calculations, and as a general reference book for researchers. The fact that it contains exercises and is available in relatively inexpensive soft cover is another welcome feature.

One comment on the organization of the material: the first part on theory of conservation laws and gas dynamics equations seems too lengthy, since most of the material can already be found in many good books. It would seem worthwhile to condense the first part and to expand the second part on numerical methods. One gets the impression, when reading through the book, that it is gradually running out of steam: towards the end, the description becomes more and more sketchy.

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43[65M60, 65N30, 65N35].—CLAUDIO CANUTO, M. YOUSSUFF HUSSAINI, ALFIO QUARTERONI & THOMAS ZANG, *Spectral Methods in Fluid Dynamics*, Springer Series in Comput. Phys., Springer, New York, 1988, xv+557 pp., 24 cm. Price $90.50.

The authors present here a comprehensive, up-to-date treatment of spectral methods: "when to use them, how to implement them, and what can be learned from their rigorous theory."

The distinguishing feature of this book is its synthesis between the description of spectral *algorithms* that are successfully implemented in Computational Fluid Dynamics (CFD) problems, and whatever rigorous theory is currently available to support their numerical results.

After the introductory material in Chapter 1, the content of the book can be grouped into three related parts.

The first part outlines the basic ingredients which are involved in spectral solution of PDE's. In includes a bird's eye view on the fundamental concepts of spectral expansions in Chapter 2. Chapter 3 is concerned with accurate spatial differentiation of these spectral expansions in the context of linear and nonlinear PDE's. Temporal discretizations of spectral approximations to time-dependent problems are treated in Chapter 4, and the solution of implicit spectral equations (which arise owing to full spectral differentiation matrices) is studied in Chapter 5.

The second part of the book is devoted to applications of spectral methods for CFD problems. It includes a detailed description of spectral algorithms for unsteady incompressible Navier-Stokes equations in Chapter 7, and for compressible Euler equations in Chapter 8.

The third part of the book deals with the rigorous analysis of spectral methods. Error estimates for spectral expansions are presented in Chapter 9, which is followed by the linear stability and convergence analysis of spectral methods in

Chapter 10. Chapters 11 and 12 demonstrate applications of the general theory to steady Navier-Stokes and time-dependent problems, respectively. The final Chapter 13 addresses the issue of spectral computations in general geometries via domain decomposition methods.

The main text concludes with an impressive list of references.

The class of spectral methods is an umbrella for a family of certain *projection* methods. I now proceed with a brief description of these methods, touching upon various aspects that are highlighted in the book.

The underlying problem consists of one or more PDE's

$$Lu = F \tag{1}$$

augmented with side conditions—initial conditions, boundary conditions, etc., which make the problem *well posed*. The N-dimensional spectral approximation of (1) reads

$$P_N L u_N = F_N, \tag{2}$$

together with appropriate N-dimensional side conditions. Different spectral methods are identified with different N-dimensional projections P_N. The distinctive character of spectral methods is their use of *global, highly accurate* projections. Typically, these projections are represented by N-term orthogonal expansions, e.g., Fourier, Chebyshev, Legendre, etc. Chapters 2–5 describe how to proceed with efficient solution of (2). In particular, the FFT can be used to carry out algebraic and differential operations with the spectral expansions. Preconditioning and multigrid techniques are used for the solution of the full, often ill-conditioned linear systems associated with the linearized equation (2).

The behavior of the approximate spectral solution, u_N, depends on the familiar notions of consistency, stability, and convergence. In the linear case, one obtains from (1) and (2) that $e_N = u_N - P_N u$ satisfies the error equation

$$P_N L P_N e_N = P_N L(I - P_N)u + F_N - P_N F. \tag{3}$$

Consistency requires that the approximation error on the right-hand side of (3) becomes small as N increases. An attractive feature of the spectral projections, analyzed in Chapter 9, is their so-called spectral (or "infinite-order") accuracy, which means that the approximation error decays as fast as the global smoothness of the exact solution u permits. The spectral method (2) is *stable* if $P_N L P_N$ is boundedly invertible. In this case, the error equation (3) implies that u_N is spectrally accurate with $P_N u$ and hence, by consistency, u_N *converges* spectrally to u itself, provided the latter is sufficiently smooth. The consistency, stability, and convergence of spectral methods hinge on several key issues which make the spectral algorithm work successfully. Particular attention should be paid to the aliasing phenomenon; the use of filtering techniques in the presence of nonsmooth data; the appropriate choice of a spectral projection for a given problem, and the choice of discrete collocation points to realize this projection; the correct treatment of boundary conditions; and the behavior of the eigenproblems associated with the linearized equations (2). The fundamental role of these and other issues in the context of spectral methods is explained in Chapters 3–5. Practical implementations are demonstrated in Chapters 7–8. The importance of these issues is reemphasized in the stability analysis presented in Chapters 9–13.

The book is *not* easy to read. This is in part due to its large scope, and in part due to the subject itself: the details involved in spectral algorithms are inherently less 'clean' than those of, say, finite difference methods. The reader is therefore required to be familiar with spectral methods. As the authors indicate in the preface, they intend to reach audiences of both users and theorists of spectral methods. In this respect, the expert will find *Spectral Methods in Fluid Dynamics* to be a valuable source of material. The classical 1977 monograph of Gottlieb and Orszag [1] is, naturally, out of date in view of the enormous amount of activity that took place during the last decade in the area of spectral methods, and the present book will most likely replace it as the standard reference on the subject.

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1. D. Gottlieb and S. Orszag, *Numerical analysis of spectral methods: Theory and applications*, SIAM, Philadelphia, PA, 1977.

44[65-02, 65N05, 65N10, 65N15].—BERND HEINRICH, *Finite Difference Methods on Irregular Networks: A Generalized Approach to Second Order Elliptic Problems*, International Series of Numerical Mathematics, vol. 82, Birkhäuser, Basel, 1987, 206 pp., 24 cm. Price $49.00.

The subtitle of this work is: A generalized approach to second order elliptic problems. Actually, the book is basically concerned with one particular method of approximation, viz., the "finite volume method," also known as the "box integration method," the "box method," the "finite control volume method," or, as the author prefers to call it, the "balance method." Mainly selfadjoint problems in the plane are treated.

In brief, the balance method investigated in this book is as follows: Given a primary mesh, write down the weak formulation of the elliptic partial differential equation against a constant function in a control volume, locally associated with the primary mesh. This procedure has a "physical" interpretation; hence the name balance method. Then approximate the resulting relation using values of the unknown function at nodal points of the primary mesh; equations for these nodal values ensue. Variations of the basic scheme differ according to how the control volumes are constructed from the primary mesh and also according to whether a triangular or rectangular primary mesh is used.

Three types of primary meshes are considered: A. Uniform (regular), B. almost uniform, i.e., uniform except for a small layer around curved boundaries,