27[35–02, 73–02, 76–02, 22Exx].—C. ROGERS & W. F. AMES, *Nonlinear Boundary Value Problems in Science and Engineering*, Mathematics in Science and Engineering, vol. 183, Academic Press, Boston, 1989, xiii + 416 pp., 23 $\frac{1}{2}$ cm. Price \$69.96.

Has the ascendancy of the computer and numerical analysis in the late twentieth century signalled the end of the era of exact solutions of differential equations? On the contrary! Not only is the subject alive, but it is currently in the midst of an incredible renaissance, with many new methods, new solutions and new insights appearing in the last 10–20 years. Indeed, exact solutions still remain the touchstone of physical applications, and, even more to the point, are also crucial in the proper development of numerical methods. Explicit exact solutions (even nonphysical ones) are crucial tools for checking the accuracy of numerical integration schemes, and serve as key benchmarks for comparison and evaluation of competing numerical packages. New physical phenomena (e.g., shock waves, black holes, interacting solitons, cavitation of elastic materials, scattering phenomena, etc.) are often first detected or are epitomized in their simplest form by suitable exact solutions of particular model systems. (Nobel prizes have been awarded for exact solutions of the equations of physics!) Moreover, exact solutions often serve as asymptotics for more complicated solutions after the decay of irrelevant transients. The discovery of solitons, the application of Lie group methods, Bäcklund and reciprocal transformations, canonical form theory, Bergman series, etc., etc., have all vastly enlarged the researcher's arsenal of techniques. Moreover, the recent arrival of sophisticated and powerful computer algebra systems will provide yet another stimulus to the continued development and expansion of the methods to yet more complicated problems.

The book under review is a paean to the exact solution, supplied with a cornucopia of examples, methods, and physical applications. It contains a wealth of interesting and unusual examples of special solution techniques for a variety of boundary value problems arising in a wide range of physical systems, including fluid mechanics, elasticity, heat conduction, gas dynamics, metereology and so on. It is divided into four chapters: Bäcklund and reciprocal transformations, Bergman expansion methods in moving boundary and Stefan problems, model constitutive laws of elasticity, and applications of symmetry group methods, although the different methods often interact in unexpected ways. Of particular interest to those interested in computations are sections on the use of group methods to compute eigenvalues of nonlinear boundary value problems by exact shooting, and to devising numerical finite difference schemes incorporating group invariance. There is a useful appendix listing the symmetry groups of a large number of physically interesting problems, and an excellent list of references to recent research papers.

In the broad spectrum of mathematical exposition, this book lies almost at the polar opposite to the Bourbaki approach: instead of empty theory and abstract generalizations, the book develops its topics by relying almost exclusively on particular examples. Each chapter consists of a number of sections, each of which presents a particular physical problem, some special techniques to generate solutions, and, often, physical consequences of the results. My own pedagogical tastes certainly run towards the particular, well-chosen example, although I felt this was perhaps carried to an extreme here. What is often lacking is any kind of general framework for the particular methods introduced, or a discussion of how to ever decide which of the many techniques available to apply to a new problem. Of course, in many cases, this approach is necessitated by the nature of the subject; many of the methods only work in particular instances, and (as in much of applied mathematics) one learns primarily through example. Only in the final section on symmetry groups is there an attempt to develop a general theory which can be readily ported to other contexts. Students especially will profit from the wide repertoire of methods and applications, although I would find it hard to use this book in a course other than as a supplement to more systematic texts. Nevertheless, I can recommend the book to anyone seeking to enlarge their "bag of tricks" for tackling complicated nonlinear problems.

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28[15–02, 65F50].—I. S. DUFF, A. M. ERISMAN & J. K. REID, *Direct Methods for Sparse Matrices*, Monographs on Numerical Analysis, Clarendon Press, Oxford University Press, New York, 1989, xiv + 341 pp., 23 $\frac{1}{2}$ cm. Price $22.50 paperback.

This is a paperback edition (with corrections) of the 1986 edition of the book. See [1] for a review of the original edition.

W. G.

1. K. Turner, Review 3, Math. Comp. **52** (1989), 250–252.

29[53–01, 65D05, 65D07, 65D10, 68U05].—GERALD FARIN, *Curves and Surfaces for Computer Aided Geometric Design—A Practical Guide*, Computer Science and Scientific Computing, Academic Press, Boston, 1988, xv + 334 pp., 23 $\frac{1}{2}$ cm. Price $39.95.

This book consists of a collection of material on parametric curves and surfaces used in fields known variously as "Computer Aided Design" and "Computer Aided Geometric Design". The topics covered are, in order of the chapters: the de Casteljau algorithm, Bézier curves, polynomial interpolation, B-spline curves, geometric continuity for curves, conic sections, rational Bézier and B-spline curves, tensor product and composite surface patches. Included