

Supplement to
GENERALIZED EULER CONSTANTS
FOR ARITHMETICAL PROGRESSIONS

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TABLE 1. Generalized Euler constants

r	m	$\gamma_1(r, m)$	$\gamma_2(r, m)$	$\gamma_3(r, m)$	r	m	$\gamma_4(r, m)$	r	m	$\gamma_5(r, m)$
1	1	-.072815845484	-.009690363193	.002053834420	1	1	.002325370065	1	1	.000793323817
1	2	-.116342374613	-.037531477876	-.003680057906	1	2	-.010161154086	1	2	.012654115737
2	1	.043526529129	.027841114683	.005733892326	2	1	-.0078335784021	2	1	-.011860791920
2	2	-.140713732357	-.068200237651	-.022593835249	2	2	.010432667873	2	2	.032776359833
3	1	.081949254612	.030177519449	.0112323516171	3	1	-.033353821366	3	1	-.0359393355979
3	2	.014051367739	.028332350059	.0359883185840	3	2	.025246523599	3	2	.003410319963
3	3	-.054621867705	-.095836601153	.049281459556	3	3	-.0020757930311	3	3	.007452120746
4	1	.101909292109	.025427232255	.033365103088	4	1	-.066296210822	4	1	-.068345462519
4	2	.038279471092	.058305123277	.045601400650	4	2	.012236884397	4	2	-.034798005009
4	3	.058287629719	.002413882428	.039103402715	4	3	.058460426802	4	3	.056484670600
4	4	-.162593875615	-.114887169778	-.079268389317	4	4	-.025999192928	4	4	.048930547182
5	1	.114887169778	.019061176323	.056912306261	5	1	-.104142089763	5	1	-.1111424685061
5	2	.070722057769	.04481944199	.043950453133	5	2	.0126506429526	5	2	.087718356593
5	3	.08037543869	.043877910670	.070607447936	5	3	.072204793736	5	3	.038237272265
5	4	.087936395346	.027711304787	.023676628929	5	4	.072768286348	5	4	.112486546025
6	1	.154621867705	.139176874753	.109883221798	6	1	-.058509568679	6	1	-.034944311005
6	2	.124515444932	.012690432118	.08037058182	6	2	-.145231266671	6	2	-.163430958929
6	3	.093246167243	.084158309547	.03716817871	6	3	.043206725340	6	3	-.150327796218
6	4	.026308619180	.070976637102	.087389386549	6	4	.068942233752	6	4	-.002167951172
6	5	.042566190320	.017487087330	.069134992021	6	5	.111877445305	6	5	.128037600950
6	6	-.107297534982	-.055825954538	-.001284986031	6	6	.068453248899	6	6	.153738116181
7	1	.169244255291	.155734869524	.140073621837	7	1	-.096995439181	7	1	.00602878013
7	2	.132263831829	.006840433572	.0130357981772	7	2	-.188324668925	7	2	-.224287941863
7	3	.110937327314	.006840433572	.027939678028	7	3	-.077162572802	7	3	-.220088414373
7	4	.151495860096	.090396586899	.086087108183	7	4	.056188338638	7	4	-.056040532668
7	5	.009698991982	.049673576979	.099104662583	7	5	.130793869811	7	5	.114935077849
7	6	.067315366104	.010384009168	.052496076896	7	6	.128101070319	7	6	.20558685979
7	7	.120412739865	.080502442255	.030442087660	7	7	.049724775606	7	7	.174709305480
8	1	.17004726263	.169631361277	.168689893410	8	1	-.139407020606	8	1	-.036260631276
8	2	.138828447499	.001660411771	.124568423040	8	2	-.232497357815	8	2	-.292554546869
8	3	.12337832888	.094678160158	.017590007666	8	3	-.112883255608	8	3	-.295351113458
8	4	.070538259022	.104141353195	.100011190046	8	4	.037773451619	8	4	-.118833661513
8	5	.054251353580	.073794760124	.119588434854	8	5	.137331292025	8	5	.087712752023
8	6	.085098911796	.023766820484	.091289912651	8	6	.162001146992	8	6	.224209084350
8	7	.129321022001	.036373036882	.028011392984	8	7	.125120140005	8	7	.2605535108449
9	1	.1469916580856	.101727470766	.06090787331	9	1	-.18422404012	9	1	-.089897835619
9	2	.144579141797	.118341951960	.196074119696	9	2	-.277068725434	9	2	-.367041247266
9	3	.134258462330	.002842530326	.145043800180	9	3	.149460268433	9	3	-.374921180375
9	4	.086529530967	.097789619582	.006827957362	9	4	.015868820895	9	4	-.187849453407
9	5	.035370528883	.009258926335	.115061526404	9	5	.135856913040	9	5	.040327486400
9	6	.012927460984	.050422039116	.133913741232	9	6	.190136194051	9	6	.220766525708
9	7	.05732662468	.001919812094	.072494107659	9	7	.178787890951	9	7	.310523648858
9	8	.098000416068	.059569213576	.000105457223	9	8	.071857991027	9	8	.291320404887
9	9	.135382369085	.119879303689	-.091153858091	9	9	-.015429402059	9	9	.157564974630

TABLE 1 (continued)

ε	m	γ_6 (r,m)	ε	m	γ_7 (r,m)	ε	m	γ_8 (r,m)	ε	m	γ_9 (r,m)	ε	m	γ_{10} (r,m)
1	1	-0.00238769345	1	1	-0.00527289567	1	1	-0.00352123354	1	1	-0.00034394774	1	1	0.00205332815
1	2	0.08223513327	1	2	0.0015355901	1	2	-0.008384099565	1	2	-0.014073779844	1	2	-0.013711459557
2	1	-0.008462282673	2	1	-0.00680845468	2	1	-0.008031976212	2	1	0.014039395632	2	1	0.013916792372
2	2	0.41822252695	2	2	0.32971198360	2	2	0.001913180171	2	2	-0.051769595632	2	2	-0.118801948194
2	3	-0.18572400632	2	3	-0.13764130707	2	3	-0.054187729501	2	3	0.088446310048	2	3	-0.093011704921
3	1	-0.023488621408	3	1	-0.047262618635	3	1	-0.056453033026	3	1	-0.036711109191	3	1	0.025995576088
3	2	-0.093066361189	3	2	-0.118571056674	3	2	-0.048159232224	3	2	-0.020644330571	3	2	-0.274389304064
3	3	-0.033560681737	3	3	-0.042751376390	3	3	-0.048453232224	3	3	0.283703933148	3	3	-0.353831740207
4	1	-0.084842847862	4	1	-0.118447500772	4	1	-0.156998808279	4	1	0.00657050527	4	1	0.260677844507
4	2	-0.025098390664	4	2	-0.04343221858	4	2	-0.103200022789	4	2	-0.0690948079	4	2	-0.339914947835
4	3	-0.14612975380	4	3	-0.04343221858	4	3	-0.148966832067	4	3	0.269890948079	4	3	-0.339914947835
5	1	-0.058829800552	5	1	0.2491840455173	5	1	0.306876656406	5	1	0.206540139321	5	1	0.952217161182
5	2	-0.164667076451	5	2	0.077917086439	5	2	-0.319228560814	5	2	0.649761468871	5	2	0.794875423022
5	3	-0.045229227978	5	3	-0.20595658789	5	3	-0.139482317503	5	3	0.146664550476	5	3	-0.444035983328
5	4	-0.123257560256	5	4	-0.186780365675	5	4	-0.370966974090	5	4	-0.522069354506	5	4	-0.444035983328
5	5	-0.188500231563	5	5	0.064747602341	5	5	-0.116008048981	5	5	-0.480931198936	5	5	-0.444035983328
6	1	-0.098442719308	6	1	0.405254852094	6	1	0.638101193356	6	1	0.716652106517	6	1	0.219295803214
6	2	-0.260147038912	6	2	-0.310295493364	6	2	-0.1700882046278	6	2	1.210745568984	6	2	0.033064031978
6	3	-0.146677978868	6	3	-0.108569933537	6	3	0.530590976145	6	3	1.210745568984	6	3	0.033064031978
6	4	-0.079870318676	6	4	-0.372283653734	6	4	-0.6361888013185	6	4	0.391573372575	6	4	1.707045064286
6	5	-0.236658415504	6	5	-0.094805802829	6	5	-0.476403246644	6	5	-0.768421702148	6	5	-0.338097751408
6	6	-0.212581929718	6	6	0.263032874729	6	6	1.13629013252	6	6	-0.12229925894	6	6	-0.338097751408
7	1	-0.154705233391	7	1	0.566673587657	7	1	0.067031192738	7	1	1.543166526721	7	1	1.94005232706
7	2	-0.370248054740	7	2	-0.124815013393	7	2	-0.204990050605	7	2	1.543166526721	7	2	1.94005232706
7	3	-0.269690372017	7	3	-0.435876760682	7	3	-0.30307062925	7	3	1.965735882503	7	3	3.700770890869
7	4	-0.06152259761	7	4	-0.59089948347	7	4	-0.937886168265	7	4	0.734016142610	7	4	0.20843660443
7	5	-0.242868174400	7	5	-0.321972469715	7	5	-0.30307062925	7	5	-0.187128770902	7	5	-0.82124273861
7	6	-0.34510704646	7	6	0.139228674371	7	6	0.539721726817	7	6	-0.140466782184	7	6	-0.82124273861
7	7	-0.214574572377	7	7	0.517504613756	7	7	0.539721726817	7	7	0.033317808677	7	7	-0.82124273861
8	1	-0.228515370208	8	1	0.716854675261	8	1	1.563757633028	8	1	2.678358323847	8	1	3.363694383613
8	2	-0.494243662483	8	2	0.11871618264	8	2	0.029136524923	8	2	2.895276701692	8	2	6.011782786229
8	3	-0.40935223352	8	3	-0.585558738664	8	3	-0.258623747032	8	3	1.156470758138	8	3	4.828907696363
8	4	-0.12550821188	8	4	-0.839008465119	8	4	-0.27698118031	8	4	-0.122379544221	8	4	6.45384176880
8	5	-0.194954688472	8	5	-0.598283618587	8	5	-0.46894170980	8	5	-0.26900265442	8	5	-0.63808368768
8	6	-0.409400814621	8	6	-0.075964806874	8	6	0.872137716644	8	6	-0.261154368854	8	6	-0.63808368768
8	7	-0.434451622416	8	7	0.467141237871	8	7	0.152623724243	8	7	-0.11499020741	8	7	-0.65795104602
8	8	-0.193162528793	8	8	0.795576243261	8	8	0.128014348246	8	8	0.954102494133	8	8	4.56822985186
9	1	-0.319805864559	9	1	0.843462291645	9	1	0.2097408443044	9	1	4.088699769393	9	1	6.254540406751
9	2	-0.631364928179	9	2	0.084461139666	9	2	1.277630061813	9	2	3.968502624856	9	2	8.979103893567
9	3	-0.562707598048	9	3	-0.761836587613	9	3	-0.33402392695	9	3	1.635280584249	9	3	7.042243913143
9	4	-0.258163136510	9	4	-0.11500641823	9	4	-0.65723222885	9	4	-0.145219158000	9	4	4.37413865120
9	5	-0.10986202890	9	5	-0.913749614683	9	5	-0.260798845929	9	5	-0.35968221842	9	5	1.526683063737
9	6	-0.411367321950	9	6	-0.357044522770	9	6	-0.55589844197	9	6	-0.39297464270	9	6	-0.792745344128
9	7	-0.55936600438	9	7	0.304515324950	9	7	-0.43826234021	9	7	-0.68927781203	9	7	-0.9002541868
9	8	-0.498090103881	9	8	0.843052605724	9	8	-0.844546126975	9	8	-0.2833231096383	9	8	-0.904205104226
9	9	-0.498090103881	9	9	1.071618491749	9	9	1.8334682194259	9	9	2.320982949259	9	9	-0.904205104226

TABLE I (continued)

Γ	m	$\gamma_{11}(r,m)$	Γ	m	$\gamma_{12}(r,m)$	Γ	m	$\gamma_{13}(r,m)$	Γ	m	$\gamma_{14}(r,m)$	Γ	m	$\gamma_{15}(r,m)$
1	1	.000270184440	1	1	.000167272912	1	1	-.000027463807	1	1	-.000202920262	1	1	-.000283468655
1	2	-.004972466941	1	2	.012281471347	1	2	.034262249392	1	2	.051521179298	1	2	.048245777323
2	2	.005242653381	2	2	-.012114198435	2	2	-.034289713199	2	2	-.051730388561	2	2	-.048529245978
1	3	-.172117762506	1	3	-.158228544040	1	3	.006137160656	1	3	.418926603563	1	3	1.134802733348
2	3	.035138822840	2	3	-.120239342090	2	3	-.389595040618	2	3	-.726281802145	2	3	-.944289793547
3	3	.137249624106	3	3	.278635159042	3	3	.383430416156	3	3	.307145989320	3	3	-.190796408456
1	4	-.685629001784	1	4	-.117176137502	1	4	-.140573988637	1	4	-.5935029579167	1	4	2.771811721673
2	4	.226511275189	2	4	-.331485282674	2	4	-.161940953495	2	4	-.383707242535	2	4	-.6.60570527798
3	4	.68065632843	3	4	1.184042846370	3	4	1.440001115761	3	4	.645030758465	3	4	-.2.72336594435
4	4	-.221268621808	4	4	-.319371084239	4	4	1.585119821753	4	4	3.785342036792	4	4	6.557176032002
1	5	-.1.36809231760	1	5	-.3.41224601178	1	5	-.6.25896633633	1	5	-.8.37382933590	1	5	-.4.71115459539
2	5	.885164199210	2	5	-.300325269110	2	5	-.3.94289518325	2	5	-.11.8895127493	2	5	-.10.7035191613
3	5	1.899491631717	3	5	3.282764077152	3	5	3.983707441559	3	5	1.292012705955	3	5	-.25.4111176868
4	5	-.257808881789	4	5	2.227581213888	4	5	6.233832708795	4	5	12.50033732111	4	5	18.72855024625
5	5	-.1.67410221068	5	5	-.1.79760673724	5	5	-.1570609458135	5	5	6.470782828902	5	5	22.09695772867
1	6	-.1.70291378171	1	6	-.6.44689565691	1	6	-.15.6406658852	1	6	-.29.3436877637	1	6	-.40.3768649882
2	6	2.420637414169	2	6	.741334154255	2	6	-.6.64857809037	2	6	-.26.4270133923	2	6	-.67.8346681310
3	6	4.08344040102	3	6	7.320750624605	3	6	9.415945084885	3	6	3.694477352860	3	6	-.26.4652675719
4	6	1.530796019208	4	6	6.288667112873	4	6	15.64680304590	4	6	29.76261436729	4	6	41.51166772159
5	6	-.2.38549909133	5	6	-.861573496345	5	6	6.258983049750	5	6	25.70073159017	5	6	66.89037833746
6	6	-.3.94619078000	6	6	-.7.04211546556	6	6	-.9.03251466873	6	6	-.3.38733136354	6	6	26.27447116345
1	7	-.1.0399737655	1	7	-.9.11822636466	1	7	-.28.8237884006	1	7	-.67.5665740006	1	7	-.125.720984436
2	7	5.255040852722	2	7	3.865343988922	2	7	-.7.93244823086	2	7	-.46.6583972714	2	7	-.142.286127996
3	7	7.501998973766	3	7	14.16272335406	3	7	19.76183912749	3	7	11.07921473540	3	7	-.49.2206708256
4	7	3.799820149942	4	7	13.21340373739	4	7	31.80399799333	4	7	59.94889303002	4	7	81.66619564592
5	7	-.2.52813961648	5	7	2.424944699302	5	7	19.55830129789	5	7	62.39753130196	5	7	148.3459416373
6	7	-.6.79284511092	6	7	-.9.74492190898	6	7	-.6.61956739212	6	7	18.84589838422	6	7	103.7916134528
7	7	-.6.19560768804	7	7	-.14.8033002331	7	7	-.27.7485976589	7	7	-.38.0467753888	7	7	-.16.5762509474
1	8	1.243554316101	1	8	-.9.86103354137	1	8	-.43.3477314008	1	8	-.122.975001395	1	8	-.277.407559897
2	8	9.759064805996	2	8	10.27812954893	2	8	-.5.07278846759	2	8	-.68.9123278000	2	8	-.251.608470378
3	8	12.37779168746	3	8	24.70221759259	3	8	37.52961399098	3	8	28.58969892298	3	8	-.73.3373610440
4	8	7.215067665544	4	8	23.68155136115	4	8	56.91054684692	4	8	108.664023942	4	8	149.7786850078
5	8	-.1.92918331789	5	8	6.889272166349	5	8	41.94199253447	5	8	122.3814918155	5	8	280.1793716182
6	8	-.9.53255353081	6	8	-.10.6096148316	6	8	3.453378932636	6	8	65.07525537461	6	8	245.0007651003
7	8	-.11.6971351546	7	8	-.23.5181747462	7	8	-.36.0896128752	7	8	-.27.9446681645	7	8	478.9886475132
8	8	-.7.43633628735	8	8	-.23.3621802769	8	8	-.55.3254270252	8	8	-.104.87900357	8	8	88.143.221508976
1	9	5.661333921339	1	9	-.6.99253135359	1	9	-.55.4992538150	1	9	-.191.128550311	1	9	-.502.7239914100
2	9	16.22062794904	2	9	21.17803065405	2	9	5.261380928630	2	9	-.86.7667135421	2	9	-.391.123901371
3	9	18.87378029227	3	9	39.78710747741	3	9	65.51783997270	3	9	63.09668212667	3	9	-.87.5080860071
4	9	11.87717263746	4	9	38.31081948984	4	9	93.28739107073	4	9	182.5864178257	4	9	260.6196913594
5	9	-.48737791607	5	9	18.44744157255	5	9	75.42023079870	5	9	212.0191734363	5	9	478.9886475132
6	9	-.11.9583082436	6	9	-.9.01160003573	6	9	23.25332448098	6	9	65.07525537461	6	9	468.1879536521
7	9	-.17.7106243213	7	9	-.31.4765166803	7	9	-.81.0715667679	7	9	8.961059088790	7	9	243.2450254744
8	9	-.15.6981118346	8	9	-.37.7457115687	8	9	-.81.0715667679	8	9	-.125.970741696	8	9	-.88.8090359362
9	9	-.6.77822242461	9	9	-.30.4968722826	9	9	-.88.3877380375	9	9	-.204.5873998617	9	9	-.380.8706664053

TABLE 1 (continued)

ε	m	γ_{16} (r,m)	ε	m	γ_{17} (r,m)	ε	m	γ_{18} (r,m)	ε	m	γ_{19} (r,m)	ε	m	γ_{20} (r,m)
1	1	-.00019969858	1	1	.000026277037	1	1	.000307368408	1	1	.000503605453	1	1	.000466343562
1	2	.00449974400	1	2	-.096180724564	1	2	-.24956296862	1	2	-.404603661751	1	2	-.428627343578
2	2	-.004649671258	2	2	.096207001601	2	2	.249965665270	2	2	.405107267204	2	2	.429093687139
1	3	.027626469882	1	3	.2534271419615	1	3	1.27793732848	1	3	4.29718599785	1	3	17.78397995762
2	3	-.615749373982	2	3	1.011204307555	2	3	4.934291975083	2	3	11.84568639208	2	3	20.53897539590
3	3	-1.41207679256	3	3	-3.54544945013	3	3	-6.21137811482	3	3	-7.54799678878	3	3	-2.75452947618
1	4	10.92403104638	1	4	25.8952111647	1	4	46.06129272078	1	4	57.25808369044	1	4	14.41188139517
2	4	-7.90551663500	2	4	-2.15590673785	2	4	-22.43845757122	2	4	-85.60007476840	2	4	209.8377721064
3	4	-10.9195810720	3	4	-25.9913918410	3	4	-46.3109510176	3	4	-57.6626873522	3	4	-14.8405087387
4	4	7.900866963745	4	4	2.252113739449	4	4	-22.1884919059	4	4	-85.1949675012	4	4	-209.4086784919
1	5	16.89626587612	1	5	80.13397310595	1	5	220.5148917284	1	5	465.2641808714	1	5	750.1772407941
2	5	-41.4179649452	2	5	-42.7183962347	2	5	-22.54630432954	2	5	-275.8790814819	2	5	947.6350964155
3	5	-42.3974867488	3	5	-107.028816798	3	5	-208.198268501	3	5	-297.928565803	3	5	-168.890503535
4	5	15.50632035125	4	5	-22.4740353240	4	5	-149.389732807	4	5	-457.564047119	4	5	-1050.30707922
5	5	51.41266576977	5	5	92.08730152725	5	5	114.5270766193	5	5	14.34985417473	5	5	-478.551288113
1	6	-20.2052738495	1	6	110.3424536240	1	6	529.4277880367	1	6	1561.218239155	1	6	3609.854215520
2	6	-135.734026202	2	6	-204.435305071	2	6	-144.686849064	2	6	444.7053668418	2	6	2556.016152362
3	6	-114.908553004	3	6	-315.885143727	3	6	-679.298587148	3	6	-1128.76316057	3	6	-1074.80566590
4	6	22.23290031921	4	6	-107.808182204	4	6	-528.150394304	4	6	-1565.51542515	4	6	-3627.63819510
5	6	135.1182768279	5	6	205.4465093785	5	6	149.6211408142	5	6	-432.859682250	5	6	-2535.477171697
6	6	113.4964762114	6	6	312.3396942769	6	6	673.0872090329	6	6	1121.215163779	6	6	1072.051136421
1	7	-166.495319616	1	7	-32.8516127388	1	7	744.4414849071	1	7	3311.760082182	1	7	9978.316035806
2	7	-333.558296967	2	7	-623.398603319	2	7	-810.280943334	2	7	-5750.46096080	2	7	4571.286789308
3	7	-247.218188051	3	7	-745.676085925	3	7	-1766.02727767	3	7	-3344.59989192	3	7	-4343.51369052
4	7	29.16517301746	4	7	-296.998393922	4	7	-1375.35074745	4	7	-4149.99762558	4	7	-9982.10607112
5	7	279.4839175053	5	7	371.6264249304	5	7	53.44120691526	5	7	-1809.09778705	5	7	-8042.47109236
6	7	317.6063263786	6	7	753.3725444649	6	7	1425.535755126	6	7	1865.744715989	6	7	-79.5413485848
7	7	121.0161880360	7	7	573.9259527860	7	7	1728.240828879	7	7	4126.756056091	7	7	7898.029843815
1	8	-497.860213110	1	8	-580.189815984	1	8	335.2725405840	1	8	4895.156100787	1	8	19853.76364760
2	8	-675.251654956	2	8	-1470.62059598	2	8	-2494.9937748	2	8	-2298.78707201	2	8	4890.244929758
3	8	-451.327957583	3	8	-1498.6868576	3	8	-3881.85006436	3	8	-8211.33554313	3	8	-13093.2277739
4	8	45.17464941094	4	8	-629.055609799	4	8	-2959.40935866	4	8	-9269.63438733	4	8	-23394.4013517
5	8	508.7842441562	5	8	606.0850271007	5	8	-289.211247863	5	8	-4837.89801710	5	8	-19839.3517662
6	8	667.3461383206	6	8	1468.464692846	6	8	2517.432235056	6	8	2384.387146775	6	8	-4680.40715765
7	8	440.4083765106	7	8	1472.695463915	7	8	3835.539113340	7	8	8154.439419827	7	8	13078.38726517
8	8	-37.2737824472	8	8	631.3077235382	8	8	2937.220866756	8	8	9184.439419827	8	8	23184.99267323
1	9	-1082.95356818	1	9	-1806.76966716	1	9	-1506.43364715	1	9	4490.193006927	1	9	30612.22133312
2	9	-1189.25820350	2	9	-2925.47733057	2	9	-5870.51953085	2	9	-8392.98491574	2	9	-893.747991678
3	9	-726.99568620	3	9	-2668.12258320	3	9	-7510.25203129	3	9	-17458.7462453	3	9	-32285.1024927
4	9	90.15716936638	4	9	1126.22713632	4	9	-5569.12955166	4	9	-18267.3009004	4	9	-48535.6146470
5	9	856.4231440942	5	9	943.2821581536	5	9	4009.969880104	5	9	-10398.7785931	5	9	-41765.3397218
6	9	1206.770826425	6	9	2539.194083292	6	9	409.078664592	6	9	2289.147964095	6	9	-15398.5570832
7	9	994.8240252861	7	9	2935.531074897	7	9	7076.840592537	7	9	13772.81070744	7	9	17905.60933434
8	9	332.2193100303	8	9	1983.206356726	8	9	6860.423702705	8	9	18803.60919525	8	9	42679.62668891
9	9	-481.187246398	9	9	125.3830504623	9	9	3494.961988586	9	9	15162.05028440	9	9	47680.90504639