these and other issues in the context of spectral methods is explained in Chapters 3–5. Practical implementations are demonstrated in Chapters 7–8. The importance of these issues is reemphasized in the stability analysis presented in Chapters 9–13.

The book is *not* easy to read. This is in part due to its large scope, and in part due to the subject itself: the details involved in spectral algorithms are inherently less 'clean' than those of, say, finite difference methods. The reader is therefore required to be familiar with spectral methods. As the authors indicate in the preface, they intend to reach audiences of both users and theorists of spectral methods. In this respect, the expert will find *Spectral Methods in Fluid Dynamics* to be a valuable source of material. The classical 1977 monograph of Gottlieb and Orszag [1] is, naturally, out of date in view of the enormous amount of activity that took place during the last decade in the area of spectral methods, and the present book will most likely replace it as the standard reference on the subject.

EITAN TADMOR

School of Mathematical Sciences
Tel-Aviv University
Tel-Aviv 69978 Israel

1. D. Gottlieb and S. Orszag, *Numerical analysis of spectral methods: Theory and applications*, SIAM, Philadelphia, PA, 1977.

44[65-02, 65N05, 65N10, 65N15].—BERND HEINRICH, *Finite Difference Methods on Irregular Networks: A Generalized Approach to Second Order Elliptic Problems*, International Series of Numerical Mathematics, vol. 82, Birkhäuser, Basel, 1987, 206 pp., 24 cm. Price $49.00.

The subtitle of this work is: A generalized approach to second order elliptic problems. Actually, the book is basically concerned with one particular method of approximation, viz., the "finite volume method," also known as the "box integration method," the "box method," the "finite control volume method," or, as the author prefers to call it, the "balance method." Mainly selfadjoint problems in the plane are treated.

In brief, the balance method investigated in this book is as follows: Given a primary mesh, write down the weak formulation of the elliptic partial differential equation against a constant function in a control volume, locally associated with the primary mesh. This procedure has a "physical" interpretation; hence the name balance method. Then approximate the resulting relation using values of the unknown function at nodal points of the primary mesh; equations for these nodal values ensue. Variations of the basic scheme differ according to how the control volumes are constructed from the primary mesh and also according to whether a triangular or rectangular primary mesh is used.

Three types of primary meshes are considered: A. Uniform (regular), B. almost uniform, i.e., uniform except for a small layer around curved boundaries, say, and C. quasi-uniform but globally irregular meshes. Given the title of the book, the last case is clearly the most important.

The book is to a large extent a summary of the work of the author and others on how this balance method behaves.

There is at present a discussion among numerical analysts about the role of truncation error analysis for irregular meshes in "ordinary" finite difference and finite element methods. Some of the issues are very nicely illustrated in this monograph, as I hope the following outline of some main ideas will show.

First, the ensuing equations for nodal values alluded to above are treated in a classical manner, well known to anyone who has studied the five-point operator on a uniform mesh. A maximum principle is derived and thereafter the exact solution is put through its paces through the equation for nodal values, resulting in a "truncation error." Using the maximum principle, any bound on the maximum norm of the "truncation error" yields a maximum norm bound on the actual error.

This analysis works all right for meshes of type A, and, with some exertions inspired by finite difference work in the late sixties, for meshes of type B, provided the solution is sufficiently smooth.

However, in the case C of totally irregular meshes, the "truncation error" is merely bounded in the maximum norm; thus the above analysis via the maximum principle does not even show convergence. This not unusual observation for irregular meshes is made on page 107 in this book.

Noting that the pointwise analysis via the maximum principle does not work for irregular meshes, to rescue the situation, the author switches to an analysis in L_2 dual norms, basically, with minor technical deviations now and then, using that the actual H_1 error is bounded by the H_{-1} error in the "truncation error." The program is carried through with aplomb for the three mesh situations A, B, and C. In particular, in the case A of globally regular meshes, so-called superconvergent results, $O(h^2)$ for first-order difference quotients, are obtained, as also, to a lower order, in the case of meshes of type B.

As for Chapter 6, nonsymmetric problems, I have little to say. Although the author uses upwinding techniques for the first-order convection terms, the results there say little about a truly singularly perturbed, convection-dominated case, as far as I could ascertain.

The style of this book is readable, although I suspect that some numerical analysts will find the collections of notation for meshes and geometry of domains in the beginning somewhat overpowering. Many technical points are relegated to the literature. I note with pleasure that the author very carefully takes into account the treatment of curved boundaries. The typography is pleasant, with hand-written symbols carefully executed.

In conclusion, this is a careful monograph on error analysis of the balance method.

L. B. W.