

at the end are not the same as the x and y at the beginning, and the second because of missing quantifiers. Moreover, the proof that follows contains a gap: one needs to know that a nontrivial divisibility $p|x^2 + y^2$ implies that $p \equiv 1 \pmod{4}$, since otherwise (middle of p. 11) one cannot rule out the possibility that N is the product of p and a prime $q \equiv 3 \pmod{4}$.

This type of imprecision, while not likely to discourage a sophisticated reader, does diminish the book's value for undergraduates.

To summarize, David Cox's book is an excellent textbook and reference for people at the graduate level and above.

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20[11-00, 65-00].—JONATHAN BORWEIN & PETER BORWEIN, *A Dictionary of Real Numbers*, Wadsworth & Brooks/Cole Advanced Books & Software, Pacific Grove, California, 1990, viii + 424 pp., 28½ cm. Price \$69.95.

This book is quite accurately described by its title. The authors have catalogued approximately 100,000 "interesting" real numbers, indexing them lexicographically according to the first eight digits following the decimal point. Both pure mathematical constants and some constants of physics are included.

Such a listing could be quite valuable as a reference book for a mathematician, computer scientist or physicist who in the course of a calculation, either theoretical or empirical, produces some number whose identity is not known. If such a number is found in this listing, and computation to higher precision confirms its equality to the constant in the list, then one can pursue a rigorous proof with a high level of confidence. In other words, a listing of this sort could enable mathematicians to employ "experimental" techniques in their research.

To their credit, the authors give a definition in the Introduction of exactly what numbers are included in the list. It is worth briefly summarizing this definition. Their listing is based on a "standard domain" of 4,258 numbers. These numbers consist of

1. Certain simple rational numbers.
2. Rational multiples of certain irrational and transcendental constants.
3. Square roots and cube roots of small integers and simple rational numbers.
4. Elementary functions evaluated at certain simple values.
5. Sums and differences of square roots of certain small integers.
6. Rational combinations of certain constants.
7. Euler's constant, Catalan's constant, and some constants from physics.

All other numbers in the listing are either combinations of values of elementary functions, real roots of cubic polynomials with small coefficients, or special functions evaluated at numbers in the "standard domain" defined above.

As an exercise in evaluating how useful this listing might be in actual practice, this reviewer made a listing of 16 real constants (before studying the authors' definition of the book's contents). Of these, all but four were actually indexed in the book. It is worth mentioning, and briefly discussing, the four exceptions:

1. 4.6692016091 = Feigenbaum's δ constant (from the theory of chaos). This constant was not included in the standard domain.
2. 14.1347251417 = the imaginary part of the first zero of Riemann's ζ function. Note the "Real" in the title of the book.
3. $0.4619397663 = \frac{1}{2} \sin(3\pi/8) = \frac{1}{4} \sqrt{2 + \sqrt{2}}$. Twice this constant is included.
4. $0.9772498680 = P(2)$, where $P(\cdot)$ denotes the cumulative Gaussian probability function. Twice this constant is included, since $P(x) = \frac{1}{2}[1 + \operatorname{erf}(x/\sqrt{2})]$, and the values of erf are indexed.

This exercise underscores both the strengths and the weaknesses of the book. On one hand, it appears that most numbers ordinarily encountered in mathematics are included. However, there are some holes in the list. Most notable are those cases where the constant one is looking for differs from some special function result by a simple rational factor, such as one-half.

Twenty years ago, tables of mathematical functions were widely used by both pure and applied mathematicians. In the intervening years such compilations have been rendered obsolete by the widespread availability of scientific calculators and subroutine libraries that can evaluate even fairly esoteric functions. One wonders if eventually the same fate will come to a book such as this. Already part of this table, namely the tabulation of roots of simple polynomials, has been rendered obsolete by the availability of relation-finding algorithms in packages such as Mathematica. These routines detect polynomial roots by searching for small integer relations in the vector $(1, x, x^2, x^3, \dots, x^{n-1})$. Perhaps eventually such routines can be expanded in power to search for many other possible relationships. But in the meantime the Borwein book is all we have.

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21[11-01, 11B39].—S. VAJDA, *Fibonacci & Lucas Numbers, and the Golden Section: Theory and Applications*, Ellis Horwood Series in Mathematics and its Applications, Wiley, New York, 1989, 190 pp., $24\frac{1}{2}$ cm. Price \$64.95.

Although there is a journal devoted to Fibonacci numbers and their generalizations, few monographs deal with this subject alone. There is a short book about Fibonacci and Lucas numbers by V. E. Hoggatt, Jr. [1] and a little book