approximation by splines with free knots. The second section outlines ways of defining bivariate splines, including tensor-products, blending, simplex splines, and piecewise polynomial spaces on triangulations and other partitions. The final section deals with the solution of ODE's by collocation. This part of the book contains no proofs, but does cite a considerable number of references. A bibliography of approximately 400 references is included.

This book should be of interest to researchers in the theory of splines, as well as to users of splines in approximation and numerical analysis. It can be read by anyone with a good background in elementary analysis. The material is well organized, and the text reads very smoothly.

L.L.S.

25[94-01, 65T05, 68Qxx, 94A11].—RICHARD TOLIMIERI, MYOUNG AN & CHAO LU, *Algorithms for Discrete Fourier Transform and Convolution*, Springer, New York, 1989, xv + 350 pp., 24 cm. Price $59.00.

Discrete Fourier transforms and finite convolutions form a mainstay of digital signal processing algorithms. Ever since the discovery of the Cooley-Tukey fast Fourier transform, there has been a flurry of activity in designing efficient algorithms for finite harmonic analysis, and these algorithms have found applications far beyond the realm of digital signal processing. There is now a growing list of monographs devoted to these algorithms; we mention the widely used references Blahut [1] and Nussbaumer [3].

The book under review presents a unified approach to many fast Fourier transform and convolution algorithms, using matrix factorizations and tensor products of matrices as the common themes. The individual steps in such an algorithm are viewed as matrix operations, and the full algorithm amounts then to a matrix factorization, typically involving diagonal matrices, permutation matrices, and tensor products of relatively simple matrices. Conversely, matrix factorizations of an appropriate type can be translated into algorithms. This provides a systematic and mathematically appealing framework for the design of discrete Fourier transform and convolution algorithms. This approach also makes it easy to switch from parallelized to vectorized algorithms and vice versa, as one can move from one to the other essentially by transposition of the matrix factorization and by application of a commutation theorem for tensor products of matrices. This allows an adaptation of the algorithms to the available computer architecture.

After the necessary background on ring and field theory and on tensor products of matrices, a detailed discussion of the Cooley-Tukey FFT algorithm from the viewpoint described above is given. Several variants of the Cooley-Tukey algorithm, including those of Gentleman-Sande, Pease, and Korn-Lambiotte, are also presented. These algorithms make use of the additive structure of residue class rings of integers. The Good-Thomas algorithm is described as an example of an algorithm based on the multiplicative structure of such rings. Two chapters are devoted to linear and cyclic convolution. Linear convolution is identified with polynomial multiplication, and cyclic convolution is identified with polynomial multiplication modulo a binomial $x^N - 1$. The basic tools for the design of convolution algorithms are the convolution theorem and the Chinese remainder theorem. The Cook-Toom algorithm, the Winograd small convolution algorithm, and the Agarwal-Cooley algorithm are the principal convolution algorithms that are discussed. Convolution algorithms can be applied to the design of multiplicative Fourier transform algorithms. This was first observed by Rader who showed that for a prime p, a p-point Fourier transform can be computed by a $(p-1)$-point cyclic convolution. This principle can be extended to N-point Fourier transforms for various values of N, such as prime powers N or squarefree N. The duality between periodic and decimated data established by the Fourier transform is studied and applied to the computation of N-point Fourier transforms, where N is an odd prime power. Fourier transforms of multiplicative characters $\bmod N$ and orthogonal bases diagonalizing Fourier transform matrices are also discussed.

The underlying idea of the book, namely to present the algorithms for finite harmonic analysis from the systematic viewpoint of matrix factorizations, is certainly attractive. However, the execution of this idea leaves a lot to be desired. The writing is repetitious and tedious, and the approach is numbingly slow. For instance, an algorithm might first be presented for $N = 15$, then for N a product of two distinct primes, then for N a product of three distinct primes, and finally for the general case of squarefree N, always with the same arguments and usually with the same wording. The book could easily be streamlined to half its size without any loss of information. A further irritant is the sloppy attitude that pervades the book. On p. 79 the authors speak of the "following diagram" and on pp. 271 and 274 of the " following table", but the reader will look in vain for any of these. On p. 279, Problems 6 and 7 refer to tables that do not exist. The definition of a subfield on p. 23 is worth repeating: "F is a subfield of K in the sense that F is a subset of K containing 1, and it is closed under the addition and multiplication in K". According to this "definition", the natural numbers would form a subfield of the reals! The distinction between "relatively prime" and "pairwise relatively prime" is often not made. In the statement of the unique factorization theorem for polynomials on p. 22 one has to assume that the irreducible factors are monic in order to guarantee uniqueness up to the ordering of factors. On p. 16 the authors speak of the "Euler quotient function" instead of the Euler totient function. There are many misprints and even some systematic spelling errors. No serious attempt is made to assign proper credit for the results that are presented. For instance, with respect to the construction of orthogonal bases diagonalizing Fourier transform matrices only papers from the 1980's are quoted, although it is well known that this was first achieved by McClellan and Parks [2] and that the basic ideas go back to Schur [4]. Overall, the careless and repetitious style takes away a lot of the pleasure one might have had in reading this book.

H. N.

1. R. E. Blahut, *Fast algorithms for digital signal processing*, Addison-Wesley, Reading, MA, 1985.
2. J. H. McClellan and T. W. Parks, *Eigenvalue and eigenvector decomposition of the discrete Fourier transform*, IEEE Trans. Audio Electroacoust. **20** (1972), 66–74.
3. H. J. Nussbaumer, *Fast Fourier transform and convolution algorithms*, Springer, Berlin, 1981.
4. I. Schur, *Über die Gaußschen Summen*, Göttinger Nachr. **1921**, 147–153.

26[60-02, 49D37, 60G15, 60G17, 60G35, 60G60, 62C10].—Jonas Mockus, *Bayesian Approach to Global Optimization—Theory and Applications*, Mathematics and Its Applications (Soviet Series), Kluwer Academic Publishers, Dordrecht, 1989, xiv + 254 pp., 24 $\frac{1}{2}$ cm. Price $59.00 /Dfl 190.00.

In the Bayesian approach to global optimization an objective function f is a priori modelled as a realization of a stochastic process (also called random function), which can be viewed as a probability distribution on a class of functions. The objective function is evaluated in certain points, and the posterior stochastic process is computed, conditional on the observed function values. The posterior information is used to determine the location of the next point where f will be evaluated.

The stochastic processes are taken to be Gaussian. A Gaussian process is characterized by a mean and covariance function. The covariance function specifies how the correlation of the function values $f(x)$ and $f(y)$ depends on the originals x and y. Gaussian stochastic processes have the very attractive property that the posterior process, conditional on a number of observed function values, is Gaussian as well. However, the determination of the posterior mean and covariance function involves the time-consuming operation of the inversion of a matrix, whose size is equal to the number of observations.

On page 12 the author argues that the Bayesian approach and a stochastic model of f were first applied to global optimization in his reference of 1963. However, H. J. Kushner already in 1962 published a paper on this subject [1].

The author first defines an optimal n-step optimization strategy which minimizes the expected deviation from the global optimum. The strategy can be computed by solving n Bellman equations. However, it is well known that serious problems arise from a practical point of view, even for moderate values of n. Therefore, a one-step approximation is introduced, where the next observation is considered as the last one. Even this approximation, however, is hard to implement, since at each step the matrix referred to above has to be inverted, and since the next observation point is the optimum value of a function, say ϕ, which also will have different local optima. Several simplifications are carried out to achieve a method which is able to process a reasonable number of function evaluations. Unfortunately, these simplifications all ruin