHED.
                          -------   -------   --------------
0.[illegible]e-01 0.[illegible]e-06  1 0 0 0   0 1 0 0   10  13   3  06 106   3     954      735

0.[illegible]e-01               1 0 0 0   0 1 0 1   56  13  55  04 107   3     963      742
             0.[illegible]e-06  1 0 0 1   0 1 1 0   76  00  62 101 107   3     963      742

0.[illegible]e-01               1 0 0 0   0 1 0 0   70  03  04  09 109   3     981      756
             0.[illegible]e-06  1 0 0 0   0 1 0 1   26  37  25  32 109   3     981      756

0.[illegible]e-01 0.[illegible]e-06  1 0 0 0   0 1 1 1   30   4 107  12 110   3     990      763

0.[illegible]e-01               1 0 0 0   0 1 1 1   00  19  62  32 112   3    1000      777
             0.[illegible]e-06  1 0 0 0   0 1 1 1   30  30   0  16 112   3    1000      777

0.[illegible]e-01 0.[illegible]e-06  1 0 0 0   0 1 0 1    2  24 106  56 113   3    1017      784

0.[illegible]e-01               1 0 0 1   0 1 0 1   67  30  76  63 115   3    1035      798
             0.211220e-06  1 0 0 0   0 1 0 0   17  71  36  40 115   3    1035      798

0.[illegible]e-01               1 0 0 0   0 1 1 1   22  74  15  30 110   3    1044      805
             0.[illegible]e-06  1 0 0 1   0 1 1 1   42 113  62  00 110   3    1044      805


0.[illegible]E-01               1 0 0 1   0 1 1 0    1  96 100  00 106   3     954      735
             0.[illegible]E-06  1 0 0 0   0 1 0 0    1  61  11  30 106   3     954      735
```

```
0.804618E-01 0.241248E-06 1 0 0 0  0 1 0 0   1  31 105  45 107  3     963     742

0.867012E-01              1 0 0 0  0 1 0 0   1  43 105  44 109  3     981     756
             0.222190E-06 1 0 0 0  0 1 0 0   1  83  22  82 109  3     981     756

0.810538E-01              1 0 0 0  0 1 0 0   1  31  81  91 110  3     990     763
             0.143048E-06 1 0 0 0  0 1 0 0   1  79  81  19 110  3     990     763

0.807108E-01              1 0 0 1  0 1 1 1   1  22  36   8 112  3    1008     777
             0.204008E-06 1 0 1 1  0 1 1 2   1  37  25  29 112  3    1008     777

0.826828E-01              1 0 0 0  0 1 0 0   1  70  41  45 113  3    1017     784
             0.151238E-06 1 0 0 0  0 1 0 0   1  43  41  60 113  3    1017     784

0.808868D-01              1 0 0 0  0 1 1 1   1  36  31  81 115  3    1035     790
             0.259908E-06 1 0 1 1  0 1 1 2   1  82  54  50 115  3    1035     790

0.735108E-01 0.880847E-07 1 0 1 1  0 1 1 2   1  30  52   4 116  3    1044     803

........................  .........................................................

                     s = 3
                   ---------

      P2          P6         y1         y2           z            r   n     N    NO.RULES SEARCHED.

                          ---------  ---------  -------------
0.677810e+00              1 0 0 0 0  0 1 0 0 0  26  36  24  31  32 106  3   954   1260
             0.202900e-04 1 0 0 0 0  0 1 1 1 1  20  44  37  88  14 106  3   954   1260

0.708812e+00              1 0 0 0 0  0 1 0 0 0  77  54  84  94  11 107  3   963   1272
             0.400140e-04 1 0 0 0 1  0 1 0 0 1  70  58  77  49  89 107  3   963   1272

0.717840e+00              1 0 0 0 0  0 1 0 0 0  91  97  84  58  10 109  3   981   1296
             0.303000e-04 1 0 0 0 1  0 1 0 1 0  15  32  87  24  61 109  3   981   1296

0.680020e+00              1 0 0 1 1  0 1 1 0 1  17  80  34  30 100 110  3   990   1308
             0.383370e-04 1 0 0 1 1  0 1 1 0 1 100  62  13  62  27 110  3   990   1308

0.680080e+00 0.280090e-04 1 0 0 0 0  0 1 0 0 0  86  43  98  70  69 112  3  1008   1332

0.687080e+00 0.281220e-04 1 0 0 0 0  0 1 0 0 1  95  46  33  95  73 113  3  1017   1344

0.680860e+00              1 0 0 0 0  0 1 0 0 0  57  67 102   5 101 115  3  1035   1368
             0.290080e-04 1 0 0 0 0  0 1 1 1 1 109  71 109  72   4 115  3  1035   1368

0.575780e+00 0.160070e-04 1 0 0 0 0  0 1 0 0 0  88  74  88  98  95 116  3  1044   1380


0.708878D+00              1 0 0 1 1  0 1 1 0 1   1  98  62  20  30 106  3   954   1260
             0.204308D-04 1 0 0 0 1  0 1 0 1 1   1  40  11  71  15 106  3   954   1260

0.707838D+00              1 0 0 0 0  0 1 0 0 0   1  22  98  98  32 107  3   963   1272
             0.613008D-04 1 0 0 0 0  0 1 1 1 1   1  86  98  58  33 107  3   963   1272

0.777080D+00              1 0 0 0 0  0 1 0 0 1   1  32  43  88 106 109  3   981   1296
             0.373048D-04 1 0 0 0 1  0 1 0 0 1   1  81  21  86   5 109  3   981   1296

0.670808D+00 0.316048D-04 1 0 0 0 1  0 1 0 1 1   1 106  16  46  36 110  3   990   1308

0.684080D+00              1 0 0 0 1  0 1 0 1 1   1  70 108   0  30 112  3  1008   1332
             0.529638D-04 1 0 0 0 1  0 1 1 1 1   1  60  61  41  68 112  3  1008   1332

0.688140D+00              1 0 0 0 0  0 1 0 0 0   1  54  91  56  30 113  3  1017   1344
             0.200008D-04 1 0 0 0 0  0 1 1 1 1   1  73  30  71  90 113  3  1017   1344
```

```
0.783668E+00             1 0 0 0 0  0 1 0 0 0     1   0  64  52  71 115  3     1035    1360
           0.310408E-04  1 0 0 0 1  0 1 1 1 1     1 102  54 103  41 115  3     1035    1360

0.590702E+00             1 0 0 0 1  0 1 1 1 0     1  44  28  12  88 116  3     1044    1380
           0.139098E-04  1 0 0 0 1  0 1 1 1 0     1  70  29 104  88 116  3     1044    1380

.......................................................  ...................................

                 s - 6
               ---------

     P2          P6          y1          y2                    s                  r    n       N     NO.RULES SEARCHED.
                         -----------  ----------- ---------------------

0.401018E+01             1 0 0 0 0 0  0 1 0 0 0 0  60  90  42  72  71  93 106   3      954     2100
           0.217048E-02  1 0 0 0 0 1  0 1 0 1 1 0  45  69  61  65  78  71 106   3      954     2100

0.400908E+01             1 0 0 0 0 0  0 1 0 0 0 1  11  36  56  35  63  72 107   3      963     2120
           0.267178E-02  1 0 0 0 1 1  0 1 1 1 1 1  11  65  35  87  47  11 107   3      963     2120

0.396878E+01             1 0 0 0 0 0  0 1 0 0 0 0  92  60  20  62  35   2 109   3      981     2160
           0.263008E-02  1 0 0 0 0 1  0 1 0 0 0 1   5  67  84  65  66 102 109   3      981     2160

0.389048E+01             1 0 0 0 0 0  0 1 0 0 1 1  62  93  44  74  74  54 110   3      990     2180
           0.180838E-02  1 0 0 0 1 1  0 1 0 0 1 1 104  10  57  72  10  42 110   3      990     2180

0.388088E+01 0.140088E-02  1 0 0 0 1 1  0 1 0 1 0 1  64  21  14  14  66  84 112   3     1000     2220

0.420108E+01             1 0 0 0 0 0  0 1 0 0 0 0  16  27  92  37  94  36 113   3     1017     2240
           0.217138E-02  1 0 0 0 0 0  0 1 0 1 1 1  64  48  66  90  75  42 113   3     1017     2240

0.418898E+01             1 0 0 0 0 0  0 1 0 0 0 0  12  39  60  37  60  77 115   3     1035     2280
           0.217708E-02  1 0 0 0 0 0  0 1 0 0 1 1  66  63  12  29  27  64 115   3     1035     2280

0.388088E+01             1 0 0 0 0 0  0 1 0 0 0 0  90  84  30  86  37   2 116   3     1044     2300
           0.114012E-02  1 0 0 0 0 0  0 1 0 0 1 1  24  88  95  14  92  54 116   3     1044     2300


0.413018E+01             1 0 0 0 0 1  0 1 0 0 0 1   1  36  24  16  80  86 106  3     954    2100
           0.180818E-02  1 0 0 0 0 1  0 1 0 0 0 1   1  70  42  98  80   4 106  3     954    2100

0.400838E+01             1 0 0 0 0 0  0 1 1 1 1 1   1  33  19  92  40  36 107  3     963    2120
           0.230408E-02  1 0 0 0 0 1  0 1 1 1 1 0   1  52  29  10  92  76 107  3     963    2120

0.437088E+01             1 0 0 0 0 0  0 1 0 0 0 0   1  72  61  32  15  99 109  3     981    2160
           0.112608E-02  1 0 0 0 0 0  0 1 0 0 0 0   1  37  61  77  15  10 109  3     981    2160

0.440888E+01 0.159018E-02  1 0 0 0 0 1  0 1 0 1 1 0   1   0  36 104  86  76 110  3     990    2180

0.398878E+01             1 0 0 0 0 1  0 1 0 0 1 1   1  98  36 104  64  60 112  3    1000    2220
           0.191378E-02  1 0 0 0 0 1  0 1 0 1 1 1   1 106  36   0  84  64 112  3    1000    2220

0.400888E+01             1 0 0 0 0 0  0 1 0 0 0 0   1  98  87  30 111   3 113  3    1017    2240
           0.209088E-02  1 0 0 0 0 1  0 1 0 0 1 1   1  60  64  35  18  71 113  3    1017    2240

0.400048E+01             1 0 0 0 0 0  0 1 0 0 0 0   1  17  59  83  31  67 115  3    1035    2280
           0.180808E-02  1 0 0 0 1 1  0 1 1 1 0 0   1  37 104  53   0 107 115  3    1035    2280

0.380108E+01             1 0 0 0 1 1  0 1 0 0 1 1   1  66  30  12  60 104 116  3    1044    2300
           0.141048E-02  1 0 0 0 0 1  0 1 0 1 1 0   1  46  20  12  60 104 116  3    1044    2300

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```

```
                        n = 7
                      ---------

      P2            P6           y1              y2                      z                     r     n      N    NO.RULES SEARCHED.
                            -------------  -------------  --------------------------

0.204018E+02               1 0 0 0 0 0 0  0 1 0 0 0 1 1  104   4 100  94  81  72  75  106   3     954   3150
             0.529039E-01  1 0 0 0 1 1 1  0 1 0 1 0 1 2   40 105  90  82  88   8  94  106   3     954   3150

0.224198E+02               1 0 0 0 0 0 0  0 1 0 0 0 0 0   53  49  61   9  58  22  94  107   3     963   3180
             0.576238E-01  1 0 0 0 1 1 1  0 1 0 1 0 1 2   70  95  71  41  44  55  89  107   3     963   3180

0.223000E+02               1 0 0 0 0 0 0  0 1 0 0 0 0 0  107   5 103  97  63  74  72  109   3     901   3240
             0.508738E-01  1 0 0 0 0 0 1  0 1 0 1 1 1 1   29  85  55  56  83  21  23  109   3     901   3240

0.197540E+02               1 0 0 0 0 0 0  0 1 1 1 1 1 1   56  91  34  90 102  86   8  110   3     990   3270
             0.533908E-01  1 0 0 0 0 1 1  0 1 0 1 1 0 0   94  26  80  80 104  87  54  110   3     990   3270

0.190198E+02               1 0 0 0 0 0 0  0 1 0 0 0 0 0   36  30  20 106  36   1  46  112   3    1000   3330
             0.565518E-01  1 0 0 0 1 1 1  0 1 0 1 0 1 2   45  16  65  64  13  75  52  112   3    1000   3330

0.215300E+02               1 0 0 0 0 0 0  0 1 0 0 0 0 0   49  30  69  90  61   2  93  113   3    1017   3360
             0.061618E-01  1 0 0 0 1 1 1  0 1 1 1 0 0 1   70  15  75  74   6  93 100  113   3    1017   3360

0.205780E+02               1 0 0 0 0 0 0  0 1 0 0 0 0 0    5  32 100  37  19  55 105  115   3    1035   3420
             0.504278E-01  1 0 0 0 0 0 0  0 1 1 1 1 1 1   50  06  76  87 105 106 110  115   3    1035   3420

0.200000D+02               1 0 0 0 0 0 0  0 1 0 0 0 0 0   66 102 115  50  22  56  66  116   3    1044   3450
             0.503798E-01  1 0 0 0 1 1 1  0 1 1 1 0 1 1   26  42  76  60  53  81 100  116   3    1044   3450



0.207822E+02 0.282078E-01  1 0 0 0 0 0 1  0 1 0 0 1 1 1    1  06  82  56  66  34  62 106  3     954    3150

0.212032E+02               1 0 0 0 0 0 0  0 1 0 0 0 0 0    1  10   3  94   9  55  27 107  3     963    3180
             0.610008E-01  1 0 0 0 0 0 1  0 1 1 1 1 1 1    1   7  69  22  47   0  56 107  3     963    3180

0.200018E+00               1 0 0 0 0 0 0  0 1 0 0 0 0 0    1  10 106  55   9  53  82 109  3     901    3240
             0.000098E-01  1 0 0 0 1 1 1  0 1 1 1 0 1 2    1  78 106  86  25   6  93 109  3     901    3240

0.100080D+02               1 0 0 0 0 0 1  0 1 0 0 1 1 0    1  10 106   2  36  90   4 110  3     990    3270
             0.432178E-01  1 0 0 0 0 1 1  0 1 0 1 1 0 0    1  26  10  86  36  54  26 110  3     990    3270

0.100000E+00               1 0 0 0 0 0 1  0 1 0 1 1 1 1    1  62  36 106  64  40  64 112  3    1000    3330
             0.420008E-01  1 0 0 0 0 0 1  0 1 0 0 0 1 0    1  90  36   0  64  64  64 112  3    1000    3330

0.211678E+02               1 0 0 0 0 0 0  0 1 0 0 0 0 0    1  32   7 111  49  99   4 113  3    1017    3360
             0.320008E-01  1 0 0 0 0 0 0  0 1 1 1 1 1 1    1  26  11  20   0  79  80 113  3    1017    3360

0.200000D+02               1 0 0 0 0 0 0  0 1 0 0 0 0 0    1   7  69 113 101  17   4 115  3    1035    3420
             0.316198E-01  1 0 0 0 0 0 1  0 1 0 1 1 1 0    1   0  26 101  31  71  81 115  3    1035    3420

0.200000D+02               1 0 0 0 0 0 1  0 1 0 0 0 1 1    1  70  52 112  36  26  10 110  3    1044    3450
             0.470648E-01  1 0 0 0 1 1 1  0 1 1 1 0 1 2    1  30  52   0  36  98  10 110  3    1044    3450

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                        n = 8
                      ---------

      P2            P6           y1              y2                      z                            r    n     N    NO.RULES SEARCHED.
```

```
                         ---------------  --------------  ------------------  --------
0.990032E+02             1 0 0 0 0 0 0 0  0 1 0 0 0 0 0 0   88   4  48  60  90 100  65  50 106   3    954   4620
        0.355947E+00     1 0 0 0 0 0 0 0  0 1 0 1 1 1 1 1   63  40  79 102  90  62  72   2 106   3    954   4620

0.103560E+03             1 0 0 0 0 0 0 0  0 1 0 0 0 0 0 0   47  60  77  47  82  87  43  72 107   3    963   4664
        0.521798E+00     1 0 0 0 0 1 1 1  0 1 1 1 1 0 0 1   12  76  85  96  44  18 101  72 107   3    963   4664

0.101298E+03             1 0 0 0 0 0 0 0  0 1 0 0 0 0 0 0   90 106  17  83  18  15  94  88 109   3    981   4752
        0.574108E+00     1 0 0 0 0 1 1 1  0 1 1 1 1 0 0 0   25  30  18  89  20  24  16  90 109   3    981   4752

0.870478E+02             1 0 0 0 0 0 0 0  0 1 0 0 0 0 0 0   24  60   3  96  70   0 104  94 110   3    990   4796
        0.439018E+00     1 0 0 0 0 0 1 1  0 1 0 1 1 1 0 1  104  92  62  92  69  60   8  12 110   3    990   4796

0.867348E+02             1 0 0 0 0 0 0 0  0 1 0 0 0 0 0 0  104  96   6  70  53  76  20   6 112   3   1000   4884
        0.369168E+00     1 0 0 0 0 0 0 1  0 1 0 1 1 1 1 1   62  48  81  32   6  38  88  20 112   3   1000   4884

0.960807E+02             1 0 0 0 0 0 0 0  0 1 0 0 0 0 0 0   83  65  65  38   0 108  72  59 113   3   1017   4928
        0.463208E+00     1 0 0 0 0 0 0 1  0 1 0 0 0 1 1 0    4  23  62 108  59  88  50  35 113   3   1017   4928

0.876828E+02             1 0 0 0 0 0 0 0  0 1 0 0 0 0 0 0   65  90  60  38  60  28  55  85 115   3   1035   5016
        0.422988E+00     1 0 0 0 0 1 1 1  0 1 1 1 1 0 1 2   43   9  53 107  12  38  59  24 115   3   1035   5016

0.887388E+02             1 0 0 0 0 0 0 0  0 1 0 0 0 0 0 0    2  96  60  30  24  79  60   4 114   3   1044   5060
        0.363678E+00     1 0 0 0 0 1 1 1  0 1 1 1 1 0 1 1    2  96  60  30  24  79  60   4 116   3   1044   5060



0.98863e+02              1 0 0 0 0 0 0 1  0 1 0 1 1 1 1 0    1  92  90  12  44  20  30 104 106   3    954   4620
        0.40309e+00      1 0 0 0 0 0 1 1  0 1 0 0 1 1 0 0    1 108   4  96  16  74  64  84 106   3    954   4620

0.10309e+03              1 0 0 0 0 0 0 0  0 1 0 0 0 0 0 0    1  53  27  40  87  10 102  56 107   3    963   4664
        0.43001e+00      1 0 0 0 0 0 0 0  0 1 0 0 0 1 1 1    1  53  27  40  87  10 102  56 107   3    963   4664

0.98888e+02              1 0 0 0 0 0 0 0  0 1 0 0 0 0 0 0    1   2   4   8  16  32  64  19 109   3    981   4752
        0.40600e+00      1 0 0 0 0 0 0 0  0 1 0 0 1 1 1 1    1  54  82  88  75  17  48  86 109   3    981   4752

0.98089e+02              1 0 0 0 0 0 0 1  0 1 0 0 0 0 0 1    1  82  14  40  86  12 104  58 110   3    990   4796
        0.38121e+00      1 0 0 0 0 0 0 1  0 1 0 0 0 1 1 0    1 108   4 102  16  78  64  92 110   3    990   4796

0.88830e+02              1 0 0 0 0 0 0 1  0 1 0 0 1 1 1 1    1  82   4 104  16  88  64  96 112   3   1000   4884
        0.32067e+00      1 0 0 0 0 0 1 1  0 1 0 0 0 1 0 1    1  82   4 104  16  88  64  96 112   3   1000   4884

0.88788e+02              1 0 0 0 0 0 0 0  0 1 0 0 0 0 0 0    1  26 111  61   6 104 100  10 113   3   1017   4928
        0.37880e+00      1 0 0 0 0 0 0 0  0 1 0 0 0 0 0 0    1  87 111  53   4   9 105  95 113   3   1017   4928

0.90200e+02              1 0 0 0 0 0 0 0  0 1 0 0 0 0 0 0    1   2   4   8  16  32  64  13 115   3   1035   5016
        0.36722e+00      1 0 0 0 0 0 1 1  0 1 0 0 0 1 0 1    1  57  29  43  36  97   8  53 115   3   1035   5016

0.88671e+02              1 0 0 0 0 0 0 1  0 1 0 0 0 0 1 1    1  14  88  76  20  40  92  12 116   3   1044   5060
        0.30148e+00      1 0 0 0 0 0 0 1  0 1 0 0 1 1 1 1    1  62  16  64  24  96  36  28 116   3   1044   5060
--------------------------------------------------------------------------------
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10,000 POINT DATA
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---------------------------------------------------------------------------------------------
----------------------------------------------------------------------------------
REDUCED SEARCH.


                e = 3
              ---------

      P2          P6      Y1   Y2          G            F    R    E    NO.RULES SEARCHED.
```

```
                                          --- ---      --------------
  0.11782E-03 ( 0.00000E+00)100 010      496    42   303    1100 3    9972       554
( 0.22000E-03)  0.00000E+00 100 010      203   194  1044    1100 3    9972       554

  0.12082E-03 ( 0.00000E+00)100 010        1    77   309    1100 3    9972       554
( 0.19047E-03)  0.00000E+00 100 010        1    50   204    1100 3    9972       554

                         n = 4
                       ---------

       P2            P6          y1   y2                   s                r    n     N    NO.RULES SEARCHED.
                                 ---- ----    -------------------

  0.27300E-02 ( 0.11553E-10)1000 0100     207    960    926    403    1100 3    9972      554
( 0.28123E-02)  0.65512E-11 1000 0100      92    787    343    644    1100 3    9972      554

  0.26009E-02 ( 0.86260E-11)1000 0100       1    329    765    169    1100 3    9972      554
( 0.28000E-02)  0.37963E-11 1000 0100       1    409    901    713    1100 3    9972      554

                         n = 5
                       ---------

       P2            P6          y1    y2                    s                    r    n     N    NO.RULES SEARCHED.
                                 ----- -----    -------------------------

  0.[illegible]-01 ( 0.[illegible]-00)10000 01000    270    313    309    362    440    1100 3    9972      554
( 0.[illegible]-01)  0.[illegible]-00 10000 01000    180   1037    883    845    515    1100 3    9972      554

  0.[illegible]-01 ( 0.[illegible]-00)10000 01000      1    183    309    130   1061    1100 3    9972      554
( 0.[illegible]-01)  0.[illegible]-00 10000 01000      1    347     69    423    329    1100 3    9972      554

                         n = 6
                       ---------

       P2            P6          y1         y2                      s                          r    n     N    NO.RULES SEARCHED.

                                 ------ ------    -----------------------------
  0.[illegible]+00 ( 0.[illegible]-0[illegible])100000 010000    [illegible]    [illegible]    [illegible]    276     96    112    1100 3    9972      554
( 0.[illegible]+00)  0.[illegible]-0[illegible] 100000 010000   1007    [illegible]    [illegible]    [illegible]     21    [illegible]    1100 3    9972      554

  0.[illegible]+00    0.71112E-06 100000 010000      1     10    [illegible]    211    [illegible]    [illegible]    1100 3    9972      554

                         n = 7
                       ---------

       P2            P6          y1          y2                      s                                  r    n     N    NO.RULES SEARCHED.

                                 ------- -------    -----------------------------------
  0.[illegible]+01 ( 0.[illegible]-0[illegible])1000000 0100000    670    [illegible]    771    [illegible]    426    [illegible]    [illegible]    1100 3    9972      554
( 0.[illegible]+01)  0.[illegible]-04 1000000 0100000    [illegible]    767    152    644    [illegible]    [illegible]     22    1100 3    9972      554

  0.[illegible]+01 ( 0.[illegible]-0[illegible])1000000 0100000      1    [illegible]    161    771    437     36    [illegible]    1100 3    9972      554
( 0.[illegible]+01)  0.[illegible]-04 1000000 0100000      1    273    [illegible]    213    [illegible]    [illegible]   1009    1100 3    9972      554

                         n = 8
                       ---------

       P2            P6          y1           y2                           s                                    r    n     N    NO.RULES SEARCHED.

                                 ------- -------    -----------------------------------------
  0.[illegible]+01 ( 0.[illegible]-0[illegible])10000000 01000000    410    [illegible]    [illegible]   [illegible]     34    [illegible]    152    516    1100 3    9972      554
( 0.[illegible]+01)  0.[illegible]-02 10000000 01000000    [illegible]    231    179    [illegible]    [illegible]    161    [illegible]   1067    1100 3    9972      554

  0.[illegible]+01 ( 0.[illegible]-0[illegible])10000000 01000000      1    219    317    727    [illegible]   1203     13    [illegible]    1100 3    9972      554
( 0.[illegible]+01)  0.[illegible]-02 10000000 01000000      1     29    [illegible]     13    377    [illegible]    100    [illegible]    1100 3    9972      554
```

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                   n = 3
                 ---------

       P2            P6       y1   y2           s             r    n    N    NO.RULES SEARCHED.

                              ---  ---    ---------------

  0.109877D-03 ( 0.000000E+00)100 010    674   326   993   1124 3  10116      562
( 0.105748D-03)  0.000000E+00 100 010    965   899   564   1124 3  10116      562

  0.122000D-03 ( 0.000000E+00)100 010      1   367   933   1124 3  10116      562
( 0.147300D-03)  0.000000E+00 100 010      1    85   401   1124 3  10116      562

  0.115700D-03 ( 0.000000E+00)100 010   1049    71   990   1132 3  10100      566
( 0.130040D-03)  0.000000E+00 100 010    707  1012   401   1132 3  10100      566

  0.100210D-03 ( 0.000000E+00)100 010      1   309   765   1132 3  10100      566
( 0.153000D-03)  0.000000E+00 100 010      1    61   540   1132 3  10100      566

  0.127700D-03 ( 0.000000E+00)100 010    796   282   741   1100 3  10332      576
( 0.158900D-03)  0.000000E+00 100 010     63   534     4   1100 3  10332      576

  0.120000D-03 ( 0.000000E+00)100 010      1   195  1009   1100 3  10332      576
( 0.190000D-03)  0.000000E+00 100 010      1    59    37   1100 3  10332      576

  0.111000D-03 ( 0.000000E+00)100 010    300   520   277   1156 3  10404      570
( 0.177000D-03)  0.000000E+00 100 010     57   905    60   1156 3  10404      570

  0.120000D-03 ( 0.142110D-13)100 010      1    97   161   1156 3  10404      570
( 0.140000D-03)  0.000000E+00 100 010      1    41   575   1156 3  10404      570

  0.112000D-03 ( 0.000000E+00)100 010    117   535   901   1172 3  10540      586
( 0.140400D-03)  0.000000E+00 100 010    269   604   771   1172 3  10540      586

  0.100000D-03 ( 0.000000E+00)100 010      1    61   206   1172 3  10540      586
( 0.101310D-03)  0.000000E+00 100 010      1    37   107   1172 3  10540      586

  0.100000D-03 ( 0.000000E+00)100 010    961   572   063   1180 3  10600      590
( 0.100210D-03)  0.000000E+00 100 010   1130   621  1079   1180 3  10600      590

  0.112000D-03 ( 0.000000E+00)100 010      1   467   960   1180 3  10600      590
( 0.100010D-03)  0.000000E+00 100 010      1    46   900   1180 3  10600      590

  0.112000D-03 ( 0.142110D-13)100 010    306  1122   330   1180 3  10704      598
( 0.130400D-03)  0.000000E+00 100 010    313   180   607   1180 3  10704      598

  0.101400D-03 ( 0.000000E+00)100 010      1   367   600   1196 3  10704      598
( 0.100400D-03)  0.000000E+00 100 010      1    62   296   1196 3  10704      598

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                   n = 4
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       P2            P6       y1   y2                 s                 r   n    N    NO.RULES SEARCHED.
                              ---- ----     ---------------------
  0.200000D-02  0.796300D-11 1000 0100    973   600   366   149   1124 3  10116     562

  0.207070D-02  0.302300D-11 1000 0100      1   262   601   149   1124 3  10116     562
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  0.271[illegible]D-02 ( 0.23[illegible]D-10)1000 0100   1106    34   339   [illegible]   1132 3  10100     566
( 0.270[illegible]D-02)  0.9[illegible]D-11 1000 0100    939   931  1027   [illegible]   1132 3  10100     566

  0.2[illegible]D-02 ( 0.16[illegible]D-10)1000 0100      1   513   545  1113   1132 3  10100     566
( 0.20[illegible]D-02)  0.[illegible]D-11 1000 0100      1   501   429  1017   1132 3  10100     566

  0.2[illegible]D-02 ( 0.[illegible]D-11)1000 0100    312   330   [illegible]   102   1140 3  10332     574
( 0.2[illegible]D-02)  0.3[illegible]D-11 1000 0100    [illegible]   [illegible]   051   215   1140 3  10332     574

  0.2[illegible]D-02   0.7[illegible]D-11 1000 0100      1   223   365  1035   1140 3  10332     574

  0.2[illegible]D-02 ( 0.4[illegible]D-11)1000 0100    [illegible]   700    82   [illegible]   1156 3  10404     578
( 0.2[illegible]D-02)  0.254[illegible]D-11 1000 0100     87  1043   334   196   1156 3  10404     578

  0.25[illegible]D-02   0.[illegible]D-11 1000 0100      1   [illegible]  1001   [illegible]   1156 3  10404     578

  0.2[illegible]D-02 ( 0.11[illegible]D-10)1000 0100    120  1110   315   110   1172 3  10540     586
( 0.2[illegible]D-02)  0.7[illegible]D-1[illegible] 1000 0100    157   107   [illegible]   [illegible]   1172 3  10540     586

  0.2[illegible]D-02   0.[illegible]D-11 1000 0100      1   107   [illegible]   [illegible]   1172 3  10540     586

  0.2[illegible]D-02   0.[illegible]D-11 1000 0100    [illegible]   [illegible]   [illegible]  1091   1180 3  10620     590

  0.2[illegible]D-02   0.[illegible]D-11 1000 0100      1   457  1160   [illegible]   1180 3  10620     590

  0.2[illegible]D-02 ( 0.[illegible]D-10)1000 0100    270   674   [illegible]   [illegible]   1196 3  10764     598
( 0.2[illegible]D-02)  0.10[illegible]D-10 1000 0100    [illegible]    89  1170   127   1196 3  10764     598

  0.2[illegible]D-02   0.[illegible]D-11 1000 0100      1   [illegible]    [illegible]   [illegible]   1196 3  10764     598

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                        n - 5
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        P2             P6        y1    y2                   a                      r   n    N    NO.RULES SEARCHED.

                                ----- -----    -------------------------
  0.3[illegible]D-01   0.[illegible]D-08 10000 01000    [illegible]   [illegible]   120     0   [illegible]   1136 3  10110     562

  0.3[illegible]D-01   0.[illegible]D-08 10000 01000      1   [illegible]   [illegible]  1119   717   1136 3  10110     562

  0.2[illegible]D-01   0.[illegible]D-08 10000 01000    [illegible]   [illegible]   [illegible]  [illegible]   [illegible]   1132 3  10100     566

  0.3[illegible]D-01 ( 0.7[illegible]D-08)10000 01000      1   167   [illegible]   415   [illegible]   1132 3  10100     566
( 0.3[illegible]D-01)  0.[illegible]D-08 10000 01000      1   [illegible]   141   [illegible]   637   1132 3  10100     566

  0.[illegible]D-01 ( 0.[illegible]D-08)10000 01000   1076   [illegible]   [illegible]   [illegible]   [illegible]   1140 3  10332     574
( 0.3[illegible]D-01)  0.3[illegible]D-08 10000 01000    [illegible]   [illegible]   130     0   [illegible]   1140 3  10332     574

  0.3[illegible]D-01   0.[illegible]D-08 10000 01000      1   [illegible]   [illegible]   [illegible]   [illegible]   1140 3  10332     574

  0.3[illegible]D-01   0.3[illegible]D-08 10000 01000     16   333   [illegible]   [illegible]   [illegible]   1156 3  10404     578

  0.2[illegible]D-01 ( 0.31[illegible]D-08)10000 01000      1   [illegible]  [illegible]   [illegible]   [illegible]   1156 3  10404     578
( 0.[illegible]D-01)  0.2[illegible]D-08 10000 01000      1   263   [illegible]   [illegible]   [illegible]   1156 3  10404     578

  0.3[illegible]D-01 ( 0.3[illegible]D-07)10000 01000     70   [illegible]  1111   [illegible]   [illegible]   1172 3  10540     586
( 0.3[illegible]D-01)  0.7[illegible]D-08 10000 01000    [illegible]  1075   [illegible]   137   [illegible]   1172 3  10540     586

  0.3[illegible]D-01 ( 0.7[illegible]D-08)10000 01000      1   [illegible]   [illegible]   110    [illegible]   1172 3  10540     586
( 0.3[illegible]D-01)  0.[illegible]D-08 10000 01000      1    43   677   [illegible]    77   1172 3  10540     586

  0.2[illegible]D-01 ( 0.12[illegible]D-07)10000 01000    [illegible]    52   [illegible]   179    22   1180 3  10620     590
( 0.3[illegible]D-01)  0.41[illegible]D-08 10000 01000    773   [illegible]    74   [illegible]    17   1180 3  10620     590
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  0.322006E-01 ( 0.104298E-07)10000 01000       1    211   861  1131   201   1100 3  10620       590
( 0.327738E-01)  0.947068E-08 10000 01000       1    453  1069   457   321   1100 3  10620       590

  0.299938E-01   0.639308E-08 10000 01000   1090  1044   547    23   566   1196 3  10764       590

  0.300538E-01   0.502378E-08 10000 01000       1    107   685   330   393   1196 3  10764       590
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                  n = 6
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         P2           P6        y1      y2                   s                          r   n    N    NO.RULES SEARCHED.

                                ------  ------   ---------------------------------
  0.266238E+00 ( 0.195478E-05)100000 010000   690   316   609   302   900   464   1126 3  10116      562
( 0.280008E+00)  0.122748E-05 100000 010000   907   172   560   904   763   316   1126 3  10116      562

  0.250038E+00 ( 0.506248E-06)100000 010000     1   361  1061   861   597   833   1126 3  10116      562
( 0.250008E+00)  0.440618E-06 100000 010000     1   221   569    89   561   361   1126 3  10116      562

  0.239058E+00   0.120718E-05 100000 010000   148   962   670   104   747   570   1132 3  10100      566

  0.280008E+00 ( 0.130178E-06)100000 010000     1   511   761   595   669  1127   1132 3  10100      566
( 0.280008E+00)  0.000008E-06 100000 010000     1   431   113    27   317   707   1132 3  10100      566

  0.230638E+00   0.120168E-05 100000 010000   314   202   562    56   836   865   1140 3  10332      574

  0.253718E+00 ( 0.961238E-06)100000 010000     1   137   601   901    61   765   1140 3  10332      574
( 0.256318E+00)  0.626578E-06 100000 010000     1   657  1061   421   601   109   1140 3  10332      574

  0.240108E+00 ( 0.415378E-05)100000 010000    30    05   435   790   806   834   1156 3  10404      578
( 0.280008E+00)  0.150618E-05 100000 010000   620   844  1030   503   750   903   1156 3  10404      578

  0.250008E+00 ( 0.120618E-06)100000 010000     1    71   417   797   689    39   1156 3  10404      578
( 0.266728E+00)  0.700618E-06 100000 010000     1   325   429   705   237   729   1156 3  10404      578

  0.237778E+00 ( 0.300618E-05)100000 010000   760   936   140   376   590   507   1172 3  10540      586
( 0.241308E+00)  0.050008E-06 100000 010000  1005   652   607   400   212   834   1172 3  10540      586

  0.260138E+00 ( 0.503778E-06)100000 010000     1   691   821  1115   141    83   1172 3  10540      586
( 0.280108E+00)  0.505008E-06 100000 010000     1   659   893   850   609   509   1172 3  10540      586

  0.230018E+00   0.100008E-05 100000 010000  1000  1176   492  1002   376   807   1180 3  10620      590

  0.247208E+00 ( 0.116618E-05)100000 010000     1   277    38   953   801   697   1180 3  10620      590
( 0.268018E+00)  0.606318E-06 100000 010000     1   960   501   109   861   329   1180 3  10620      590

  0.243578E+00 ( 0.174308E-05)100000 010000   160   202   962   300   570   309   1196 3  10764      590
( 0.257328E+00)  0.018308E-06 100000 010000   790   665   856   955  1036   200   1196 3  10764      590

  0.250918E+00   0.977958E-06 100000 010000     1   215   777   811   965  1051   1190 3  10764      590

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                  n = 7
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         P2           P6        y1       y2                    s                              r   n    N    NO.RULES SEARCHED.

                                -------  -------   -----------------------------------------
  0.104282E+01 ( 0.500308E-08)1000000 0100000   42  1000   689   800   982   980  1090   1126 3  10116     562
( 0.100148E+01)  0.672008E-06 1000000 0100000   21   306    77   710   763    64   826   1126 3  10116     562

  0.107308E+01   0.200648E-06 1000000 0100000    1   109   661   101   621   209   165   1126 3  10116     562
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  0.16000E+01 ( 0.63004E-03)1000000 0100000   306    4  959  710  702  506  374  1132 3  10100     566
( 0.16079E+01)  0.89114E-04 1000000 0100000   836  557  151 1001  612  769  605  1132 3  10100     566

  0.16523E+01 ( 0.15950E-03)1000000 0100000     1  217  677  801 1001 1005  741  1132 3  10100     566
( 0.16003E+01)  0.75274E-04 1000000 0100000     1  345  165  325   57  421  349  1132 3  10100     566

  0.16300E+01 ( 0.34735E-03)1000000 0100000   730  500  255  424 1070  302  120  1140 3  10332     574
( 0.15190E+01)  0.10000E-03 1000000 0100000   710  690  804 1022 1075  972  734  1140 3  10332     574

  0.15922E+01 ( 0.16045E-03)1000000 0100000     1  425  389   13  933  465  160  1140 3  10332     574
( 0.16000E+01)  0.12900E-03 1000000 0100000     1  207  373  295  221  975  925  1140 3  10332     574

  0.16054E+01 ( 0.18249E-03)1000000 0100000    72  514  206  824  654 1010  123  1156 3  10404     570
( 0.16200E+01)  0.98000E-04 1000000 0100000  1125  269  810 1130 1074  300  914  1156 3  10404     570

  0.16054E+01 ( 0.11509E-03)1000000 0100000     1  501  140  665  237  825  633  1156 3  10404     570
( 0.17100E+01)  0.10490E-03 1000000 0100000     1   73  705  601 1101  860  529  1156 3  10404     570

  0.16500E+01 ( 0.15120E-03)1000000 0100000   370  276 1151 1006 1010  700  320  1172 3  10540     586
( 0.16100E+01)  0.82190E-04 1000000 0100000   265  420  771  806  537  827 1109  1172 3  10540     586

  0.16030E+01 ( 0.23299E-03)1000000 0100000     1   25  625  309  349  521  133  1172 3  10540     586
( 0.16216E+01)  0.11700E-03 1000000 0100000     1  227 1133  523  340  609  653  1172 3  10540     586

  0.13000E+01 ( 0.10429E-03)1000000 0100000   300  576  424  806  536  506  170  1180 3  10620     590
( 0.15970E+01)  0.8151+E-04 1000000 0100000    72  500   87  404  161   83  525  1180 3  10620     590

  0.15019E+01 ( 0.14512E-03)1000000 0100000     1  809  901  349 1141  569  261  1180 3  10620     590
( 0.15000E+01)  0.79400E-04 1000000 0100000     1  477  969  833  861   57   40  1180 3  10620     590

  0.14207E+01 ( 0.14106E-03)1000000 0100000   132  804  940  500  904  906 1005  1196 3  10764     598
( 0.15579E+01)  0.58097E-04 1000000 0100000   773 1195  720  822  309  500  275  1196 3  10764     598

  0.15000E+01 ( 0.18000E-03)1000000 0100000     1  509   81 1000  301  153  417  1196 3  10764     598
( 0.16000E+01)  0.94200E-04 1000000 0100000     1  237 1153  573  653  477  625  1196 3  10764     598

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                  m = 8
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        F2            F6         y1       y2                    c                              r  s   n    NO.RULES SEARCHED

                                ------- -------    -----------------------------------------
  0.70000E+01 ( 0.36735E-02)10000000 01000000   832  736  500   29  370   62 1024 1102  1124 3  10116     562
( 0.88470E+01)  0.22970E-02 10000000 01000000   816  231 1076 1090  744  806  304  723  1124 3  10116     562

  0.87200E+01 ( 0.40005E-02)10000000 01000000     1    9   81  729  961  801  913  240  1124 3  10116     562
( 0.80087E+01)  0.41053E-02 10000000 01000000     1  323  307  815  209  967 1065  615  1124 3  10116     562

  0.77500E+01 ( 0.32182E-01)10000000 01000000   202  170  407 1000   54  300  512  800  1132 3  10100     566
( 0.88130E+01)  0.10630E-02 10000000 01000000   900  220  400  500  626  913 1067  297  1132 3  10100     566

  0.80070E+01   0.19874E-02 10000000 01000000     1  343    0  539   81  323  729  863  1132 3  10100     566

  0.72751E+01 ( 0.63737E-02)10000000 01000000   200 1020  272  936  722  760  790   77  1140 3  10332     574
( 0.80070E+01)  0.24000E-02 10000000 01000000   994  335  487  500  825  926 1082  381  1140 3  10332     574

  0.80126E+01   0.38730E-02 10000000 01000000     1   10  361 1110  587 1011  861 1065  1140 3  10332     574

  0.72000E+01 ( 0.44040E-02)10000000 01000000   974 1140 1134  802  196  296 1070  306  1156 3  10404     570
( 0.80000E+01)  0.26041E-02 10000000 01000000  1000  802  821  684  883  805  779  474  1156 3  10404     570

  0.80000E+01 ( 0.50000E-02)10000000 01000000     1  440  457  881  709  758    0  573  1156 3  10404     570
( 0.80090E+01)  0.10035E-02 10000000 01000000     1   53  487  989  781  833  807  149  1156 3  10404     570
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  0.735308E+01 ( 0.262087E-02)10000000 01000000    240   154  1050  1030   631   802   104   200  1172 3  10540      506
( 0.819827E+01)  0.220718E-02 10000000 01000000    701   700  1138   977   157   631   620  1031  1172 3  10540      506

  0.832038E+01 ( 0.724004E-02)10000000 01000000      1   339    85   939   709    91   377    55  1172 3  10540      506
( 0.835018E+01)  0.220656E-02 10000000 01000000      1   499   537   747    57   315   137   307  1172 3  10540      506

  0.716308E+01 ( 0.680087E-02)10000000 01000000    640   720   614   600   896  1043   534   814  1180 3  10620      590
( 0.804438E+01)  0.279908E-02 10000000 01000000    640  1030   974   703   871   620   154   661  1180 3  10620      590

  0.799838E+01 ( 0.200618E-02)10000000 01000000      1   423   749   507   501   703     9   267  1180 3  10620      590
( 0.813058E+01)  0.173038E-02 10000000 01000000      1   347    49   403    41    67   829   823  1180 3  10620      590

  0.708998E+01 ( 0.351638E-01)10000000 01000000   1150    60   720   470  1044   993   272   640  1196 3  10764      590
( 0.881778E+01)  0.224708E-02 10000000 01000000    107   765   801   791  1065   967   320   270  1196 3  10764      590

  0.788728E+01 ( 0.282540E-02)10000000 01000000      1   191   801  1171     0   523   625   971  1196 3  10764      590
( 0.788748E+01)  0.147068E-02 10000000 01000000      1   499   233   255   469   811   461  1181  1196 3  10764      590

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