4. Roland Glowinski, *Numerical methods for nonlinear variational problems*, Springer-Verlag, Berlin, 1984.

5. John Heywood and Rolf Rannacher, *Finite element approximation of the nonstationary Navier-Stokes problem. Part* I: *regularity of solutions and second-order error estimates for spatial discretization*, SIAM J. Numer. Anal. **19** (1982), 275–311; *Part* II: *stability of solutions and error estimates uniform in time*, SIAM J. Numer. Anal. **23** (1986), 750–777; *Part* III: *smoothing property and higher order error estimates for spatial discretization*, SIAM J. Numer. Anal. **25** (1988), 489–512; *Part* IV: *error analysis for second-order time discretization*, SIAM J. Numer. Anal. **27** (1990), 353–384.

6. A. S. Lodge, W. G. Pritchard, and L. R. Scott, *The hole-pressure problem*, IMA J. Appl. Math. **46** (1991), 39–66.

41[65-01, 35A15, 65N25, 65N30].—M. Křížek & P. Neittaanmäki, *Finite Element Approximation of Variational Problems and Applications*, Pitman Monographs and Surveys in Pure and Applied Mathematics, vol. 50, Wiley, New York, 1990, viii+239 pp., 24 cm. Price $95.00.

According to the authors' introduction the purpose of this book is two-fold: to present a condensed and elementary form of the theory of the finite element method, and to extend standard treatises to cover a number of recent special developments.

The first part of the book (Chapters 2–7) is an introduction to the mathematics of the finite element method. The starting point is Chapter 2, on the variational formulation of second-order elliptic problems, including a concise section on Sobolev spaces. Chapter 3 introduces the finite element method and describes several standard finite element spaces, and Chapter 4 discusses their approximation properties and associated convergence results in energy and L_2-norms. Chapters 5 and 6 address quadrature aspects and the generation of the stiffness matrix, and the relatively lengthy Chapter 7 is devoted to the solution of the resulting systems of linear equations.

In this part of the book some basic results are proved, but for the more sophisticated ones the reader is referred to other treatises. For instance, the Bramble-Hilbert lemma is stated and used extensively, but for a proof the authors quote the book of Ciarlet, which also is frequently referenced in the sections on quadrature and elsewhere. In the chapter on solution of the linear algebraic systems, different subsections treat the Jacobi method, the Gauss-Seidel method, SOR, steepest descent, and conjugate gradient methods. Preconditioning is mentioned without being pursued, and more modern notions, such as methods based on FFT, domain decomposition, and multigrid methods, are not mentioned.

The remaining part of the book, corresponding to the second purpose stated, has twelve chapters on topics such as methods for increasing the accuracy in standard finite element approximation (Chapter 8), fourth-order elliptic problems (Chapter 9), parabolic and hyperbolic problems (Chapters 10–11), methods and problems associated with divergences and rotations, including solution of Stokes, Maxwell's, and Helmholtz's equations (Chapters 12–14), nonlinear problems (Chapters 15 and 18), and eigenvalue and bifurcation problems (Chapters 16–17).

The selection of topics appears motivated mainly by the authors' own research interests, and no attempt at a wide coverage of recent developments appears to have been made. The chapters are often quite short and sometimes characterized by a certain lack of substance and precision, with simple computed examples carrying the burden of persuasion. In view of the expressed purpose of being accessible to readers with limited background, the presentation is often on the terse side.

Although the list of references is extensive, many household names and relevant works from the field are missing, conveying the impression that the authors are somewhat less than well informed about some of the areas they describe, whereas no less than 19 papers by the authors themselves are quoted.

To give some examples of the above points, in a short chapter on eigenvalue problems, the authors reproduce an incorrect error estimate for eigenvalues from the early literature. The subsequent related four-page chapter on a bifurcation problem is too short to accomplish its purpose of explaining the rather difficult problem, describing a method for its solution, and presenting numerical evidence. In the chapter containing a discussion of superconvergence, on which the authors have written an exhaustive survey article, one is surprised to find the book by Axelsson and Barker from 1984 to be the basic reference on nodal superconvergence, the only other reference being the survey just mentioned.

The short chapter on hyperbolic problems considers only two specialized topics, perhaps not the obvious first choices, namely the solution of the wave equation with nonhomogeneous Dirichlet boundary conditions for $t > 0$, and a stationary, convection-dominated diffusion problem. In the former a penalty approximation is presented, where the nonhomogeneous boundary condition is replaced by a Newton-Robin type condition with a small coefficient in front of the normal derivative. The resulting equations are then discretized by standard finite elements in space and a symmetric difference scheme in time. No analysis of the method is presented, nor any claim about its properties, but some computations are described with tables and pictures. In the latter part of the chapter a Petrov-Galerkin method is applied to a linear one-dimensional singularly perturbed problem. Again, no analysis or convergence statement is given, only a computation hinting at the superiority of the Petrov-Galerkin method with the appropriate trial space over the standard Galerkin method.

In summary, the reviewer feels that the two purposes of being both an elementary introduction and a survey of the authors' research interests have turned out not to be quite compatible. In the first part, for simplicity and brevity, the analysis has been cut down to such a minimum that the second part would be very difficult to penetrate for someone with only this as a background. However, the appearance of a new book in such a central and active field of numerical analysis as finite elements always arouses interest and curiosity. The present one will certainly serve as a useful survey of work of the authors and their collaborators and compatriots, often not easily accessible otherwise.

V.T.

42[65-02, 35L65, 76-08].—RANDALL J. LEVEQUE, *Numerical Methods for Conservation Laws*, Lectures in Mathematics, ETH Zürich, Birkhäuser, Basel, 1990, ix+214 pp., 24cm. Price: Softcover $24.50.

The title seems too general. The book does not cover all the aspects of numerical methods for conservation laws. For example, shock-fitting methods, or finite elements and spectral methods, are not discussed. However, the text does provide an up-to-date coverage on recent developments of shock-capturing finite difference methods, which is one of the most active research areas in numerical solutions for conservation laws.

The first part of this book summarizes the mathematical theory of shock waves. It also covers topics related to gas dynamics equations. Although the material is available in other books listed in the references, a somewhat more elementary approach with the help of graphs is provided here. It should prove helpful to students with a limited background in partial differential equations, but who want to get some feeling about the theory in order to read the second part regarding numerical methods.

Part two of this book is about recent developments of shock-capturing finite difference methods. In the past fifteen years there has been a lot of activity in designing and analyzing stable and accurate shock-capturing finite difference methods for conservation laws whose solutions contain shocks. The developments have been following quite a different path than the traditional linear stability analysis based on smooth solution assumptions. Tools for nonlinear stability such as TVD (total-variation-diminishing) methods have been developed, and high-order nonlinearly stable methods have been designed to resolve shocks and other complicated flow structures. Unfortunately, most of the results have been available only in isolated, sometimes hard-to-read journal articles. Books or Lecture Notes in this area are notably rare. This book is therefore rather unique and should prove to be a valuable reference and textbook in this area.

The text is carefully prepared. I have used a report version of it for a graduate reading course at Brown University, and students feel that it is well written and on the whole easy to understand. Misprints are rare, although the first sentence in the Preface misses an "are" right after the first two words. Another not-so-obvious mistake is in Exercise 17.1 on page 199: the minmod function in (16.51) would have to be changed to a minimum-in-absolute-value function to establish the claimed agreement.

In view of its limited scope, this book is not suitable for a general numerical analysis course for partial differential equations, or for computational fluid dynamics. However, it would serve as an excellent textbook for a graduate seminar course for mathematics or engineering students who are interested in shock