

## CORRIGENDA

CHARLES CHUI & TIAN XIAO HE, *On the dimension of bivariate superspline spaces*, Math. Comp. **53** (1989), 219–234.

We are grateful to Carla Manni for pointing out an error in the proof of Lemma 1. This necessitates corrections in the statement of Lemma 1 and its consequences. In addition to the notations used in the paper, we set

$$\begin{aligned} \beta_d^r &= \left[ \binom{d-r+1}{2} - 2 \binom{r+1}{2} + \binom{(r-(d-2r)_+ + 1)_+}{2} \right]_+, \\ \gamma_d^r &= \left[ \binom{d+2}{2} - \binom{2r+2}{2} \right]_+, \\ \sigma_d^r(n) &= \begin{cases} \sum_{j=r+1}^{d-r} [r+j+1-nj]_+ & \text{for } d \geq 3r+1, \\ \sum_{j=r+1}^{d-r} [r+j+1-n(j+d-3r-1)]_+ & \text{for } 2r+1 \leq d \leq 3r, \\ 0 & \text{otherwise.} \end{cases} \end{aligned}$$

The correct formulation of Lemma 1 is then as follows.

**Lemma 1.** *Let  $\Delta_0$  be a rectilinear grid partition, as described. Then*

$$\dim \widehat{S}_d^r(\Delta_0) = \binom{d+2}{2} + [n_0 \beta_d^r + \sigma_d^r(N_0) - \gamma_d^r]_+.$$

In particular, it is clear that for  $d \geq 4r+1$  or  $d \leq 2r$  the formulation of this lemma reduces to the formulation of Lemma 1 in the original paper. For  $2r+1 \leq d \leq 4r$ , the mistake in the proof of Lemma 1 occurs in the formulation of equation (24). A correct statement is the following. For  $d \geq 3r+1$ , we have

$$\begin{aligned} \text{rank } H' &= \min \left( \sum_{j=r+1}^{d-2r} \text{rank } H_j + \sum_{j=d-2r+1}^{d-r} \text{rank}[B_{r+1}^j \cdots B_{j-1}^j \overline{H}_j], n_0 \beta_d^r \right) \\ &= \min \left( \sum_{j=r+1}^{d-r} (r+j+1 - [r+j+1 - N_0 j]_+), n_0 \beta_d^r \right) \\ &= n_0 \beta_d^r - [n_0 \beta_d^r - \gamma_d^r + \sigma_d^r(N_0)]_+. \end{aligned}$$

For  $2r < d < 3r + 1$ , we have

$$\begin{aligned} \text{rank } H' &= \min \left( \text{rank } \overline{H}_{r+1} + \sum_{j=r+2}^{d-r} \text{rank}[D_{r+1}^j \cdots D_{j-1}^j \overline{H}_j], n_0 \beta_d^r \right) \\ &= n_0 \beta_d^r - [n_0 \beta_d^r - \gamma_d^r + \sigma_d^r(N_0)]_+. \end{aligned}$$

The above reformulation of Lemma 1 entails changes in the statements of Lemma 2, Theorems 1, 2, 3, 4 and three of the corollaries. The correct statements of these results are given below.

**Lemma 2.** *Let  $\Delta_1$  be a rectilinear grid partition and  $\varepsilon = 0$  or  $1$  as described. Then*

$$\dim \widehat{S}_d^r(\Delta_1) = \binom{d+2}{2} + E_I \beta_d^r - \varepsilon[\gamma_d^r - \sigma_d^r(N_0)] + (\varepsilon[\gamma_d^r - \sigma_d^r(N_0)] - n_0 \beta_d^r)_+.$$

**Theorem 1.** *Let*

$$D_d^r = \dim \widehat{S}_d^r(\Delta) - \binom{d+2}{2} - E_I \beta_d^r + V_I \gamma_d^r.$$

*Then*

$$\sum_{i=1}^{V_I} \sigma_i \leq D_d^r \leq \sum_{i=1}^{V_I} \tilde{\sigma}_i,$$

where  $\sigma_i = \sigma_d^r(e_i) + (\gamma_d^r - \sigma_d^r(e_i) - d_i \beta_d^r)_+$ ,  $\tilde{\sigma}_i = \sigma_d^r(\tilde{e}_i) + (\gamma_d^r - \sigma_d^r(\tilde{e}_i) - \tilde{d}_i \beta_d^r)_+$  with  $d_i$  and  $e_i$  denoting the number of edges and the number of edges with different slopes attached to the vertex  $A_i$ , respectively, and  $\tilde{d}_i$  and  $\tilde{e}_i$  denoting the number of edges and the number of edges with different slopes attached to the vertex  $A_i$  but not  $A_j$ ,  $j < i$ , respectively, where  $i = 1, \dots, V_I$ . In particular, for  $d \geq 3r + 1$  and all  $\tilde{e}_i \geq 2$ ,

$$\dim \widehat{S}_d^r(\Delta) = \binom{d+2}{2} + E_I \beta_d^r - V_I \gamma_d^r.$$

**Theorem 2.** *Let*

$$F_d^r(n) := (n \beta_d^r + \sigma_d^r(n) - \gamma_d^r)_+.$$

*Then*

$$\dim \widehat{S}_d^r(\Delta_c) = \binom{d+2}{2} + L \beta_d^r + \sum_{i=1}^{V_I} F_d^r(l_i).$$

**Corollary 1.** *Let  $d \geq 3r + 1$ . Then*

$$\begin{aligned} \dim \widehat{S}_d^r(\Delta_{mn}^{(2)}) &= mn \left[ 6 \binom{d-r+1}{2} - 12 \binom{r+1}{2} - 2 \binom{d+2}{2} + 2 \binom{2r+2}{2} \right] \\ &+ (m+n) \left[ 5 \binom{d-r+1}{2} - 10 \binom{r+1}{2} - \binom{d+2}{2} + \binom{2r+2}{2} \right] \\ &+ \binom{2r+2}{2} + 4 \binom{d-r+1}{2} - 8 \binom{r+1}{2}. \end{aligned}$$

**Corollary 2.** Let  $\Delta_{mn}^{(1)}$  be a uniform type-1 triangulation of  $\Omega_R$ . Then

$$\dim \widehat{S}_d^r(\Delta_{mn}^{(1)}) = \binom{d+2}{2} + mn[3\beta_d^r - \gamma_d^r + \sigma_d^r(3)] + (2m + 2n + 1)\beta_d^r.$$

**Corollary 3.** Let  $\Delta_{mn}^{(2)}$  be a uniform type-2 triangulation of  $\Omega_R$ . Then

$$\begin{aligned} \dim \widehat{S}_d^r(\Delta_{mn}^{(2)}) &= \binom{d+2}{2} + mn[6\beta_d^r - 2\gamma_d^r + \sigma_d^r(2) + \sigma_d^r(4)] \\ &\quad + (m+n)[5\beta_d^r - \gamma_d^r + \sigma_d^r(2)] + 4\beta_d^r - \gamma_d^r + \sigma_d^r(2). \end{aligned}$$

In the following, we need the notations:

$$\begin{aligned} \beta_d^{r,\rho} &= \left[ \binom{d-r+1}{2} - 2 \binom{\rho-r+1}{2} + \binom{(\rho-r-(d-\rho)_++1)_+}{2} \right]_+, \\ \gamma_{d,\rho} &= \left[ \binom{d+2}{2} - \binom{\rho+2}{2} \right]_+, \\ \sigma_d^{r,\rho}(n) &= \begin{cases} \sum_{j=\rho-r+1}^{d-r} (r+j+1-nj)_+ & \text{for } d > 2\rho-r, \\ \sum_{j=\rho-r+1}^{d-r} [r+j+1-n(j+d-2\rho+r-1)]_+ & \text{for } \rho < d \leq 2\rho-r, \\ 0 & \text{otherwise.} \end{cases} \end{aligned}$$

**Theorem 3.** Let

$$D_d^{r,\rho} = \dim S_d^{r,\rho}(\Delta) - \binom{d+2}{2} + V_I \gamma_{d,\rho} - E_I \beta_d^{r,\rho}.$$

Then

$$\sum_{i=1}^{V_I} \sigma_i^\rho \leq D_d^{r,\rho} \leq \sum_{i=1}^{V_I} \tilde{\sigma}_i^\rho,$$

where  $\sigma_i^\rho = \sigma_d^{r,\rho}(e_i) + (\gamma_{d,\rho} - \sigma_d^{r,\rho}(e_i) - d_i \beta_d^{r,\rho})_+$ ,  $\tilde{\sigma}_i^\rho = \sigma_d^{r,\rho}(\tilde{e}_i) + (\gamma_{d,\rho} - \sigma_d^{r,\rho}(\tilde{e}_i) - \tilde{d}_i \beta_d^{r,\rho})_+$ , and  $e_i, d_i, \tilde{e}_i$  and  $\tilde{d}_i$  have been defined in Theorem 1.

**Theorem 4.** Let  $\rho \geq r$  and

$$F_d^{r,\rho}(n) := (n\beta_d^{r,\rho} + \sigma_d^{r,\rho}(n) - \gamma_{d,\rho})_+.$$

Then

$$\dim S_d^{r,\rho}(\Delta_c) = \binom{d+2}{2} + L\beta_d^{r,\rho} + \sum_{i=1}^{V_I} F_d^{r,\rho}(l_i).$$

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