imization of this function in the space of connection strength is shown to be equivalent to determining the optimal set of connection strengths for the desired equilibrium distribution. In the special case where the Boltzmann machine has no hidden units, it is proven that $D(q|q')$ is a strictly convex function with a single local minimum. This implies that a steepest descent approach to the minimization of the divergence function is guaranteed to converge. If a Boltzmann machine does have hidden units, $D(q|q')$ is no longer guaranteed to be convex, and heuristic approaches to its minimization are presented.

All in all, the presentation of the material in this book is very balanced. Rigorous results are presented, and an indication of what the authors believe to be the important open problems in the field are included. The Boltzmann machine serves as a fairly rigorous intellectual springboard into the much less rigorous field of neural networks and neural computing. For myself, I found this book an intellectually comforting introduction to this seemingly chaotic new discipline, which clearly marks out the firm ground and the quicksand.

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13[65-01, 65Fxx, 65Kxx].—PHILIPPE G. CIARLET, *Introduction to Numerical Linear Algebra and Optimisation*, Cambridge University Press, Cambridge, 1989, xiv+436 pp., 22 $\frac{1}{2}$ cm. Price $29.95.

This is what appears to be a straight translation of the French original, entitled "Introduction à l'analyse numérique matricielle et à l'optimisation", except that the exercises, which originally were published separately, are now incorporated in the same volume at the end of each subsection. For a review of the original text, see [1].

W. G.

1. V. Thomée, Review 5, Math. Comp. **42** (1984), 713–714.

14[65-00, 65-01, 65-04, 41-00, 41-01, 33-00].—B. A. POPOV & G. S. TESLER, *Computation of Functions on Electronic Computers–Handbook* (in Russian), Naukova Dumka, Kiev, 1984, 599 pp., 21 cm. Price 1 Ruble, 90 Kopecks.

For the user of modern computers or calculators of all sizes, the computation of values of elementary functions—and even of some special functions—has become a simple and common task. This fact, however, should not make us forget that a good deal of mathematics has had to be developed over the last few decades in order to establish the methods which ensure that these computations can be performed in a fast and accurate manner. Several handbooks have been published in English which treat the numerical computation of functions under different aspects, for example the manual of Cody and Waite [1], and the translation from the Russian of the handbook of Lyusternik et al. [2], for elementary functions, or the various books by Luke, in particular [3]. More than twenty years after the appearance of [2], Popov and Tesler have published, in Russian, another handbook for the computation of functions. Although some of the information contained in it can also be found elsewhere, its collection in one place and its concise presentation together with a treatment of the underlying theory, is undoubtedly valuable. This is even more true when one takes into account that many of the original papers published in Russian are not likely to be easily accessible elsewhere. It should be noted, however, that the authors were apparently unaware of the book by Cody and Waite [1].

The handbook consists essentially of two quite distinct parts. One part (Chapter 1 and Appendices 2 and 3) treats theoretical aspects of procedures which are useful for the approximation of functions, or is concerned with error analysis. The other part (Chapters 2 to 6) consists essentially of a collection of formulae, algorithms, and expansions (with their corresponding coefficients, graphs, etc.) for different classes of functions. Apart from the few short sections of text, a knowledge of Russian is not essential for understanding this part of the handbook.

In Chapter 1 (168 pages), a number of approximation methods are presented including approximation by polynomials or rational functions (e.g., Padé approximation), continued fractions, iterative processes, and piecewise approximation (e.g., splines). Nonlinear approximation and approximation by asymptotic series are also considered, as well as questions of economization and convergence acceleration. A fairly large part of this chapter is devoted to the presentation of a method which the authors call "expansion by residual values", which seems to have been developed mainly by one of the authors. Examples and tables of various kinds are interspersed in the text to illustrate the theory and facilitate applications.

Chapter 2 (109 pages) deals with the algorithmic computation of elementary functions, and Chapter 3 (61 pages) with the so-called "integral" functions, i.e., those which are obtained by nonelementary integration of elementary function, such as the error function. The gamma function is also discussed in this chapter, as well as some functions useful in statistics (the χ^2-, F-, and Student's t-distributions). Chapter 4 (62 pages) is concerned with the family of Bessel functions (of integer and fractional order). Chapter 5 (36 pages) discusses the elliptic integrals and elliptic functions (Jacobi, Weierstrass, and Theta). The last Chapter 6 (39 pages) gives information about some classes of orthogonal polynomials (Jacobi, Gegenbauer, Legendre, Laguerre, Hermite, and Chebyshev). In connection with Chebyshev polynomials, methods for the economization of power series and the Lanczos τ-method are discussed. Euler and Bernoulli polynomials complete this chapter.

It is important to note that most of the computational methods presented in Chapters 2 to 6 refer to functions of real variables. Those algorithms for the elementary functions which are actually implemented on a large number of computers (in particular on those in use in the socialist countries) are given in Appendix 1. Appendix 2 discusses questions of error analysis and Appendix 3 problems of optimization. Appendix 4 gives some tables of useful constants.

An impressive bibliography of more than 400 references, about 60 percent of them referring to publications in Russian, and an index, complete the volume.

Unfortunately, access to this useful handbook is likely to be difficult outside the Soviet Union. In view of the amount of information it contains, one could imagine that a (perhaps updated) English edition would be a valuable complement to the already existing handbooks.

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1. W. J. Cody and W. Waite, *Software manual for the elementary functions*, Prentice-Hall, Englewood Cliffs, N.J., 1980.
2. L. A. Lyusternik, O. A. Chervonenkis, and A. R. Yanpol′skii, *Handbook for computing elementary functions*, Pergamon Press, Oxford, 1965.
3. Y. L. Luke, *Mathematical functions and their approximations*, Academic Press, New York, 1975.

15[68Mxx, 68Q20, 65V05, 65-04].—Jean-Michel Muller, *Arithmétique des ordinateurs—Opérateurs et fonctions élémentaires*, Etudes et recherches en informatique, Masson, Paris, 1989, 214 pp., 25 cm. Price 215 FF.

This book provides a good introduction to the subject of computer arithmetic, covering most of the major areas in an easy manner. The style is pleasantly narrative—but like many narrative novels—it leaves the reader somewhat disappointed at its lack of depth and detail in places. There are chapters on each of the important topics—Boolean logic, number representations, addition, multiplication, division, and elementary function evaluation. Each has its strengths and its weaknesses.

There are also major omissions. Most notably, perhaps, are the very scant treatment of noninteger representations and arithmetic, and the consequent almost total absence of a discussion of errors in computer arithmetic. Another serious lack, since the book is purported to be a text for courses in this subject, is that there are *no* exercises for the student/reader.

Nonetheless, the overall blend of mathematical theory with some of the practicalities of hardware implementation of algorithms is pleasing. I just wish it could have been expanded to a full and comprehensive treatment of these important topics.

The first chapter (9 pages) provides the reader with a very brief review of Boolean algebra and the various logic gates that are used in integrated circuits.