of integration is infinite) when the trapezoidal rule gives very accurate results which cannot be improved by means of Richardson extrapolation. On the other hand, the authors give an interesting treatment of adaptive quadrature.

Chapter 8 gives a very good introduction to the numerical treatment of linear systems of equations and Gaussian elimination. The cases of general, i.e., nonstructured matrices, positive definite and band matrices are dealt with in detail. Iterative improvement is also discussed. It would perhaps have been worthwhile for the authors to include a short description of the Gauss-Seidel and Jacobi iteration schemes, which now are of interest in the context of the popular multigrid methods. We also find in Chapter 8 a discussion of perturbation, for which the authors introduce vector and matrix norms. The discussion of high-performance computers is also valuable.

In Chapter 9 normed function spaces are introduced in an elementary way, and some classical results on orthogonal polynomials are presented. The reader is also informed that interpolation at the Chebyshev points in general gives a good polynomial approximation.

Chapter 10 is devoted to ordinary differential equations and describes a few important methods for initial value and boundary value problems.

The book is well written. The discussion is clear and easy to follow. The authors present central topics in numerical analysis and the book should be useful for anyone who is interested in numerical calculations.

F.S.

40[65N30, 76D05].—MAX D. GUNZBURGER, *Finite Element Methods for Viscous Incompressible Flows: A Guide to Theory, Practice, and Algorithms*, Computer Science and Scientific Computing, Academic Press, Boston, 1989, xvii +269 pp., $23\frac{1}{2}$ cm. Price $44.50.

The book by Gunzburger will appeal to a broad range of people interested in understanding algorithms for solving the model equations of fluid flow. This could be someone who wants to write new software, someone interested only in using existing software, but wanting to use it more intelligently, or someone looking for research problems in the field. Gunzburger has attempted to present the body of mathematical results currently available on the subject to such people.

The casual reader might comment that the book lacks proofs. This is not an accurate statement, even though the preface says that "no detailed proofs are given." For example, a fairly complete outline of a key issue, "divergence stability," is given (Chapter 2); details are in the papers cited. Even research mathematicians may appreciate this approach—it tells them what is important and where to find out more. However, the approach may make the book inappropriate as a text for a graduate course in mathematics.

There are several books with subject matter related (and complementary) to the book under review. The monographs of Girault and Raviart [2, 3] present more mathematical details, but without the scope available in Gunzburger's book. The forthcoming book by Brezzi and Fortín [1] will also offer a more mathematical perspective. Glowinski's monograph [4] is closer in spirit, in that it addresses many algorithmic details essential in a complete implementation of a computer code. However, [4] is more advanced technically, whereas Gunzburger has attempted something more expository.

Gunzburger addresses many of the algorithmic issues related to model complexities that arise in practical applications, such as mixed boundary conditions, non-Newtonian models, nonsimply connected domains, to name just three. Often ignored in theoretical treatments, such complexities lead to nontrivial perturbations in an algorithm and may cause significant problems for code modifications if not anticipated in the planning stage. The inclusion of such topics makes the book a valuable reference.

Not all topics critical to successful simulations are covered as fully as one might like. Indeed, the last chapter is devoted to a discussion of a number of "omitted" topics, and it provides some excellent guidance for future research topics. One of the topics discussed there that may surprise theoretically oriented computational mathematicians, yet is of crucial practical importance, concerns test problems. Software for solving partial differential equations in complex geometries is difficult to write and debug, so the testing of it is very important. One test problem for two-dimensional Navier-Stokes codes has recently been proposed in [6].

One cannot assume that all available mathematical theory regarding the modelling of "viscous incompressible flows" has been represented in the book. As an example, one of the earlier chapters is devoted to time discretizations. The material presented is the essential introductory theory for the most basic time-stepping techniques, yet much more could have been said on the subject. For example, there is no reference to the important series of papers by Heywood and Rannacher [5] which discuss some severe limitations to the accuracy of standard methods unique to incompressible flows, nor is there a discussion of the techniques introduced by Glowinski and coworkers [4].

Whether for getting an introduction to the subject, finding a detailed algorithm for a complicated model, or just seeking the philosophical viewpoint of one of the experts in the field, this book will be very valuable. It has appeared at a time when the subject is maturing and some definitive guidance can be given regarding code development and use. It also proposes numerous directions for further research in this continually developing field.

R. Scott

1. Franco Brezzi and Michel Fortin, *Mixed and hybrid finite element methods* (to appear).

2. Vivette Girault and Pierre-Arnaud Raviart, *Finite element approximation of the Navier-Stokes equations*, Lecture Notes in Math., vol. 749, Springer-Verlag, Berlin, 1979.

3. ——, *Finite element methods for Navier-Stokes equations, theory and algorithms*, Springer-Verlag, Berlin, 1986.

4. Roland Glowinski, *Numerical methods for nonlinear variational problems*, Springer-Verlag, Berlin, 1984.

5. John Heywood and Rolf Rannacher, *Finite element approximation of the nonstationary Navier-Stokes problem. Part* I: *regularity of solutions and second-order error estimates for spatial discretization*, SIAM J. Numer. Anal. **19** (1982), 275–311; *Part* II: *stability of solutions and error estimates uniform in time*, SIAM J. Numer. Anal. **23** (1986), 750–777; *Part* III: *smoothing property and higher order error estimates for spatial discretization*, SIAM J. Numer. Anal. **25** (1988), 489–512; *Part* IV: *error analysis for second-order time discretization*, SIAM J. Numer. Anal. **27** (1990), 353–384.

6. A. S. Lodge, W. G. Pritchard, and L. R. Scott, *The hole-pressure problem*, IMA J. Appl. Math. **46** (1991), 39–66.

41[65-01, 35A15, 65N25, 65N30].—M. KŘÍŽEK & P. NEITTAANMÄKI, *Finite Element Approximation of Variational Problems and Applications*, Pitman Monographs and Surveys in Pure and Applied Mathematics, vol. 50, Wiley, New York, 1990, viii+239 pp., 24 cm. Price $95.00.

According to the authors' introduction the purpose of this book is two-fold: to present a condensed and elementary form of the theory of the finite element method, and to extend standard treatises to cover a number of recent special developments.

The first part of the book (Chapters 2–7) is an introduction to the mathematics of the finite element method. The starting point is Chapter 2, on the variational formulation of second-order elliptic problems, including a concise section on Sobolev spaces. Chapter 3 introduces the finite element method and describes several standard finite element spaces, and Chapter 4 discusses their approximation properties and associated convergence results in energy and L_2-norms. Chapters 5 and 6 address quadrature aspects and the generation of the stiffness matrix, and the relatively lengthy Chapter 7 is devoted to the solution of the resulting systems of linear equations.

In this part of the book some basic results are proved, but for the more sophisticated ones the reader is referred to other treatises. For instance, the Bramble-Hilbert lemma is stated and used extensively, but for a proof the authors quote the book of Ciarlet, which also is frequently referenced in the sections on quadrature and elsewhere. In the chapter on solution of the linear algebraic systems, different subsections treat the Jacobi method, the Gauss-Seidel method, SOR, steepest descent, and conjugate gradient methods. Preconditioning is mentioned without being pursued, and more modern notions, such as methods based on FFT, domain decomposition, and multigrid methods, are not mentioned.

The remaining part of the book, corresponding to the second purpose stated, has twelve chapters on topics such as methods for increasing the accuracy in standard finite element approximation (Chapter 8), fourth-order elliptic problems (Chapter 9), parabolic and hyperbolic problems (Chapters 10–11), methods and problems associated with divergences and rotations, including solution