# REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1980 Mathematics Subject Classification (1985 Revision) can be found in the December index volumes of Mathematical Reviews.

**24[41-02, 65Dxx].**—GÜNTHER NÜRNBERGER, *Approximation by Spline Functions*, Springer, Berlin, 1989, xi + 243 pp., 25 cm. Price $39.50.

In the past 30 years, there has been an explosive development in the theory of spline functions (piecewise polynomials and generalizations thereof) and their applications. This great interest is due in no small part to the fact that splines are ideal tools for use in approximating functions, and for designing methods for solving problems numerically. While a number of monographs on splines have appeared in the past 10 years, much of the theory has not yet appeared in book form. This book helps fill this gap by providing a comprehensive development of *best approximation* by splines, along with some related topics.

The book is divided into three parts. The first part (approx. 80 pp.) is devoted primarily to various types of Chebyshev systems and their properties. The topics discussed include divided differences, interpolation, existence, uniqueness, strong uniqueness, characterization, and computation of best approximations in each of the $L_1$, $L_2$, and $L_\infty$ norms. Much of this material has been treated in other books, but the unified treatment given here (most of the results are given full and detailed proofs) provides a useful basis for the study of the rest of the book.

The second part of the book (approx. 100 pp.) deals with various types of Weak Chebyshev systems, and in particular with polynomial splines as their most important example. First, the basic properties of splines (including B-splines, zero properties, etc.) are developed. Then several special interpolation methods are discussed. The most interesting sections deal with best approximation by splines with fixed knots. In addition to the usual questions of existence, uniqueness, strong uniqueness, and characterization, the problems of continuity of the metric projection, the existence of a continuous selection for it, one-sided $L_1$ approximation and its connection with Gauss quadrature formulae, and the optimal approximation of linear functionals are all treated in detail. A Remez algorithm for finding best spline approximants in the uniform norm is also presented. Many of the results in this part of the book are quite new.

The third part of the book is in the form of an extended Appendix (approx. 30 pp.), divided into three sections. The first section sketches the theory of best


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approximation by splines with free knots. The second section outlines ways of defining bivariate splines, including tensor-products, blending, simplex splines, and piecewise polynomial spaces on triangulations and other partitions. The final section deals with the solution of ODE's by collocation. This part of the book contains no proofs, but does cite a considerable number of references. A bibliography of approximately 400 references is included.

This book should be of interest to researchers in the theory of splines, as well as to users of splines in approximation and numerical analysis. It can be read by anyone with a good background in elementary analysis. The material is well organized, and the text reads very smoothly.

L.L.S.

**25[94-01, 65T05, 68Qxx, 94A11].**—RICHARD TOLIMIERI, MYOUNG AN & CHAO LU, *Algorithms for Discrete Fourier Transform and Convolution*, Springer, New York, 1989, xv + 350 pp., 24 cm. Price $59.00.

Discrete Fourier transforms and finite convolutions form a mainstay of digital signal processing algorithms. Ever since the discovery of the Cooley-Tukey fast Fourier transform, there has been a flurry of activity in designing efficient algorithms for finite harmonic analysis, and these algorithms have found applications far beyond the realm of digital signal processing. There is now a growing list of monographs devoted to these algorithms; we mention the widely used references Blahut [1] and Nussbaumer [3].

The book under review presents a unified approach to many fast Fourier transform and convolution algorithms, using matrix factorizations and tensor products of matrices as the common themes. The individual steps in such an algorithm are viewed as matrix operations, and the full algorithm amounts then to a matrix factorization, typically involving diagonal matrices, permutation matrices, and tensor products of relatively simple matrices. Conversely, matrix factorizations of an appropriate type can be translated into algorithms. This provides a systematic and mathematically appealing framework for the design of discrete Fourier transform and convolution algorithms. This approach also makes it easy to switch from parallelized to vectorized algorithms and vice versa, as one can move from one to the other essentially by transposition of the matrix factorization and by application of a commutation theorem for tensor products of matrices. This allows an adaptation of the algorithms to the available computer architecture.

After the necessary background on ring and field theory and on tensor products of matrices, a detailed discussion of the Cooley-Tukey FFT algorithm from the viewpoint described above is given. Several variants of the Cooley-Tukey algorithm, including those of Gentleman-Sande, Pease, and Korn-Lambiotte, are also presented. These algorithms make use of the additive structure of residue class rings of integers. The Good-Thomas algorithm is described as an exam-