

**Supplement to**  
**COMPUTATION OF THE ZEROS OF  $p$ -ADIC  $L$ -FUNCTIONS**

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Here we present two supplementary sections, one (§11) providing further details of the computations and another (§12) giving the proofs of some propositions. These are followed by the numerical tables from Programs A and B as announced in the main text.

11. DETAILS OF THE COMPUTATIONS

As mentioned before, our computation of  $L_p(s, \theta) = \sum_{i=0}^{\infty} u_i s^i$ , i.e., the approximation of  $u$ , by  $\bar{u}$ , mod  $p^M$ , was based on the expression given in (5.1). To study the structure of this expression, we write it in the form

$$L_p(s, \theta) = -\frac{1}{dp} \sum_{\substack{a=1 \\ p \nmid a}}^{dp} \theta(a) (S_1(a) + S_2(a) + S_3(a)),$$

where

$$S_1(a) = \sum_{j=1}^{\infty} \frac{1}{1-s} \binom{1-s}{j} \tau_1(a)^j \quad (\tau_1(a) = \langle a-1 \rangle),$$

$$S_2(a) = \sum_{j=1}^{\infty} \frac{1}{1-s} \binom{1-s}{j} B_j \tau_2(a)^j \quad (\tau_2(a) = dp/a),$$

$$S_3(a) = (1-s) S_1(a) S_2(a)$$

(note that the sum of  $\theta(a)$  with  $a = 1, \dots, dp$ ,  $p \nmid a$ , vanishes). Setting, moreover,

$$\frac{1}{1-s} \binom{1-s}{j} = \sum_{i=0}^{j-1} \frac{c_{ji}}{j!} s^i$$

with  $c_{ji} \in \mathbb{Z}$ , we have

$$S_\nu(a) = \sum_{i=0}^{\infty} R_{\nu i} s^i, \quad R_{\nu i} = \sum_{j=i+1}^{\infty} \frac{c_{ji}}{j!} b_{\nu j} \tau_\nu(a)^j \quad (\nu = 1, 2),$$

where  $b_{1j} = 1$  and  $b_{2j} = B_j$ . Note that  $v_p(\tau_\nu(a)) \geq 1$ .

From this it is seen that the principal task is the calculation of (rational) numbers  $\bar{R}_{\nu i}$ , satisfying

$$(11.1) \quad R_{\nu i} \equiv \bar{R}_{\nu i} \pmod{p^{M+1}}.$$

These numbers are computed from the series of  $R_{\nu i}$  by way of truncations of two kinds: the truncation of the series itself and that of the  $p$ -adic expansions of  $c_{ji}$ ,  $j!$ ,  $b_{\nu j}$ , and  $\tau_\nu(a)$ . Let  $\eta_1$  and  $\eta_2$  be parameters such that the series for  $R_{\nu i}$  are computed up to  $j = \eta_1$  and the above  $p$ -adic numbers are computed mod  $p^{\eta_2}$ . An optimal choice appears to be  $\eta_1 = \eta_2 - 1 = \eta$ , say. Between  $M$  and  $\eta$  there is the following relationship, proved in the next section.

PROPOSITION 12. If  $p - 1 \nmid \eta + 1$ , then (5.3) and (5.4) are satisfied with

$$M = \eta - \left\lfloor \frac{\eta}{p-1} \right\rfloor, \quad i_M = \eta - 1.$$

In particular, to obtain

$$\bar{L}_p(s, \theta) = \sum_{i=0}^{i_M} \bar{u}_i s^i,$$

we thus have to compute  $\bar{R}_m$  up to  $i = \eta - 1$ .

One check for the calculation of  $\bar{u}_i$  is provided by Proposition 6. By way of another check, we compared the values of  $\bar{u}_0$  with the class number tables of quadratic fields. Let  $h(\sqrt{m})$  denote the class number of  $\mathbf{Q}(\sqrt{m})$ . Since

$$u_0 = L_p(0, \theta) = \frac{\theta \omega^{-1}(p) - 1}{f} \sum_{\alpha=1}^f \theta \omega^{-1}(\alpha) a,$$

where  $f$  stands for the conductor of  $\theta \omega^{-1}$ , we have

$$u_0 = \begin{cases} 2h(\sqrt{m}) & \text{for } \theta = \theta_m \omega, m = -2 \text{ or } m < -3, \text{ if } \theta_m(p) = -1, \\ h(\sqrt{mp}) & \text{for } \theta = \theta_m \omega^{(p+1)/2}, m < 0, p \equiv 1 \pmod{4}, \\ h(\sqrt{-mp}) & \text{for } \theta = \theta_m \omega^{(p+1)/2}, m > 1, p \equiv 3 \pmod{4}. \end{cases}$$

For  $m > 0$  and  $\theta = \theta_m$ , a third check is given by the  $p$ -adic class number formula

$$L_p(1, \theta_m) = 2 \left( 1 - \frac{\theta_m(p)}{p} \right) \frac{h(\sqrt{m}) \log \varepsilon_m}{\sqrt{d}},$$

where  $\varepsilon_m$  denotes the fundamental unit of  $\mathbf{Q}(\sqrt{m})$ . In fact, we made this check in the cases when  $\sqrt{d} \in \mathbf{Z}_p$  and  $\varepsilon_m$  is included in the table by Ince [6].

The Iwasawa series  $f_\theta(T)$  was computed in the way described in §6. Checks were provided by the facts that all its coefficients are integral and the number of the first coefficients vanishing mod  $p$  is  $\lambda_\theta$ . A final check was obtained upon converting the zeros  $T_0$  of  $f_\theta(T)$  into the zeros  $s_0$  of  $L_p(s, \chi)$ : then we verified that  $L_p(s_0, \chi)$  vanishes to the expected accuracy.

In the computation of  $T_0$  by the Newton algorithm note that the convergence in fact is faster than indicated by (7.4). It is easy to see that the number of correct places in  $t_n$  grows exponentially in  $n$ . Working with the polynomial  $\bar{f}_\theta(T)$  (see Proposition 10), we ran the algorithm until the desired number of places of  $T_0$  remained the same on two successive iterations. Then  $\bar{f}_\theta(T_0)$  was checked to make sure it vanishes. In view of (8.1), one concludes that this quantity should be congruent to 0 (mod  $\pi^{eM-\tau}$ ), but its actual value was much closer to 0.

In order to do the computations implied by the Newton algorithm, we had to prepare computer programs which perform arithmetic in various extensions of  $\mathbf{Q}_p$ . These

programs were also required in the computation, according to (8.2), of the approximate zeros  $\bar{s}_0$  of  $L_p(s, \theta \psi_n)$ .

If  $\bar{s}_0$  approximates a zero  $s_0$  of  $L_p(s, \theta)$ , our check about the vanishing of  $L_p(\bar{s}_0, \theta)$  is based on the formula

$$\bar{L}_p(\bar{s}_0, \theta) - L_p(s_0, \theta) = \sum_{i=0}^{i_M} ((\bar{u}_i - u_i) \bar{s}_0^i + u_i (\bar{s}_0^i - s_0^i)) - \sum_{i>1} u_i s_0^i.$$

If  $v_\pi(s_0) \geq 0$ , this implies by (5.3), (5.4), and Proposition 11 that  $\bar{L}_p(\bar{s}_0, \theta) \equiv 0 \pmod{\pi^{i_M}}$  with

$$M_1 = \min \left( eM, \rho M - \gamma - e + \min_{1 \leq j \leq 0} v_\pi(u_j) \right).$$

In the case  $v_\pi(s_0) < 0$ , this estimate has to be modified. In particular, when estimating  $v_\pi(\sum_{j>1} u_j s_0^j)$  it suffices for the present purpose to look at the numbers  $v_\pi(u_i)$  just for a few values of  $i$  near  $i_M$ .

There is a useful way for checking that the theoretical estimates for the number of correct digits (both for the coefficients and for the zeros) are good enough: run the same example anew with a bigger  $\eta$ . We applied this check in several cases that did not require too much computation time.

### 12. PROOFS

This section contains the proofs of Propositions 7, 9, 11, and 12.

PROOF OF PROPOSITION 7. For  $j = 0$ , the assertion follows from (5.3), since  $a_0 = u_0$  and  $\bar{s}_0 = \bar{u}_0$ . If  $0 < j < M$ , it is enough to show (see (6.3)) that, for  $i = 1, \dots, j$ ,

$$p^{j-i} \sum_{t_1+\dots+t_i=j} e_{t_1} \dots e_{t_i} \equiv p^{j-i} \sum_{t_1+\dots+t_i=j} \bar{e}_{t_1} \dots \bar{e}_{t_i} \pmod{p^M}.$$

Since  $e_j \equiv \bar{e}_j \pmod{p^M}$  and  $j e_j$  is integral, we have

$$p^S e_{t_1} \dots e_{t_i} \equiv p^S \bar{e}_{t_1} \dots \bar{e}_{t_i} \pmod{p^M}$$

with  $S = v_p(t_1) + \dots + v_p(t_i)$ , for any set  $\{t_1, \dots, t_i\}$  of positive indices  $t_k$ . Hence, we are done once we prove that  $S \leq j - i$  whenever  $t_1 + \dots + t_i = j$ .

In a set  $\{t_1, \dots, t_i\}$  summing up to  $j$ , every  $t_k$  satisfies  $1 \leq t_k \leq j - i + 1$ . Setting

$$n_i = \text{card}\{k \in \mathbf{Z} : 1 \leq k \leq i, t_k = t\},$$

we find that

$$S = \sum_{i=1}^{j-i+1} v_p(t) n_i \leq \sum_{i=1}^{j-i+1} (t-1) n_i = \sum_{i=1}^{j-i+1} t n_i - \sum_{i=1}^{j-i+1} n_i = j - i. \square$$

REMARK. In fact,  $v_\pi(e_j T_0^j) \geq j - v_p(j)$ , which easily yields an effective bound for  $J$ .  
 PROOF OF PROPOSITION 12. By Proposition 6,

$$v_p(u_i) \geq \eta - \left\lfloor \frac{\eta-1}{p-1} \right\rfloor \quad \text{for all } i \geq \eta.$$

Hence, with the  $M$  and  $i_M$  given in Proposition 12, we have  $v_p(u_i) \geq M$  for all  $i > i_M$ . This proves the second assertion in (5.4); the first is trivial.

To prove (5.3), it suffices to show that (11.1) holds for  $\nu = 1, 2$  and for  $i = 0, \dots, \eta-1$ . We may write

$$\bar{r}_{\nu i} = \sum_{j=i+1}^{\eta} \bar{r}_{\nu j},$$

where  $\bar{r}_{\nu j}$  approximates

$$r_{\nu j} = \frac{c_{\nu j} b_{\nu j} \tau_\nu(\alpha^j)}{j!}.$$

Thus, the assertion follows once we prove that

$$(12.4) \quad r_{\nu j} \equiv \bar{r}_{\nu j} \pmod{p^{M+1}} \quad \text{for } j = i+1, \dots, \eta,$$

$$(12.5) \quad r_{\nu j} \equiv 0 \pmod{p^{M+1}} \quad \text{for all } j > \eta.$$

Note that  $r_{\nu j}$  is expressed as a  $p$ -adic integer divided by  $j!$  and by the denominator of  $b_{\nu j}$ . The latter equals either 1 or  $D_j$ , the denominator of the Bernoulli number  $B_j$ . Staudt's theorem tells us that  $v_p(D_j) = 1$  or 0 according to whether or not  $p-1$  divides  $j$ .

Since the numerator of  $r_{\nu j}$  is computed mod  $p^{M+1}$ , it is enough for (12.4) to verify that

$$v_p(j!) + v_p(D_j) \leq \eta - M = \left\lfloor \frac{\eta}{p-1} \right\rfloor \quad (j = i+1, \dots, \eta).$$

If  $p-1 \nmid j$ , we obtain this estimate by using the inequality  $v_p(j!) \leq \left\lfloor \frac{j-1}{p-1} \right\rfloor$ . In the remaining case, say  $j = k(p-1)$ , the same inequality yields  $v_p(j!) + v_p(D_j) \leq k$ , proving the claim.

We prove (12.5) by establishing the estimate

$$v_p(r_{\nu j}) \geq \eta + 1 - \left\lfloor \frac{\eta}{p-1} \right\rfloor$$

separately for  $j = \eta + 1, j = \eta + 2$ , and  $j \geq \eta + 3$ . Since  $v_p(r_\nu(\alpha)) \geq 1$ , we know that

$$v_p(r_{\nu j}) \geq j - \left\lfloor \frac{j-1}{p-1} \right\rfloor - v_p(D_j).$$

Hence, if  $j = \eta + 1$ , we only have to observe that  $v_p(D_\eta) = 0$  by the assumption in the proposition. For  $j = \eta + 2$ , the same assumption implies that  $\left\lfloor \frac{j-1}{p-1} \right\rfloor = \left\lfloor \frac{\eta+1}{p-1} \right\rfloor =$

$$\left\lfloor \frac{\eta}{p-1} \right\rfloor. \text{ If } j \geq \eta + 3, \text{ then}$$

$$v_p(r_{\nu j}) \geq \eta + 2 - \left\lfloor \frac{\eta+2}{p-1} \right\rfloor.$$

This yields the desired estimate.  $\square$

PROOF OF PROPOSITION 9. Let  $n \geq 0$ . Note that  $t_n \equiv 0 \pmod{\pi^w}$ . The assumptions imply that  $c_j t_n^j \equiv e_j T_0^j \pmod{\pi^{N+\gamma+1}}$  for  $j = 0, \dots, N'$  and  $t_n^{N'+1} \equiv 0 \pmod{\pi^{N+\gamma+1}}$ . Consequently,

$$(12.1) \quad f(t_n) \equiv \bar{f}(t_n) \pmod{\pi^{N+\gamma+1}}, \quad f'(t_n) \equiv \bar{f}'(t_n) \pmod{\pi^N}.$$

Recalling that

$$(12.2) \quad v_\pi(f'(t_n)) = \gamma, \quad f(t_n) \equiv 0 \pmod{\pi^{2\gamma+n+1}}$$

(see (7.2) and (7.3)), we thus find, since  $N > \gamma$ , that

$$(12.3) \quad v_\pi(\bar{f}'(t_n)) = \gamma, \quad \bar{f}(t_n) \equiv 0 \pmod{\pi^{2\gamma+n+1}}.$$

For  $n = 0$ , this is our assertion (i).

To prove (ii), we suppose that  $t_n$  is given in the form  $x_w \pi^w + \dots + x_{\gamma+n} \pi^{\gamma+n}$  (with the coefficients  $x_i$  in a fixed residue system mod  $\pi$ ), and similarly for  $\bar{t}_n$ . This is no restriction, for the higher powers of  $\pi$  do not affect the algorithm. We show that in fact  $t_n = \bar{t}_n$  for  $n = 1, \dots, N - \gamma$ . Then the assertion, i.e., the congruence (7.6), follows from (7.4).

For  $t_n$  and  $\bar{t}_n$  modified in this way, we have to replace the recursive equations by the congruences

$$t_n \equiv t_{n-1} - \frac{f(t_{n-1})}{f'(t_{n-1})}, \quad \bar{t}_n \equiv \bar{t}_{n-1} - \frac{\bar{f}(\bar{t}_{n-1})}{\bar{f}'(\bar{t}_{n-1})} \pmod{\pi^{\gamma+n+1}} \quad (n = 1, 2, \dots).$$

Since  $t_0 = \bar{t}_0$ , our claim is obtained by induction once we know that

$$\frac{f(t_{n-1})}{f'(t_{n-1})} \equiv \frac{\bar{f}(\bar{t}_{n-1})}{\bar{f}'(\bar{t}_{n-1})} \pmod{\pi^{N+1}} \quad (n = 1, \dots, N - \gamma).$$

In view of (12.2) and (12.3), this follows from (12.1).  $\square$

PROOF OF PROPOSITION 11. Let us first verify that

$$e_j T_0^j \equiv \bar{e}_j \bar{T}_0^j \pmod{\pi^{\rho M - \gamma}}$$

for  $j = 1, \dots, J$ . Since  $e_j \equiv \bar{e}_j \pmod{\pi^{\rho M}}$  and  $v_\pi(\bar{T}_0) > 0$ , this assertion reduces to the form

$$e_j T_0^j \equiv e_j \bar{T}_0^j \pmod{\pi^{\rho M - \gamma}}.$$

By (8.1),  $T_0 = \bar{T}_0 + \pi^{\rho M - \gamma} \tau$  with  $v_\pi(\tau) \geq 0$ . Hence,

$$e_j (T_0^j - \bar{T}_0^j) = \pi^{\rho M - \gamma} \sum_{k=1}^j e_j j(j-1) \dots (j-k+1) \bar{T}_0^{j-k} \tau^k \frac{\pi^{\rho(M-\gamma)(k-1)}}{k!}.$$

The sum  $\sum_{k=1}^j$  on the right-hand side is  $\pi$ -integral, since  $v_p(j e_j) = 0$ ,  $v_p(k!) < \frac{k}{p-1} \leq \frac{k}{2} \leq k-1$  ( $k = 2, 3, \dots$ ), and  $e \leq \rho M - \gamma$ . This proves the claim.

We have  $e_j T_0^j \rightarrow 0$  as  $j \rightarrow \infty$ . Therefore, to obtain the congruence of Proposition 11, simply choose  $J$  so that

$$v_\pi(e_j T_0^j) \geq \rho M - \gamma \quad \text{for all } j > J. \square$$

TABLE I.  $\lambda = 2, a_0 = 0$

(3, -47, 1)	(5, -51, 1)	(7, -1087, 1)	(11, -19, 1)
0 0.0020 0.0120 0.0120 0.0012 0.0010 0.0000 0.0001	0 0.0004 3.2432 0.0342 0.0044 2.00 0.0000 4.00	0 0.0001 3.13201 0.0601 0.0031 0.0003 3.656 5.00	0 0.024 $\alpha$ 0.0135 0.0016 0.0001 0.0000 0.0000
$T_1 = 0, s_1 = 0$ $T_2 = 0.0200$ $s_2 = 0.1002$	$T_1 = 0, s_1 = 0$ $T_2 = 0.0021284$ $s_2 = 0.022013$	$T_1 = 0, s_1 = 0$ $T_2 = 0.0084003$ $s_2 = 0.010110$	$T_1 = 0, s_1 = 0$ $T_2 = 0.6736700$ $s_2 = 9.832164$

TABLE II.  $\lambda = 2, a_0 \neq 0$

(3, 2536, 0)	(3, -1389, 1)	(3, 2732, 0)
0.01100 0.00202 0.02221 0.00220 0.00220 0.00001 0.00021	0.0020 0.0211 0.0220 0.0021 0.0022 0.0000 0.0002	0.01100 0.01120 0.01212 0.00220 0.00110 0.00000 0.00011
$T_1 = 0.1121$ $s_1 = 1.210$ $T_2 = 0.2001$ $s_2 = 2.121$	$T_1 = 0.020$ $s_1 = 0.20$ $T_2 = 0.221$ $s_2 = 2.22$	$\pi = \sqrt{3}$ $T_{1,2} = 0.2021 \pm 0.121\pi$ $s_{1,2} = 1.102 \pm 2.02\pi$

(3, 2908, 0)	(3, -1207, 1)	(5, 37, 2)
0.01100 0.00001 0.01121 0.00211 0.00121 0.00001	0.01100 0.02222 0.02121 0.00111 0.00012 0.00102 0.00010 0.00010	0.30230 0.04414 0.01241 0.00040 0.00031 0.00002 0.00003
$\xi = \sqrt{2}$ $T_{1,2} = 0.0220 \pm 0.1121\xi$ $s_{1,2} = 0.112 \pm 1.111\xi$	$\pi = \sqrt{2} \cdot 3$ $T_{1,2} = 0.2220 \pm 0.102\pi$ $s_{1,2} = 2.120 \pm 1.20\pi$	$\pi = \sqrt{2} \cdot 5$ $T_{1,2} = 0.024 \pm 2.42\pi$ $s_{1,2} = 3.42 \pm 12.1\pi$

(5, -184, 3)	(5, 597, 0)	(5, 2504, 2)
0.40000 0.02334 0.04201 0.00222 0.00001 0.00002	0.03240 0.02404 0.02002 0.00034 0.00022 0.00001 0.00001	0.01023 0.00023 0.01201 0.00040 0.00008 0.00004 0.00003
$\pi = \sqrt{5}$ $T_{1,2} = 0.234 \pm 2.14\pi$ $s_{1,2} = 0.34 \pm 20.2\pi$	$\pi = \sqrt{5}$ $T_{1,2} = 0.440344 \pm 0.22230\pi$ $s_{1,2} = 2.12411 \pm 1.1442\pi$	$T_1 = 0.331200$ $s_1 = 2.41303$ $T_2 = 0.204143$ $s_2 = 3.00334$

REMARK. The above proof can be simplified if one is satisfied with a weaker value of  $M$ . However, we found it important to have as sharp an estimate as possible, since for large conductors  $d$  even a small increase in  $\eta$  causes a considerable growth in computation time.

13. TABLES

Tables I-V belong to Program B; they are described in §10 of the article. We now explain how to read the remaining tables which expose the main results from Program A. These tables list the first coefficients of the Iwasawa power series  $f_\theta(T) = \sum_{j=0}^{\infty} a_j T^j$  for  $p = 3, 5, 7, 11$  and for  $\theta = \theta_m \omega^i$ , where  $m$  ranges through the values given in [3, p. 287], excluding those values for which  $\lambda_\theta = 0$  or 1.

For each prime  $p$  there are two tables, the first for  $m > 0$ , and the second for  $m < 0$ . Every line in the tables gives data associated with a fixed character  $\theta$ . These data are arranged under 8 column headings as follows.

discr: The discriminant of  $\mathbf{Q}(\sqrt{m})$ , in absolute value equal to the conductor of  $\theta_m$ . The characters in each table are ordered according to the increasing value of  $|m|$ ; thus, to find a specific discriminant one has to think about the corresponding  $m$  (which is not printed).

t: The exponent  $t$ . Together with the discriminant, this identifies  $\theta$ .

lambda: The  $\lambda$ -invariant  $\lambda_\theta$  of  $f_\theta(T)$ .

cl: The value of  $\theta_m(p)$ . Note that  $f_\theta(0) = 0$  if and only if  $t = 1$  and  $\theta_m(p) = 1$ . This can happen for  $m < 0$  only; for this reason, there are many more characters in the tables with  $m < 0$  than with  $m > 0$  (cf. [3, p. 288]).

a(0), a(1), a(2), a(3): The coefficients  $a_0, a_1, a_2, a_3$ , respectively, of  $f_\theta(T)$ , written in the "decimal" form with decimal point (which is omitted) after the first digit. Thus, on the first line of the first table one has  $a_0 = 0.110, a_1 = 0.01, a_2 = 2.1, a_3 = (0)$ . The given values are approximations; however, as remarked above, one can decide whether  $a_0 = 0$  (exactly) by looking at the entries under t and cl.

(5, -5556, 1)	(5, -519, 1)	(7, -580, 1)
0 0.00341 0.00412 0.00111 0.00021 0.00001 0.00000	0 0.03124 0.00433 0.00314 0.00024 0.00000 0.00000	0 0.00436 0.0324564 0.00324 0.321434 4.54435 0.6533 0.00005 4.603
$T_1 = 0, s_1 = 0$ $T_2 = 4.3412$ $T_3 = 0.20140$ $s_3 = 2.2420$	$T_1 = 0, s_1 = 0$ $\pi = \sqrt{5}$ $T_{2,3} = 0.41 \pm 2.43\pi$ $s_{2,3} = 3.0 \pm 33.4\pi$	$T_1 = 0, s_1 = 0$ $\xi = \sqrt{3}$ $T_{2,3} = 0.42534 \pm 0.61510\xi$ $s_{2,3} = 3.22355 \pm 1.34656\xi$

TABLE III.  $\lambda = 3, a_0 \neq 0$

(5, -456, 0)	(5, -1317, 0)
0.22332114 0.00403344 0.00044442 0.00326313 0.0026302 0.00001213 0.00000043 0.00000104 0.00000022	0.032400 0.002111 0.004104 0.003232 0.000201 0.000031 0.000004 0.000001 0.000000
$T_1 = 0, s_1 = 0$ $\pi = \sqrt{5}$ $T_2 = 0.13302 + 1.2330\pi + 0.4414\pi^2$ $T_3 = 3.0403 + 1.4343\pi + 31.424\pi^2$	$T_1 = 0, s_1 = 0$ $\pi_k = \xi_5^k \cdot \sqrt{5} \ (k = 1, 2, 3)$ $T_k = 0.344 + 0.03\pi_k + 3.12\pi_k^2$ $s_k = 4.3 + (4.)\pi_k + (41.)\pi_k^2$

(5, -2283, 3)	(7, 649, 4)
0.0400000 0.0304413 0.0044121 0.0024033 0.0002342 0.0000344 0.0000023 0.0000002	0.400000 0.4000000 0.055530 0.146053 0.001000 0.564413 0.003606 4.62125 0.009463 3.0654 0.000030 4.115 0.000006 4.63
$T_1 = 0.134304$ $\pi_1 = 2.23313$ $\pi = \sqrt{5}$ $T_{2,3} = 0.23 \pm 2.04\pi$ $s_{2,3} = 0.0 \pm 42.1\pi$	$\pi = \sqrt{7}$ $T_1 = 0.35 + 3.1\pi + 2.2\pi^2$ $s_1 = 4.4 + (25.)\pi + (35.)\pi^2$ $T_2 = 0.35 + 6.0\pi + 1.6\pi^2$ $s_2 = 4.4 + (44.)\pi + (65.)\pi^2$ $T_3 = 0.35 + 5.4\pi + 4.5\pi^2$ $s_3 = 4.4 + (14.)\pi + (61.)\pi^2$

(5, -1163, 3)	(5, -6568, 3)
0.02000 0.04301 0.01201 0.00331 0.00063 0.00004 0.00003	0.04000 0.02143 0.040314 1.11400 4.3340 1.320 3.12 1.2
$\xi = \sqrt{2}$ $T_{1,2} = 0.421433 \pm 0.304100\xi$ $s_{1,2} = 3.30113 \pm 1.0214\xi$	$\pi = \sqrt{2 \cdot 5}$ $T_{1,2} = 0.321140 \pm 0.23424\pi$ $s_{1,2} = 1.31000 \pm 4.2124\pi$

(7, 3352, 0)	(7, -136, 3)
0.02350 0.04360 0.00660 0.00932 0.00042 0.00001	0.63342 0.63342 0.01316 0.24356 0.61264 6.206 0.364 4.32
$\xi = \sqrt{3}$ $T_{1,2} = 0.4304219 \pm 0.5563426\xi$ $s_{1,2} = 3.010215 \pm 2.520260\xi$	$\pi = \sqrt{-7}$ $T_{1,2} = 0.512 \pm 6.13\pi$ $s_{1,2} = 4.54 \pm 54.5\pi$

(11, 21, 4)	(11, -479, 5)
0.69140 0.06889 0.01732 0.00641 0.00083 0.00000	0.26270 0.03042 0.6242 2.16746 1.95801 5.37260 7.0227 5.303
$\pi = \sqrt{11}$ $T_{1,2} = 0.638 \pm 4.16\pi$ $s_{1,2} = 9.63 \pm 77.2\pi$	$\pi = \sqrt{-11}$ $T_{1,2} = 0.84\alpha \pm \alpha \cdot 453\pi$ $s_{1,2} = 6.30 \pm 93.02\pi$

TABLE IIa.  $\lambda = 3, a_0 = 0$

(3, -587, 1)	(3, -6392, 1)
0 0.000122 0.000002 0.001120 0.000011 0.000001 0.000002	0 0.0211 0.010 0.0210 1.11 2.0 2.
$T_1 = 0, s_1 = 0$ $\pi = \sqrt{2 \cdot 3}$ $T_{2,3} = 0.0012 \pm 0.102\pi$ $s_{2,3} = 0.00 \pm 2.12\pi$	$T_1 = 0, s_1 = 0$ $T_2 = 0.221$ $s_2 = 1.12$ $T_3 = 0.100$ $s_3 = 2.22$

(7, 24, 0)
0.45121 0.22411 6.04132 6.6322 3.060 3.03
$\pi = \sqrt{7}$ $T_{1,2} = 0.630 \pm 2.66\pi$ $s_{1,2} = 6.02 \pm 32.2\pi$

(7, -3572, 3)
0.05342 0.05234 0.05433 0.00336 0.00024 0.00000
$T_1 = 0.4431412$ $s_1 = 2.150160$ $T_2 = 0.11065266$ $s_2 = 4.443226$

(3, -4808, 1)
0 0.0021001 0.0000121 0.0090211 0.0000212 0.0000220 0.0000021 0.0000000 0.0000000
$T_1 = 0, s_1 = 0$ $\xi = \sqrt{2}$ $T_{2,3} = 0.01229 \pm 0.10200\xi$ $s_{2,3} = 0.0100 \pm 2.212\xi$

(7, -151, 1)		(7, -307, 3)		(5, -11444, 3)	
0.20000	0.200000000	0.26641	0.26641426	0.310000000	0.310000000
0.02646	0.4252204	0.04104	0.3606001	0.0342131400	0.223410241
0.00352	0.320220	0.00451	0.632566	0.0032114204	0.22340321
0.00663	6.35605	0.00411	3.51303	0.0004414144	0.26041233
0.00014	0.3666	0.00036	2.1251	0.0001413002	1.023033
0.00001	0.300	0.00002	6.043	0.0000404113	1.34233
$\pi = \sqrt{2} - 7$		$\pi = \sqrt{3} - 7$		0.0000004302	3.2123
$T_1 = 0.15 + 1.1\pi + 3.0\pi^2$		$T_1 = 0.56 + 3.5\pi + 6.4\pi^2$		0.000000324	4.430
$s_1 = 3.2 + (23)\pi + (50)\pi^2$		$s_1 = 1.1 + (42)\pi + (20)\pi^2$		0.0000001212	2.10
$T_2 = 0.15 + 2.6\pi + 5.0\pi^2$		$T_2 = 0.15 + 6.1\pi + 3.3\pi^2$		0.000000024	2.1
$s_2 = 3.2 + (40)\pi + (65)\pi^2$		$s_2 = 1.1 + (10)\pi + (15)\pi^2$		0.0000000021	3.
$T_3 = 0.15 + 4.6\pi + 6.5\pi^2$		$T_3 = 0.15 + 5.6\pi + 5.5\pi^2$		0.0000000003	
$s_3 = 3.2 + (13)\pi + (30)\pi^2$		$s_3 = 1.1 + (24)\pi + (41)\pi^2$		0.0000000001	

TABLE IIIa.  $\lambda = 4, a_0 = 0$

(3, -1460, 1)		(5, -311, 1)	
0	0	0	0
0.000010	0.000201	0.02041033	0.2212421
0.000020	0.00202	0.00023400	0.404312
0.001102	0.2112	0.00033413	0.21301
0.000210	2.000	0.00042044	4.3044
0.000112	1.00	0.00003233	2.104
0.000122	0.2	0.00000031	3.21
0.000012	0.	0.00000010	1.3
$T_1 = 0, s_1 = 0$		0.00000040	3.
$T_2 = 0.2102$		$T_1 = 0, s_1 = 0$	
$s_2 = 1.201$		$\pi_k = \zeta_5^k \cdot \sqrt{5} \ (k = 1, 2, 3)$	
$\pi = \sqrt{-3}$		$T_{k+1} = 1. + (33_k)\pi_k + (34_k)\pi_k^2$	
$T_{3,4} = 0.0021 \pm 0.122\pi$			
$s_{3,4} = 0.022 \pm 2.21\pi$			

TABLE IV.  $\lambda = 4, a_0 \neq 0$

(3, -856, 1)		(3, 1937, 0)	
0.1100000	0.1100000	0.1100000	0.1100000
0.0220011	0.210200	0.0120202	0.220000
0.0011010	0.0121	0.0010020	0.01202
0.0011221	0.2012	0.00022110	0.2000
0.0002212	1.212	0.0000222	2.000
0.0000011	2.2	0.0000002	0.02
0.0000010	2.	0.0000220	0.2
0.0000002	2.	0.0000012	2.
0.0000021		0.0000002	
0.0000001		0.0000012	
$\pi = \sqrt{-3}$		$\pi = \sqrt{3}$	
$T_{1,2} = 0.2 \pm (1)\pi + (1)\pi^2 \pm (1)\pi^3$		$T_{1,2} = 0.1 \pm (1)\pi + (2)\pi^2 \pm (1)\pi^3$	
$s_1^2 s_2 = 0.02 \pm 0.2\pi + 0.0\pi^2 \pm 1.1\pi^3$		$s_1^2 s_2 = 0.01 \pm 0.1\pi + 0.0\pi^2 \pm 2.2\pi^3$	
$\pi' = \zeta_4 \cdot \sqrt{-3}$		$\pi' = \zeta_4 \cdot \sqrt{3}$	
$T_{3,4} = 0.2 \pm (1)\pi + (1)\pi^2 \pm (1)\pi^3$		$T_{3,4} = 0.1 \pm (1)\pi + (2)\pi^2 \pm (1)\pi^3$	
$s_1^2 s_2 = 0.02 \pm 0.2\pi + 0.0\pi^2 \pm 1.1\pi^3$		$s_1^2 s_2 = 0.01 \pm 0.1\pi + 0.0\pi^2 \pm 2.2\pi^3$	

(5, 1756, 0)		(5, -3547, 3)	
0.3001343241	0.30013432414211	0.10100000	0.1010000000
0.012143000	0.1400332023303	0.03332333	0.423032411
0.0001042122	0.211140440344	0.0044123	0.42101431
0.0000432013	0.24340021220	0.0002412	0.1312231
0.004241242	4.0013400011	0.0004324	4.101342
0.0000030402	3.223011233	0.0000430	3.10401
0.0000001341	3.01010842	0.0000034	2.1304
0.0000000440	1.3323343	0.0000000	1.014
0.0000004344	2.101040	0.0000004	3.13
0.0000000044	1.22322	$\pi = \sqrt{5}$	
0.0000000003	3.2041	$T_1 = 0.24 + 1.20\pi + 0.31\pi^2 + 3.34\pi^3$	
0.0000000000	1.223	$s_1^2 = 1.0 + 42.3\pi + 24.3\pi^2 + 3.0\pi^3$	
0.0000000004	1.33	$T_2 = 0.24 + 2.00\pi + 0.23\pi^2 + 4.40\pi^3$	
$\pi = \sqrt{3} \cdot 5$		$s_2^2 = 1.0 + 34.2\pi + 30.1\pi^2 + 4.0\pi^3$	
$T_1 = 0.4 + 1.4\pi + 2.2\pi^2 + 0.2\pi^3$		$T_3 = 0.24 + 3.44\pi + 0.23\pi^2 + 1.04\pi^3$	
$s_1^2 = 4. + (22)\pi + (11)\pi^2 + (2)\pi^3$		$s_3^2 = 1.0 + 20.2\pi + 30.1\pi^2 + 1.4\pi^3$	
$T_2 = 0.4 + 2.4\pi + 3.2\pi^2 + 0.1\pi^3$		$T_4 = 0.24 + 4.24\pi + 0.31\pi^2 + 2.10\pi^3$	
$s_2^2 = 4. + (41)\pi + (43)\pi^2 + (1)\pi^3$		$s_4^2 = 1.0 + 12.1\pi + 24.3\pi^2 + 2.4\pi^3$	
$T_3 = 0.4 + 3.0\pi + 3.2\pi^2 + 0.4\pi^3$			
$s_3^2 = 4. + (13)\pi + (43)\pi^2 + (4)\pi^3$			
$T_4 = 0.4 + 4.0\pi + 2.2\pi^2 + 0.3\pi^3$			
$s_4^2 = 4. + (32)\pi + (11)\pi^2 + (3)\pi^3$			

(7, 492, 0)		(7, 492, 0)	
0.115452236	0.1154522362	0.115452236	0.1154522362
0.060152232	0.325504443	0.060152232	0.325504443
0.000644104	0.23636204	0.000644104	0.23636204
0.000230066	0.3302406	0.000230066	0.3302406
0.000225460	1.230333	0.000225460	1.230333
0.000035030	4.13314	0.000035030	4.13314
0.000001461	4.3201	0.000001461	4.3201
0.000000641	0.551	0.000000641	0.551
0.000000061	6.31	0.000000061	6.31
0.000000002	2.4	0.000000002	2.4
0.000000006	2.	0.000000006	2.
$\pi = \sqrt{3} - 7$		$\pi = \sqrt{3} - 7$	
$T_{1,2} = 0.2 \pm 2.4\pi + 6.6\pi^2 \pm 2.2\pi^3$		$T_{1,2} = 0.2 \pm 2.4\pi + 6.6\pi^2 \pm 2.2\pi^3$	
$s_{1,2} = 5. \pm (10)\pi + (25)\pi^2 \pm (44)\pi^3$		$s_{1,2} = 5. \pm (10)\pi + (25)\pi^2 \pm (44)\pi^3$	
$\pi' = \zeta_4 \cdot \sqrt{3} - 7$		$\pi' = \zeta_4 \cdot \sqrt{3} - 7$	
$T_{3,4} = 0.2 \pm 2.4\pi + 6.6\pi^2 \pm 2.2\pi^3$		$T_{3,4} = 0.2 \pm 2.4\pi + 6.6\pi^2 \pm 2.2\pi^3$	
$s_{3,4} = 5. \pm (10)\pi + (25)\pi^2 \pm (44)\pi^3$		$s_{3,4} = 5. \pm (10)\pi + (25)\pi^2 \pm (44)\pi^3$	

TABLE V.  $\lambda = 5$  OR  $\lambda = 6$

(3, -2516, 1)		(3, -1547, 1)	
0	0	0	0
0.02212121	0.1220121	0.01222210	0.2010012
0.00011111	0.112122	0.00220010	0.100211
0.00000020	0.10010	0.00200221	0.10122
0.00000020	0.0222	0.00021010	0.1011
0.00000020	2.122	0.00002000	2.102
0.00000020	2.10	0.00002220	2.11
0.00000010	2.1	0.00000000	1.2
0.00000011	1.	0.00000011	0.
0.0000222		0.00000101	1.56
0.0000000		0.00000012	281
0.00000000		0.00000001	1288
0.00000000		0.00000002	1324
$T_1 = 0, s_1 = 0$		$T_1 = 0, s_1 = 0$	
$\pi = \sqrt{5}$		$\pi = \sqrt{-3}$	
$T_{2,3} = 0.1 \pm (1)\pi + (0)\pi^2 \pm (0)\pi^3$		$T_{2,3} = 0.2 \pm (1)\pi + (1)\pi^2 \pm (0)\pi^3$	
$3^2s_{2,3} = 0.00 \pm 0.1\pi + 0.1\pi^2 \pm 2.0\pi^3$		$3^2s_{2,3} = 0.02 \pm 0.1\pi + 0.0\pi^2 \pm 2.0\pi^3$	
$\pi' = \zeta_4 \cdot \sqrt{3}$		$\pi' = \zeta_4 \cdot \sqrt{-3}$	
$T_{4,5} = 0.1 \pm (1)\pi + (0)\pi^2 \pm (0)\pi^3$		$T_{4,5} = 0.2 \pm (1)\pi + (1)\pi^2 \pm (0)\pi^3$	
$3^2s_{4,5} = 0.00 \pm 0.1\pi + 0.1(\pi')^2 \pm 2.0(\pi')^3$		$3^2s_{4,5} = 0.02 \pm 0.1\pi' + 0.0(\pi')^2 \pm 2.0(\pi')^3$	

(5, -16104, 1)		(3, -5051, 1)	
0	0	0	0
0.021302104	0.32423110	0.00020210	0.0010022
0.002234121	0.3022201	0.00102120	0.121111
0.000202241	0.214110	0.00021002	0.00002
0.000023004	0.43142	0.00012101	0.0001
0.000012032	2.0240	0.00002212	0.012
0.00004144	3.241	0.00001101	2.20
0.00000324	2.32	0.00000022	1.1
0.00000014	4.4	0.00000021	1.
0.00000004	0.	0.00000010	
0.00000000		0.00000022	
0.00000000		0.00000000	
$T_1 = 0, s_1 = 0$		0.00000001	
$\pi = \sqrt{5}$		$T_1 = 0, s_1 = 0$	
$5s_2^2 = 0.0 + 1.1\pi + (3)\pi^2 + (3)\pi^3$		$s_2^2 = 0.021020$	
$5s_3^2 = 0.0 + 3.3\pi + (0)\pi^2 + (3)\pi^3$		$s_3^2 = 0.12222$	
$5s_4^2 = 0.1 + 2.3\pi + (2)\pi^2 + (4)\pi^3$		$\pi = \sqrt{3}$	
$5s_5^2 = 0.0 + 1.0\pi + (0)\pi^2 + (4)\pi^3$		$T_{3,4} = 0 \pm (1)\pi + (1)\pi^2 \pm (0)\pi^3$	
$T_4 = 0.1 + 3.1\pi + (2)\pi^2 + (1)\pi^3$		$3^2s_{3,4} = 0.01 \pm 0.1\pi + 0.2\pi^2 \pm 2.2\pi^3$	
$5s_4^2 = 0.0 + 4.4\pi + (3)\pi^2 + (1)\pi^3$		$\pi' = \zeta_4 \cdot \sqrt{3}$	
$T_5 = 0.1 + 4.3\pi + (0)\pi^2 + (2)\pi^3$		$T_{5,6} = 0 \pm (1)\pi + (1)\pi^2 \pm (0)\pi^3$	
$5s_5^2 = 0.0 + 2.1\pi + (0)\pi^2 + (2)\pi^3$		$3^2s_{5,6} = 0.01 \pm 0.1\pi + 0.2(\pi')^2 \pm 2.2(\pi')^3$	

Tables from Program A

P = 3		lambda ch		a(0)		a(1)		a(2)		a(3)	
discr	t	lambda	ch	a(0)	a(1)	a(2)	a(3)	a(0)	a(1)	a(2)	a(3)
248	0	2	-1	0	1	1	0	0	1	2	1
77	0	2	-1	0	1	1	0	0	1	2	1
332	0	2	-1	0	1	1	0	0	2	2	1
412	0	2	1	0	1	1	0	0	1	2	1
556	0	2	1	0	1	1	0	0	2	2	1
604	0	2	1	0	1	1	0	0	1	2	1
716	0	2	-1	0	1	1	0	0	1	2	0
786	0	2	1	0	1	1	0	0	2	2	1
281	0	3	-1	0	2	0	0	0	1	0	2
1288	0	2	1	0	2	0	0	0	1	0	2
1324	0	5	1	0	1	1	0	0	0	0	0
337	0	2	1	0	1	1	0	0	1	0	2
401	0	3	-1	0	2	0	0	0	1	0	2
1672	0	2	1	0	2	2	0	0	2	1	0
1708	0	2	1	0	2	2	0	0	2	2	1
469	0	2	1	0	2	2	0	0	2	2	1
2504	0	3	-1	0	2	2	0	0	2	2	0
2536	0	2	1	0	0	1	1	0	0	2	2
653	0	2	-1	0	2	1	0	0	2	2	1
2716	0	2	-1	0	2	2	0	0	0	1	2
2732	0	2	-1	0	2	2	0	0	2	2	2
2793	0	3	-1	0	0	1	1	0	0	0	1
2972	0	2	-1	0	1	2	1	0	2	0	1
3160	0	2	1	0	2	2	0	0	1	1	2
3404	0	2	-1	0	2	2	0	0	1	2	1
3484	0	2	1	0	2	2	0	0	2	0	0
3512	0	3	-1	0	2	1	0	0	2	1	0
3580	0	2	1	0	2	2	0	0	2	1	1
3592	0	2	1	0	2	2	0	0	2	2	0
3736	0	2	1	0	2	2	0	0	1	2	2
3916	0	4	1	0	1	2	1	0	0	2	1
4136	0	2	-1	0	2	2	0	0	1	1	0
4172	0	3	-1	0	1	2	1	0	1	0	2
4204	0	4	1	0	0	1	1	0	1	0	0
4422	0	2	1	0	1	2	1	0	1	0	2
1125	0	2	1	0	1	1	0	0	1	2	2
1165	0	2	1	0	1	1	0	0	1	2	2
4780	0	2	-1	0	1	1	1	0	2	1	2
4844	0	2	-1	0	2	2	0	0	2	1	2
4898	0	2	1	0	1	2	1	0	1	2	1
1241	0	2	-1	0	1	1	2	0	2	2	0
1249	0	2	1	0	1	1	0	0	1	1	1
1261	0	2	1	0	1	1	0	0	1	1	0
5132	0	2	-1	0	2	2	0	0	2	1	1
1297	0	2	1	0	2	2	0	0	1	1	1
5368	0	3	1	0	1	2	1	0	0	1	1
1385	0	2	-1	0	1	1	2	0	1	1	2
1397	0	3	-1	0	2	2	0	0	0	1	1
5624	0	3	-1	0	2	2	0	0	0	2	2
5660	0	2	-1	0	1	2	1	0	1	1	2
1433	0	2	-1	0	0	2	0	0	2	2	1
2932	0	2	-1	0	1	2	1	0	1	2	2
5944	0	2	1	0	1	2	1	0	1	0	0





P = 3	discr	t	lambda	ch	a(0)	a(1)	a(2)	a(3)
-56	1	1	2	1	0	0	2	2
-35	1	1	3	1	0	0	0	2
-164	1	1	2	1	0	0	2	2
-47	1	1	2	1	0	0	0	1
-260	1	1	2	1	0	1	1	1
-296	1	1	3	1	0	1	1	0
-344	1	1	3	1	0	1	0	1
-404	1	1	2	1	0	2	2	0
-107	1	1	2	1	0	0	1	2
-452	1	1	2	1	0	1	1	1
-596	1	1	2	1	0	0	1	1
-632	1	1	2	1	0	0	2	1
-692	1	1	2	1	0	0	1	1
-841	1	1	2	1	0	0	1	1
-875	1	1	4	-1	0	0	0	1
-227	1	1	2	1	0	0	1	0
-920	1	1	2	1	0	0	2	2
-239	1	1	6	1	0	0	1	0
-1096	1	1	4	-1	0	2	0	0
-1144	1	1	3	-1	0	2	2	2
-287	1	1	2	1	0	0	0	2
-1208	1	1	3	1	0	0	1	1
-1220	1	1	3	1	0	0	1	1
-311	1	1	4	1	0	0	0	0
-323	1	1	2	1	0	0	2	2
-1336	1	1	2	-1	0	2	2	0
-1448	1	1	4	1	0	0	0	0
-1460	1	1	4	1	0	0	2	0
-1496	1	1	4	-1	0	0	0	0
-1508	1	1	2	-1	0	0	1	1
-379	1	1	2	-1	0	0	2	2
-1556	1	1	2	-1	0	0	1	1
-1604	1	1	2	-1	0	0	2	2
-1652	1	1	2	-1	0	0	1	1
-419	1	1	2	1	0	2	1	1
-1748	1	1	2	1	0	0	2	2
-1832	1	1	7	1	0	0	1	1
-1844	1	1	2	1	0	0	1	1
-467	1	1	2	1	0	0	1	1
-1928	1	1	3	1	0	0	1	1
-503	1	1	2	1	0	0	0	0
-2024	1	1	3	1	0	0	0	0
-2036	1	1	2	1	0	0	0	0
-312	1	1	5	1	0	0	2	2
-2260	1	1	2	-1	0	0	1	1
-587	1	1	3	1	0	0	0	0
-599	1	1	2	1	0	0	1	1
-2408	1	1	2	1	0	0	2	2
-611	1	1	2	1	0	0	1	1
-2468	1	1	2	1	0	0	2	0
-2516	1	1	5	1	0	0	0	0
-635	1	1	2	1	0	0	1	1
-2552	1	1	2	1	0	0	0	0
-2564	1	1	3	1	0	0	1	1
-2612	1	1	2	1	0	0	1	1
-659	1	1	3	1	0	0	1	1



discr	t	lambda	ch	a(0)	a(1)	a(2)	a(3)
92	2	2	-1	0	2	4	0
37	2	2	-1	0	2	4	0
109	0	2	-1	0	2	4	0
456	0	2	-1	0	2	4	0
508	2	2	-1	0	2	4	0
149	2	2	-1	0	2	4	0
253	2	2	-1	0	2	4	0
1068	2	2	-1	0	2	4	0
1112	2	2	-1	0	2	4	0
313	2	2	-1	0	2	4	0
1256	2	2	-1	0	2	4	0
1592	2	2	-1	0	2	4	0
433	0	2	-1	0	2	4	0
1736	0	2	-1	0	2	4	0
1756	0	2	-1	0	2	4	0
1784	2	2	-1	0	2	4	0
453	0	2	-1	0	2	4	0
457	2	2	-1	0	2	4	0
1684	0	2	-1	0	2	4	0
1684	0	2	-1	0	2	4	0
1882	2	2	-1	0	2	4	0
509	2	2	-1	0	2	4	0
581	0	2	-1	0	2	4	0
2348	0	2	-1	0	2	4	0
597	0	2	-1	0	2	4	0
2492	0	2	-1	0	2	4	0
2504	2	2	-1	0	2	4	0
629	2	2	-1	0	2	4	0
633	2	2	-1	0	2	4	0
2876	0	2	-1	0	2	4	0
2936	0	2	-1	0	2	4	0
753	2	2	-1	0	2	4	0
861	2	2	-1	0	2	4	0
869	2	2	-1	0	2	4	0
1074	2	2	-1	0	2	4	0
1074	2	2	-1	0	2	4	0
4316	0	2	-1	0	2	4	0
4344	0	2	-1	0	2	4	0
4444	0	2	-1	0	2	4	0
4524	0	2	-1	0	2	4	0













