after K. Hessenberg who, in his 1941 T. H. Darmstadt dissertation, developed a similarity transformation of an arbitrary matrix to "Hessenberg form" (cf. [1, pp. 314ff]).

In spite of these minor blemishes, the book is a welcome addition to the literature on basic numerical analysis and should especially appeal to mathematically mature students (who are conversant with the German language).

W. G.

1. R. Zurmühl, *Matrizen*, Springer, Berlin, 1950.

11[65M05, 65M10, 65M20, 76-08].—F. W. WUBS, *Numerical Solution of the Shallow-Water Equations*, CWI Tract 49, Centre for Mathematics and Computer Science, Amsterdam, 1988, iv+115 pp., 24 cm. Price Dfl17.80.

This tract is the summary of a research project on the shallow water equations from 1983–1988. The tract consists of two parts. The first part describes the numerical model used and its implementation on the Cyber 205 computer. The second part is a reprint of two papers. One is by Wubs on the stabilization of explicit methods by smoothing. The other paper is by Van der Houwen, Sommeijer, and Wubs on residual smoothing. The implicit smoothing used in the first paper is similar to that proposed both by Lerat and Jameson for fluid dynamic problems. It has also been used by Chima and Jorgenson for time-dependent problems. Here, Wubs describes some theory for this residual smoothing and considers applications to the shallow water equations. The second paper is an extension of the first; the authors derive smoothers for both hyperbolic and parabolic equations.

In the first section a staggered grid is introduced and the two velocity components and height are defined at different locations. A Cartesian grid is used, and so general boundaries are approximated by polygonal approximations. There is no discussion of using body-fitted coordinates. Both second-order and fourth-order central differences are constructed for this staggered mesh. Special formulas are needed near the boundaries. The finite difference equations are integrated in time by a Runge-Kutta scheme, and there is a short description of the stability theory, including the effects of residual smoothing. There is a detailed description of the vectorization of the code and its implementation on the Cyber 205. Finally, results are presented for the Taranto bay in Italy and for the Anno Friso Polder in the Netherlands.

The tract is mainly of interest for people in oceanography. It is nice, however, to see the interplay between this field and fluid dynamics in the use of residual smoothing. This may encourage additional interaction between these close fields. Finally, the description of the vectorization complements that

described in other papers and will give the reader details of the implementation on the Cyber 205, though that machine is no longer being produced.

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12[68T05, 65K10, 92-04].—EMILE AARTS & JAN KORST, *Simulated Annealing and Boltzmann Machines: A Stochastic Approach to Combinatorial Optimization and Neural Computing*, Wiley Interscience, New York, 1989, xii+272 pp., 24 cm. Price $49.95.

This book is an excellent introduction for mathematicians and physicists to the subjects of simulated annealing and Boltzmann machines. Furthermore, the discussion of Boltzmann machines provides a rigorous foundation with which to penetrate the very trendy subjects of "neural computing" and "neural networks". The book is divided into two sections, the first concentrating on the simulated annealing algorithm, and the second on aspects of Boltzmann machines, especially those pertaining to parallel and neural computation.

The authors motivate the simulated annealing algorithm as a method for solving problems of combinatorial optimization. These problems are generally considered to be very hard to solve, and in particular, all of the examples of combinatorial optimization problems in the book are from the class of NP-complete problems. The simulated annealing algorithm is then presented using the conceptual analogy of the algorithm to metallurgical annealing. The presentation is very general and only requires a minimization problem with a well-defined objective function, C, over a finite and discrete solution space that has a neighborhood structure. Within this mathematical framework the simulated annealing algorithm consists of proposing a neighboring configuration. The proposal is accepted if it either decreases the objective function, or, when the proposed configuration increases the objective function, a uniformly distributed random number chosen in $[0, 1]$ is greater than the value of $e^{-\Delta C/c}$. This is essentially the well-known Metropolis algorithm, where the constant c in the Boltzmann factor is the simulated annealing analog of temperature. This procedure, augmented with a sequence of c values going to zero, making certain that the algorithm reaches the equivalent of thermal equilibrium at every value of c, constitutes the simulated annealing algorithm. The decreasing sequence of c values is called a cooling schedule.

The discussion of the simulated annealing algorithm then continues with practical considerations, implementations of the algorithm for the NP-complete examples, analytic results, and numerical examples. There seems to be an extensive body of results concerning the global asymptotic convergence properties of the algorithm, and two very important results are presented in great detail. The first is an asymptotic result with the assumption of thermal equilibrium at each value of c. The second, more impressive result, shows that the global