give more explicit constants than use of the Bramble-Hilbert lemma). As the author remarks, p. 127: "Waning sadism forbids us to go further", which can be taken as a nice comment on the effort-, ink-, and tree-saving role of the Bramble-Hilbert lemma. (That role was duly appreciated at the time, e.g., in [3, p. 146].)

Secondly, in the analysis of the finite element method in one space dimension, the author derives maximum norm error estimates that are off by one full order of accuracy, pp. 215 and 221–222. The case on pp. 221–222 was actually solved by Wheeler [4] in 1973, while the general case, including that on p. 215, came later [1].

Finally, reprinting good books is an idea that deserves strong applause from the mathematical community. In numerical analysis, will Richtmyer and Morton's 1967 book [2] be next?

L. B. W.

1. J. Douglas, Jr., T. Dupont, and L. B. Wahlbin, *Optimal L_∞ error estimates for Galerkin approximations to solutions of two-point boundary value problems*, Math. Comp. **29** (1975), 475–483.

2. R. D. Richtmyer and K. W. Morton, *Difference methods for initial-value problems*, 2nd ed. Interscience, New York, 1967.

3. G. Strang and G. J. Fix, *An analysis of the finite element method*, Prentice-Hall, Englewood Cliffs, N.J., 1973.

4. M. F. Wheeler, *An optimal L_∞ error estimate for Galerkin approximations to solutions of two-point boundary value problems*, SIAM J. Numer. Anal. **10** (1973), 914–917.

8[41-02, 41A29, 41A50, 41A52, 41A65, 65D15].—ALLAN M. PINKUS, *On L^1-approximation*, Cambridge Tracts in Mathematics, Vol. 93, Cambridge Univ. Press, Cambridge, 1989, x + 239 pp., $23\frac{1}{2}$ cm. Price $44.50.

We welcome this book as the first comprehensive monograph on approximation in the mean. It merits much praise for being all that such a work should be: it takes a global viewpoint and proceeds systematically and efficiently through the entire subject. All the classical results are here—often in generalized form and with improved proofs. Fully half the book is devoted to the progress made in the last ten years. The author has played a leading role in all this recent activity and is uniquely qualified to be its chronicler and interpreter.

Mean approximation (or L^1-approximation) is the problem of minimizing the expression $\int |f - u|$ by choosing u from some given class of functions. Here, f is a function to be approximated, and the integral is over a fixed measure space. A typical example occurs when f is a continuous function on a closed and bounded interval of the real line, and u is chosen from the family of cubic spline functions with a prescribed set of knots. In this case the integral could be the usual Lebesgue or Riemann integral on the interval, but much more general measures are admitted in the theory.

In the general theory, if the approximating function u is further constrained to satisfy (for all x) $f(x) \leq u(x)$, the problem is, of course, changed, but the theory for such one-sided L^1-approximations is well developed.

The first Chapter ("Preliminaries") gives a succinct account of basic approximation theory. It includes characterization theorems for best approximations in normed linear spaces, allowing arbitrary convex sets of approximants. These theorems are couched in two different forms—either with separating linear functionals or with one-sided Gateaux derivatives. The author includes the recent theory of "strong unicity".

Chapter 2 concerns approximation from finite-dimensional subspaces in a space $L^1(B, \Sigma, \nu)$, where (B, Σ, ν) is a σ-finite measure space. This includes as a special case the space l^1, and particular results for that space are mentioned. In these general spaces, best approximations are usually *not* unique, and continuous best-approximation maps usually do not exist.

The bad features of the problems discussed in Chapter 2 are largely avoided in Chapter 3, by moving the setting of the problems to the normed linear space $C_1(K, \mu)$. This space consists of continuous functions on a compact space K, but the norm is the L^1 norm induced by a nonatomic measure, μ. The topology and the measure are related by requiring that all nonempty open sets be measurable and have positive measure. Characterization and unicity of best approximations are dealt with, as are continuous best-approximation maps.

In Chapter 4, the unicity question is investigated in greater detail in order to discover subspaces that have the unicity property for large classes of measures. Splines and their generalizations provide interesting examples of the theory developed here.

Chapter 5 is devoted to one-sided approximation, and Chapter 6 to discrete L^1-approximation, i.e., approximation in l_1^m.

Chapter 7, of 40 pages, is devoted to algorithms for L^1-approximation. Here the discussion is aimed at strategies and procedures, not computer programs. Methods of descent, linear programming, discretization, and the Newton procedure are all discussed.

The book closes with two lengthy appendices, dealing principally with Chebyshev systems and weak Chebyshev systems of functions. At the end of each chapter there are copious notes and references to the literature. A bibliography of eight pages is included.

Altogether, this monograph is an important addition to the literature on approximation theory.

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