REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1991 Mathematics Subject Classification can be found in the annual subject index of *Mathematical Reviews* starting with the December 1990 issue.

6[65-01, 65Dxx, 65Fxx, 65Gxx, 65Hxx, 65Kxx].—GÜNTHER HÄMMERLIN & KARL-HEINZ HOFFMANN, *Numerical Mathematics* (Translated by Larry Schumaker), Undergraduate Texts in Mathematics, Springer, New York, 1991, xi + 422 pp., $23\frac{1}{2}$ cm. Price: Softcover \$39.95.

This is a straight translation of the first German edition (reviewed in [1]) incorporating, however, a few very minor corrections.

W. G.

1. W. Gautschi, Review **10**, Math. Comp. **55** (1990), 391–392.

7[35R30, 65M30, 93B30].—H. T. BANKS & K. KUNISCH, *Estimation Techniques for Distributed Parameter Systems*, Systems & Control: Foundations & Applications, Vol. 1, Birkhäuser, Boston, 1989, xiii + 315 pp., $23\frac{1}{2}$ cm. Price \$42.00.

A typical problem addressed in this book is the following: Given certain "observations" z of a "state" $u = u(x, t; q)$ which satisfies

$$(1) \qquad \begin{cases} u_t = (qu_x)_x \equiv A(q)u, & 0 < x < 1, \ 0 < t, \\ u(x, 0) = \phi(x), & 0 < x < 1, \\ \text{and boundary conditions}, \end{cases}$$

can one recover the unknown coefficient $q = q(x, t)$?

A typical general scheme for this problem is as follows: First, select a criterion for, hopefully, nailing down q; say, the "output least squares error criterion,"

$$(2) \qquad \operatorname*{Min!}_{q} |u(\cdot, \cdot; q) - z|^2.$$

Here, $|\cdot|$ denotes a suitable seminorm, e.g., the deviations at some discrete points (x_i, t_j). Then fix the "admissible parameter set" $\widetilde{Q}$, typically involving constraints on q motivated from the problem, e.g., $q(x, t) \geq \gamma > 0$, and perhaps even $q =$ constant. Some additional conditions, e.g., norm bounds, are also typically involved for making $\widetilde{Q}$ a compact subset of a suitable metric space. Then select finite-dimensional approximations Q_M to $\widetilde{Q}$, and also

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approximations $A^N(q)$ to $A(q)$, in some finite-dimensional state spaces; the latter lead to (semi) discrete solution schemes for (1).

Finally, solve the finite-dimensional version of (1), (2), thus, hopefully, getting $\overline{q}_M^N$.

A typical result of this book gives conditions on the setup to guarantee so-called "function space parameter convergence." Among other things, this means that a) the $\overline{q}_M^N$ exist; b) every convergent subsequence of $\overline{q}_M^N$, as $M, N \to \infty$, converges to a solution q^* of (2); and c) there exists at least one such convergent subsequence.

In practice, some basic method has to be used for the discrete optimization problem (2), and some time-discretization for (1), which of course intervenes in (2). Practical aspects of this are discussed in the book. In the brief description above I have left out an "observation operator" $\mathscr{T}$ that the authors include throughout; this means that z is compared to $\mathscr{T}u$ in (2), rather than to u itself.

The contents of the book are as follows. Chapter I gives many examples of problems such as (1) occurring in applied science. Chapter II is devoted to preliminary material from semigroup theory. The aim here is to "provide a useful summary for quick reference." It is noted that "those for whom this is new material may wish to supplement it with readings in the references." Chapter III contains a thorough general setup and analysis along the lines I have indicated above for (1), (2), with examples of concrete approximations (with more to come in Chapter V). A long Chapter IV discusses identifiability and stability, including regularization approaches. Very loosely, these are questions about existence and uniqueness of solutions to (2), and their stability under changes in z. Section 2 of this chapter contains ten carefully chosen examples which greatly adds to the understanding of the concepts and issues. In Chapter V more details, mainly on practice and implementation, are given for the linear parabolic case. Eleven numerical examples with numerically generated observations z from a "true" q^* are given, which nicely illustrates the theory. Finally, in this chapter, examples with purely experimental data are considered. A discussion of the use of the methods is given for answering, in a statistical sense, questions such as: Is convection important in the model? Is diffusion spatially and/or temporally varying?

In Chapter VI the authors consider parameter identification in linear elliptic problems, discussing many methods, not only based on (2), and again generously sprinkling the text with interesting examples.

As the authors remark, the material covered is not complete or completely up-to-date. An extra bibliography, Chapter VII, is intended to atone for this. Five short appendices on splines conclude the book.

One aspect of not being completely up-to-date is that standard finite element methodology based on weak formulations for solving (1) is not treated (although Example V.3 comes close). Most approximations to (1) use $A^N = P^N A$ with P^N a projection to a spline or eigenfunction space which is in the domain of A (in L_2). The authors give references for the standard finite element approximation, and, in the elliptic case, they are being used.

The writing is lucid and, as should be clear from the above, concepts are thoroughly motivated and discussed. Pitfalls and imperfections in theory and practice are clearly pointed out. I found only a few trivial misprints. The book is eminently understandable to someone who knows basic spline theory and a bit more than basic functional analysis. (Facts about compactness and metrizability in the weak* topology of L_∞ are used, with careful referencing.)

A particular aspect of this work is the constant interplay between a carefully erected abstract framework, in which convergence is proven, and very concrete examples which sometimes stretch the theory to its limits and beyond. In this, the book is not only of interest to people in the area of parameter estimation, but serves up a nice slice of life of present-day applied and numerical mathematics which may be enjoyed by a wider audience.

L. B. W.

8[65-06, 65P05, 65H20, 35B32, 58F14].—Dirk Roose, Bart de Dier & Alastair Spence (Editors), *Continuation and Bifurcations: Numerical Techniques and Applications*, NATO ASI Series, Series C: Mathematical and Physical Sciences, Vol. 313, Kluwer, Dordrecht, 1990, xiii + 426 pp., $24\frac{1}{2}$ cm. Price \$132.00/Dfl.220.00.

The numerical solution of parameter-dependent problems has become an extremely important branch of scientific computing and numerical analysis. The theoretical understanding of bifurcation phenomena permits the development of powerful methods automatically detecting qualitative changes in solution behavior.

This book contains 26 articles and 10 abstracts of talks given at a workshop held at the Katholieke Universiteit Leuven, Belgium, in September 1989. The authors cover a wide range from theoretical investigations to numerical algorithms and applications to real-world problems.

Several articles are concerned with low-dimensional representations of the behavior of systems described by partial differential equations. The approaches discussed by the authors include the construction of approximate inertial manifolds as well as spectral methods. The resulting finite-dimensional systems are used for the computation of bifurcation diagrams for problems such as the Kuramoto-Sivashinski equation or systems of reaction-diffusion equations.

A group of articles is concerned with bifurcation in the presence of symmetries. Apart from theoretical investigations, numerical methods using information about symmetries are presented.

Methods for the computation of heteroclinic and homoclinic orbits and their use for the detection of global bifurcations are discussed. Further numerical aspects include the effect of time-discretization on the global attractor as well as the computation of Hopf bifurcations. Continuation and bifurcation software is presented, and the desirable features of such software are discussed in an article which tries to initiate a discussion about standards for continuation codes.

Several interesting applications are presented. The author of this review learned that cubature formulae can be constructed via continuation methods. Other articles describe a classification of flow structures for the Navier-Stokes equations, the dynamics of the Maxwell-Bloch equations for passive optical systems, Marangoni convection in crystal growth problems, image processing via the dynamics of reaction-diffusion systems and the stability of a robot.