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CONSTRUCTING INTEGRAL LATTICES WITH PRESCRIBED MINIMUM. II

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Dedicated to the memory of H. Zassenhaus

ABSTRACT. Integral laminated lattices with minimum 4 which are generated by vectors of minimum length are constructed systematically together with their automorphism groups. All lattices obtained lie in the Leech lattice.

1. INTRODUCTION

In this paper we continue our investigation of the lamination process of integral lattices with prescribed minimum m started in [6]. More precisely, we consider all series $\Lambda_1 \subseteq \Lambda_2 \subseteq \Lambda_3 \subseteq \dots$ of i-dimensional lattices Λ_i in Euclidean space with scalar product $\langle\ ,\ \rangle$ such that

(i) Λ_i is integral;
(ii) $m = \min(\Lambda_i) := \min\{\langle x, x\rangle \mid 0 \neq x \in \Lambda_i\}$;
(iii) Λ_i is generated by its vectors of minimum norm;
(iv) $\det(\Lambda_i)$ is minimal amongst all such i-dimensional lattices containing Λ_{i-1}, where $\det(\Lambda_i)$ is the determinant of a Gram matrix of Λ_i.

In [6] we showed that every such sequence for minimum $m = 3$ necessarily contains a certain unimodular lattice Λ_{23} of dimension 23. Here we prove that for minimum $m = 4$, the lattice Λ_{24} must necessarily be the Leech lattice. Thus, we have an affirmative answer for $m = 2, 3, 4$ to the problem raised by John Thompson, whether each such series necessarily passes through a unimodular lattice. Essentially the same techniques as in [6] are applied. In §3 we investigate the effect of the lamination process on some lattices which are denser than the laminated lattices but which still have minimum 4, are integral, and which are generated by vectors of minimum norm.

2. MINIMUM 4: THE LAMINATED LATTICES (Λ-LATTICES)

The main result of our computations is the following theorem.

Theorem 2.1. *Let $\Lambda_1 \subseteq \Lambda_2 \subseteq \dots$ be a series of integral laminated lattices with minimum $m = 4$. Then Λ_{24} is the Leech lattice. More precisely, all possible sequences $\Lambda_1, \dots, \Lambda_{24}$ can be read off from the diagram in Figure* 1 *to the right of the dotted line. Invariants of the lattices are described in Table* 1, *and Gram matrices are given in an appendix to this section.*

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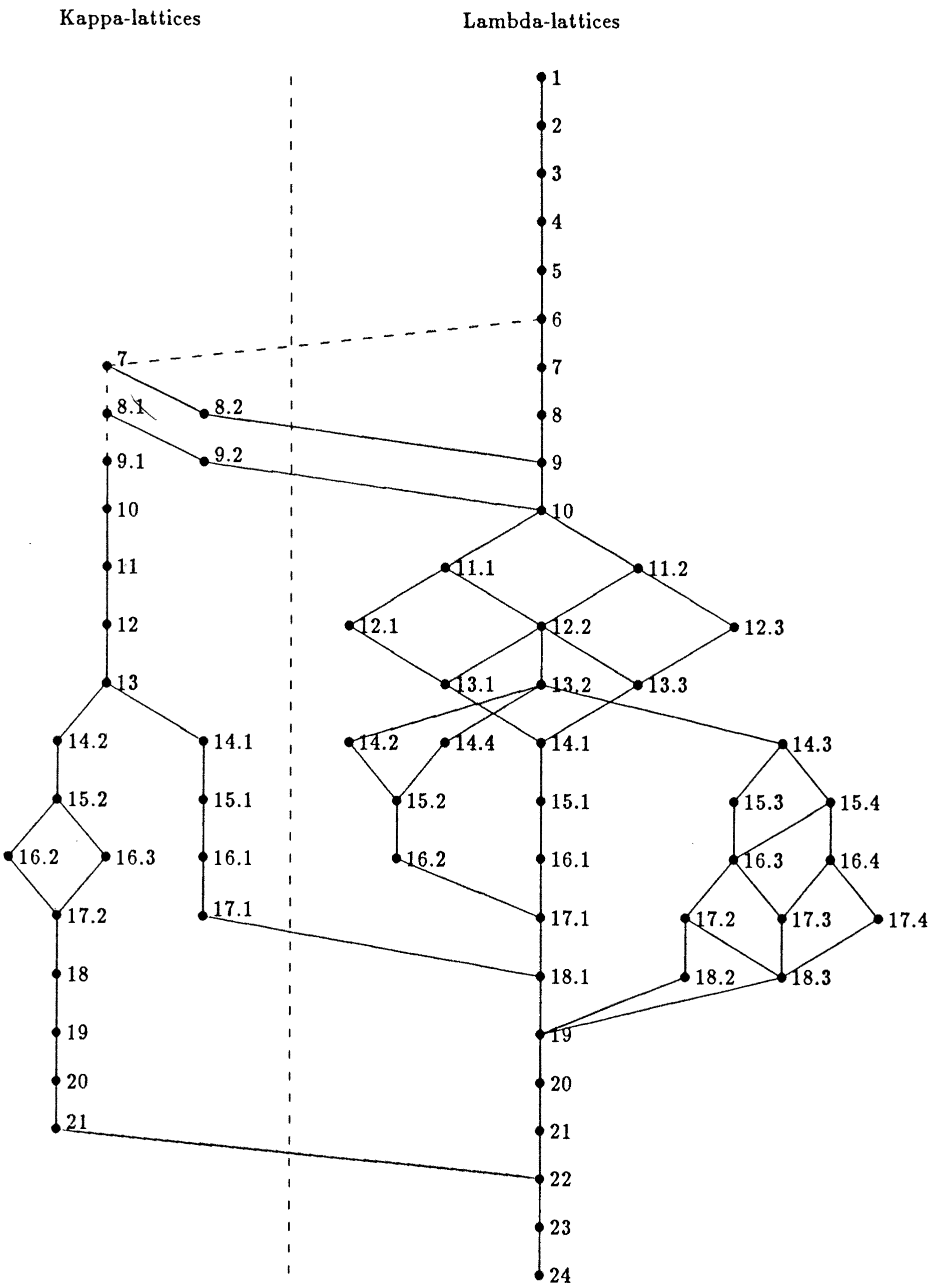
Kappa-lattices
Lambda-lattices
1
2
3
4
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11.1
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12.1
12.2
12.3
13.1
13.2
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14.1
15.2
15.1
16.2
16.3
16.1
17.2
17.1
18
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21

FIGURE 1

TABLE 1. Invariants of the laminated lattices Λ_i

dim	det	$\frac{1}{2}\lvert L_{\min}\rvert$	$\lvert\mathrm{Aut}(L)\rvert$
1	$4 = 4$	1	2
2	$12 = 2 \cdot 6$	3	$2^2 3$
3	$32 = 2^2 8$	6	$2^4 3$
4	$64 = 2^2 4^2$	12	$2^7 3^2$
5	$128 = 2^4 8$	20	$2^8 3 \cdot 5$
6	$192 = 2^5 6$	36	$2^8 3^4 5$
7	$256 = 2^6 4$	63	$2^{10} 3^4 5 \cdot 7$
8	$256 = 2^8$	120	$2^{14} 3^5 5^2 7$
9	$512 = 2^6 8$	$136 = 16 + 56 + 64$	$2^{15} 3^2 5 \cdot 7$
10	$768 = 2^4 4 \cdot 12$	$168 = 24 + 48 + 96$	$2^{15} 3^3$
11.1	$1024 = 2^4 4^3$	$219 = 3 + 24 + 2 \cdot 96$	$2^{17} 3^3$
11.2	$1024 = 2^2 4^2 16$	$216 = 8 + 48 + 64 + 96$	$2^{15} 3$
12.1	$1024 = 2^6 4^2$	$324 = 36 + 288$	$2^{20} 3^5$
12.2	$1024 = 2^2 4^4$	$316 = 4 + 8 + 48 + 64 + 192$	$2^{18} 3$
12.3	$1024 = 4^2 8^2$	$312 = 24 + 96 + 192$	$2^{14} 3^2$
13.1	$1024 = 2^4 4^3$	$453 = 1 + 12 + 24 + 128 + 288$	$2^{19} 3^2$
13.2	$1024 = 4^5$	$445 = 5 + 40 + 80 + 320$	$2^{15} 3 \cdot 5$
13.3	$1024 = 2^2 4^2 16$	$444 = 4 + 8 + 48 + 64 + 128 + 192$	$2^{14} 3$
14.1	$768 = 2^4 4 \cdot 12$	$711 = 3 + 36 + 288 + 384$	$2^{15} 3^3$
14.2	$1024 = 2^2 4^4$	$614 = 2 + 4 + 2 \cdot 8 + 16 + 32 + 2 \cdot 48 + 192 + 256$	$2^{17} 3$
14.3	$1024 = 2^2 4^4$	$606 = 6 + 120 + 480$	$2^{17} 3^2 5$
14.4	$1024 = 4^3 16$	$605 = 1 + 4 + 8 + 3 \cdot 32 + 48 + 2 \cdot 96 + 128$	$2^{12} 3$
15.1	$512 = 2^6 8$	$1170 = 210 + 960$	$2^{17} 3^2 5 \cdot 7$
15.2	$768 = 2^2 4^2 12$	$936 = 1 + 3 + 12 + 24 + 96 + 128 + 2 \cdot 144 + 384$	$2^{13} 3^2$
15.3	$1024 = 4^5$	$815 = 1 + 6 + 16 + 48 + 72 + 2 \cdot 192 + 288$	$2^{17} 3^2$
15.4	$1024 = 2^2 4^2 16$	$798 = 2 + 4 + 8 + 32 + 48 + 2 \cdot 64 + 2 \cdot 96 + 128$ $+256$	$2^{14} 3$
16.1	$256 = 2^8$	2160	$2^{21} 3^5 5^2 7$
16.2	$512 = 2^4 4 \cdot 8$	$1491 = 1 + 90 + 120 + 320 + 960$	$2^{15} 3^2 5$
16.3	$768 = 4^3 12$	$1201 = 2 \cdot 1 + 2 \cdot 2 + 3 + 3 \cdot 8 + 16 + 2 \cdot 24$ $+2 \cdot 32 + 3 \cdot 48 + 2 \cdot 64 + 2 \cdot 96 + 128 + 256$	$2^{13} 3$
16.4	$768 = 2^2 8 \cdot 24$	$1182 = 6 + 24 + 2 \cdot 96 + 3 \cdot 192 + 384$	$2^{11} 3$
17.1	$256 = 2^6 4$	$2673 = 1 + 512 + 2160$	$2^{19} 3^4 5 \cdot 7$
17.2	$512 = 2^2 4^2 8$	$1860 = 2 + 6 + 12 + 2 \cdot 24 + 2 \cdot 48 + 3 \cdot 96 + 256$ $+384 + 768$	$2^{16} 3$
17.3	$512 = 4^2 32$	$1827 = 1 + 2 + 2 \cdot 4 + 8 + 3 \cdot 16 + 7 \cdot 32 + 6 \cdot 64$ $+3 \cdot 128 + 256 + 512$	2^{11}
17.4	$512 = 2^3 8^2$	$1818 = 18 + 24 + 96 + 144 + 384 + 1152$	$2^{13} 3^2$
18.1	$192 = 2^5 6$	$3699 = 3 + 1536 + 2160$	$2^{17} 3^5 5$
18.2	$256 = 2^4 4^2$	$3250 = 2 + 64 + 240 + 1024 + 1920$	$2^{21} 3^2 5$
18.3	$256 = 2^2 8^2$	$3168 = 12 + 36 + 96 + 144 + 192 + 1152 + 1536$	$2^{15} 3^2$
19	$128 = 2^4 8$	$5334 = 6 + 96 + 240 + 1920 + 2072$	$2^{19} 3^2 5$
20	$64 = 2^2 4^2$	$8700 = 60 + 960 + 7680$	$2^{21} 3^3 5$
21	$32 = 2^2 8$	$13860 = 420 + 13440$	$2^{19} 3^3 5 \cdot 7$
22	$12 = 2 \cdot 6$	24948	$2^{17} 3^7 5 \cdot 7 \cdot 11$
23	$4 = 4$	46575	$2^{19} 3^6 5^3 7 \cdot 11 \cdot 23$
24	1	98280	$2^{22} 3^9 5^4 7^2 11 \cdot 13 \cdot 23$

Table 1 lists the dimensions of the lattices in the first column. If there is more than one lattice in dimension i, the lattices are listed as $i.1$, $i.2$ etc. as in the figure. The second column lists the determinants of the lattices as a product of the elementary divisors of the Gram matrices. Column 3 gives half of the number of (shortest) vectors of length 4 according to the lengths of orbits of the automorphism groups of the lattices on the set of shortest vectors up to sign. These orbits were computed with GAP, cf. [7]. Finally, the last column gives the order of the automorphism group of the lattices in prime factorization.

The proof of the theorem is given by computation in exactly the same way as in [6]. However, we meanwhile could use a completely automatic implementation for finding generators and the orders of the automorphism groups of the lattices due to B. Souvignier, cf. [8]. The implementation follows exactly our suggestions and is complemented by Sim's idea of strong generators for permutation groups to ensure that one has the full automorphism group, and it also obtains the order as a product of orbit lengths of stabilizers in the stabilizer chain belonging to the base. In those cases where the number of vectors of minimum length was too big, we computed the automorphism group of the dual lattice, which usually had comparatively few vectors of small length. However, the dual lattice did not always have a basis of vectors of minimum length, which forced us to consider the action of vectors of various lengths.

Remarks. 1. The lattices up to dimension 8 are 2-scaled versions of the (unique) series of integral laminated lattices for minimum $m = 2$, namely A_1, A_2, A_3, D_4, D_5, E_6, E_7, E_8.

2. For each of the lattices Λ_i of Theorem (2.1) the embedding number into Λ_{i+1} is equal to 1. (The embedding number was defined in [6] to be the number of orbits of $\mathrm{Aut}(\Lambda_i) \times \mathrm{Aut}(\Lambda_{i+1})$ on the set of (isometric) embeddings $\sigma : \Lambda_i \to \Lambda_{i+1}$.)

3. The above result can be compared to [1, Chapter 6]. There, a geometric rather than an arithmetic lamination process was considered. More precisely, conditions (i) and (iii) of the introduction are dropped and (iv) is replaced by a more restrictive condition which implies that their laminated lattices of the same dimension all have the same determinant. In that case, Conway and Sloane only obtain the lattices which are in the central part of our picture, if they scale their lattices to have minimum 4. This shows that minimum 4 plays a distinguished role for the integral lamination process. However, we obtain more than one lattice in dimension 14 to 18. In principle, some of these lattices could have a smaller determinant than the ones of the same dimension in the main branch. However, they all turn out to have bigger determinants.

4. For a relation of some of the lattices here with the ones for minimum 3 in [6] cf. the comments in that paper.

5. Theorem (2.1) does not only give the possible sequences of integral laminated lattices up to dimension 24, but indeed also beyond. This is because Λ_{24} is unimodular and hence splits off as an orthogonal direct summand of the Λ_i for $i > 24$.

6. The automorphism groups of the lattices $\Lambda_{16.1}$, Λ_{22}, Λ_{23}, and Λ_{24} are maximal finite subgroups of $GL_n(\mathbb{Q})$ for these dimensions n. In the terminology of [5] they are $[2_+^{1+8} \cdot O_8^+(2)]_{16}$, $[\pm PSU_6(2) \cdot S_3]_{22}$, $[\pm Co_2]_{23}$, and $[2.Co_1]_{24}$. The lattice Λ_{22} is the even sublattice of the lattice $\Lambda_{22}^a(3)$ of deter-

minant 3 and minimum 3 which is obtained in [6] by the lamination process for minimum 3.

Appendix. Gram matrices. Gram matrices of all lattices which come up in Theorem (2.1) are listed below. Since the matrices are symmetric, only their lower half is given explicitly. Also, the matrices are given such that for each pair of lattices $(\Lambda_i, \Lambda_{i+1})$, the first i rows of the Gram matrix belong to Λ_i and the first $i+1$ rows to Λ_{i+1}. No parts of matrices are given twice, so that complete Gram matrices have to be put together from various series according to the number in parentheses at the beginning of each series indicating with which first rows to start. No isometry is given when two Gram matrices are listed for the same lattice (though they were computed of course).

Series 1 ... 10 11.1 ... 18.1 19 ... 24

4																							
2	4																						
0	−2	4																					
0	−2	0	4																				
0	0	−2	0	4																			
−2	−2	0	0	0	4																		
0	0	0	0	0	−2	4																	
0	0	0	0	0	0	−2	4																
0	0	0	0	1	−1	0	0	4															
0	0	0	0	−1	0	0	0	0	4														
0	0	0	0	0	0	0	0	−2	0	4													
0	0	0	0	0	0	0	0	0	0	−2	4												
1	0	−1	1	1	0	0	−1	1	1	0	−1	4											
−1	−1	1	−1	0	0	1	0	1	1	−1	1	0	4										
0	0	0	0	0	0	0	0	0	−1	0	0	0	1	4									
0	0	0	0	0	0	0	0	−1	1	0	0	1	1	0	4								
0	0	1	0	−2	0	0	0	0	0	0	−1	0	0	0	0	4							
1	0	0	1	0	0	−1	0	1	0	−1	1	1	0	0	0	1	4						
0	0	0	−1	0	1	0	−1	0	0	0	0	1	1	1	0	1	0	4					
−1	−1	1	0	−1	1	0	−1	0	1	−1	1	0	1	0	0	1	1	0	4				
0	0	−1	1	1	0	0	−1	0	0	0	1	1	0	0	1	0	1	1	0	4			
0	1	−1	−1	1	−1	1	0	0	0	0	1	0	1	0	1	0	0	1	0	1	4		
0	−1	0	1	0	1	−1	1	0	1	0	−1	1	0	0	0	1	1	0	1	0	0	4	
−2	−1	0	0	0	1	0	0	0	0	0	0	−1	1	0	0	1	0	1	1	1	1	1	4

Series (11.1) 12.2 13.2 14.2 15.2 16.2 17.1

0	1	−2	0	1	0	0	−1	1	0	0	4					
−1	0	−1	−1	1	0	1	0	1	1	0	0	4				
0	0	0	0	0	0	0	0	0	1	−2	0	−1	4			
0	0	0	0	0	0	−1	1	0	1	1	0	1	0	4		
0	0	0	0	0	0	0	0	0	−1	0	1	−1	0	1	4	
0	0	0	0	0	0	0	0	−1	1	0	0	0	0	1	0	4

Series (13.2) 14.3 15.3 16.3 17.2 18.2 19

−2	−1	0	0	0	1	0	−1	1	1	0	1	0	4					
0	0	0	0	0	0	0	0	0	−1	0	1	−1	0	4				
−1	0	0	0	−1	0	0	0	1	0	0	0	1	1	1	4			
0	0	0	0	0	0	0	0	0	0	0	0	−1	1	0	0	4		
0	0	0	0	0	0	0	0	0	−1	0	1	0	0	0	1	0	4	
0	0	1	0	−1	0	−1	0	0	0	1	0	0	0	1	1	0	1	4

Series (10) 11.2 12.3 13.3 14.1

−1 2 1 1 0 1 0 −1 0 1 4
0 −1 0 0 1 0 1 0 0 1 1 4
0 0 0 0 0 0 0 0 0 −1 −1 0 4
0 0 0 0 0 0 0 0 −1 1 0 1 −2 4

Series (11.2) 12.2

0 0 0 0 0 0 0 0 0 −2 0 4

Series (17.2) 18.3 19

0 −1 1 0 0 0 1 −1 1 0 −1 0 1 0 1 1 −1 4
0 0 0 0 0 0 0 0 1 0 −2 0 0 0 −1 0 1 1 4

Series (16.3) 17.3 18.3

1 0 0 1 −1 0 0 0 −1 0 1 0 0 −1 1 0 4
0 0 0 0 0 0 0 0 −1 1 0 0 0 0 −1 0 −1 4

Series (14.3) 15.4 16.4 17.4 18.3

1 1 0 −1 0 −1 0 1 0 0 1 0 0 0 4
0 −1 0 1 0 0 1 0 −1 1 1 −1 1 0 1 4
0 0 0 0 0 0 0 0 0 −1 0 0 1 0 0 1 4
0 0 0 0 0 0 0 0 0 −1 0 1 0 0 1 −1 0 4

Series (13.2) 14.4 15.2

−1 0 −1 0 0 0 1 0 −1 0 1 1 1 4
1 0 1 −1 0 −1 1 0 1 1 0 0 0 0 4

Series (15.4) 16.3

0 0 0 0 0 0 0 0 1 −1 −2 1 0 −1 0 4

Series (16.4) 17.3

0 0 0 0 0 0 0 0 0 0 0 0 0 −1 1 −1 4

Series (12.2) 13.1

−2 −1 0 0 0 1 0 −1 1 1 0 0 4

Series (12.2) 13.3

−1 −1 −1 1 1 1 −1 1 1 1 −1 1 4

3. Minimum 4: The κ-lattices

It is well known that there are integral lattices with mimimum 4 in dimensions 11, 12, and 13, which have a smaller determinant than the laminated lattices constructed in §2, cf. [1, Chapter 6], [2], and [3]. It is therefore of interest to

TABLE 2. Invariants of the lattices K_i and K_i'

dim	det	$\frac{1}{2}\lvert L_{\min}\rvert$	$\lvert\mathrm{Aut}(L)\rvert$
7	$384 = 2^4 24$	$46 = 10 + 16 + 20$	$2^9 3 \cdot 5$
8.1	$576 = 2^2 12^2$	$66 = 6 + 12 + 48$	$2^{10} 3^2$
8.2	$512 = 2^4 4 \cdot 8$	$75 = 1 + 12 + 30 + 32$	$2^{11} 3^2 5$
9.1	$864 = 6^2 24$	$90 = 6 + 6 + 12 + 18 + 48$	$2^8 3^2$
9.2	$768 = 2^4 4^2 12$	$99 = 3 + 2 \cdot 12 + 24 + 48$	$2^{11} 3^2$
10	$972 = 3^2 6 \cdot 18$	$138 = 3 + 54 + 81$	$2^6 3^5$
11	$972 = 3^4 12$	$216 = 1 + 80 + 135$	$2^9 3^4 5$
12	$729 = 3^6$	378	$2^{10} 3^7 5 \cdot 7$
13	$972 = 3^4 12$	$459 = 81 + 108 + 270$	$2^9 3^5 5$
14.1	$972 = 3^2 6 \cdot 18$	$621 = 27 + 108 + 2 \cdot 243$	$2^6 3^6$
14.2	$972 = 3^3 6^2$	$624 = 3 + 108 + 243 + 270$	$2^{10} 3^6 5$
15.1	$864 = 6^2 24$	$873 = 3 \cdot 9 + 54 + 2 \cdot 72 + 3 \cdot 216$	$2^8 3^3$
15.2	$972 = 3^3 36$	$822 = 1 + 2 + 27 + 36 + 2 \cdot 81 + 108 + 3 \cdot 162$	$2^7 3^5$
16.1	$576 = 2^2 12^2$	$1386 = 6 + 36 + 72 + 216 + 288 + 768$	$2^{12} 3^3$
16.2	$729 = 3 \cdot 9 \cdot 27$	$1218 = 3 + 27 + 2 \cdot 54 + 81 + 108 + 162 + 243 + 486$	$2^4 3^5$
16.3	$729 = 3^4 9$	$1224 = 36 + 216 + 972$	$2^9 3^7$
17.1	$384 = 2^4 24$	$2133 = 3 + 30 + 180 + 2 \cdot 960$	$2^{13} 3^2 5$
17.2	$486 = 3^3 18$	$1872 = 36 + 26 + 648 + 972$	$2^6 3^6$
18	$243 = 3^5$	3240	$2^9 3^9 5$
19	$162 = 3^3 6$	$4698 = 3240 + 1458$	$2^7 3^7 5$
20	$81 = 3^2 9$	$7695 = 7290 + 405$	$2^7 3^8 5$
21	$36 = 3 \cdot 12$	$13041 = 8505 + 4536$	$2^{11} 3^6 5 \cdot 7$

investigate the effect of the lamination procedure on these lattices. As described in [1], these dense lattices form part of a sequence $K_1 \subset K_2 \subset \cdots \subset K_{24}$ of sublattices of the Leech lattice Λ_{24}. For $n < 7$ and $n > 17$ these K_i are isometric to the Λ_i. Applying the lamination process to K_7 resp. K_8 yields new lattices K_8' resp. K_9' (labeled 8.2 and 8.3 in the left side of the figure of §2), which are embedded into Λ_9 resp. Λ_{10} by the lamination process. The result of applying it to K_9 is shown in the left side of the figure in §2: Apart from the old κ-lattices, namely 9 to 13 and 14.1 to 17.1, one gets new lattices K_i' in dimensions 14 to 21, owing to a branching from dimension 13 to 14. However, all these lattices lie inside the Leech lattice. All the embedding numbers of the i-dimensional lattices in the $i+1$-dimensional ones are one with the exception of K_{21}' in Λ_{22}, where the embedding number is two. The main invariants of the lattices which turn up are listed in Table 2.

We note that K_{12}, K_{18}', and K_{21}' have irreducible maximal finite subgroups of $GL_n(\mathbb{Q})$ as their automorphism groups, which are in the terminlogy of [5] as follows: $[6 \cdot SU_4(3) \cdot 2^2]_{12}$, $[\pm 3_+^{1+2} SP_4(3) \cdot 2]_{18}$, and $[\pm PSU_4(3) \cdot D_8]_{21}$. Cf. also [9] concerning a common setting for K_{18}' and $\Lambda_{16.1}$.

Appendix. Gram matrices of the κ-lattices. Gram matrices of all the κ-lattices are given below in the same way as for the Λ-lattices in the first appendix.

Series $\Lambda_1 \ldots \Lambda_6\ K_7 \ldots K_{13}\ K'_{14.2} \ldots K'_{17.2}\ K'_{18} \ldots K'_{21}\ \Lambda_{22}$

4
2 4
0 −2 4
0 −2 0 4
0 0 −2 0 4
−2 −2 0 0 0 4
−2 −1 1 0 0 0 4
−2 −1 0 −1 1 2 2 4
−2 −2 0 1 1 2 2 2 4
−2 0 −2 0 1 1 0 0 0 4
1 1 0 0 0 −2 0 −1 −1 −2 4
−2 −1 0 0 0 1 1 1 1 1 −2 4
0 −1 1 1 0 −1 1 0 0 −1 1 −1 4
0 0 0 0 0 0 0 0 0 0 0 0 −2 4
0 −1 0 0 1 1 0 1 1 −1 0 0 1 −1 4
0 0 1 0 −1 0 1 0 0 0 −1 0 0 1 0 4
0 0 0 0 0 0 0 0 0 0 0 0 −1 1 0 0 4
0 0 0 0 0 0 0 0 0 0 0 0 −1 1 1 0 1 4
1 0 −1 1 1 0 −1 −1 0 0 0 0 0 0 1 −1 0 1 4
0 0 0 0 0 0 0 0 0 0 0 0 −1 1 1 1 0 1 0 4
−1 −1 1 −1 0 1 1 1 1 0 −1 1 0 0 1 0 0 0 0 1 4
0 0 0 −1 0 1 −1 0 0 0 −1 1 −1 1 0 0 0 0 0 2 2 4

Series $(K_7)\ K'_8\ \Lambda_9$

0 1 −1 0 0 0 −2 4
0 0 0 0 0 0 0 −2 4

Series $(K_8)\ K'_9\ \Lambda_{10}$

0 1 0 −1 −1 0 −1 −2 4
0 0 0 0 0 0 0 0 −2 4

Series $(K'_{15.2})\ K'_{16.3}\ K'_{17.2}$

0 0 0 0 0 0 0 0 0 0 0 0 −1 1 −1 4
0 0 0 0 −1 1 −1 −1 −1 1 −1 0 0 0 −1 1 4

Series $(K_{13})\ K_{14.1} \ldots K_{17.1}\ \Lambda_{18.1}$

0 0 0 0 0 0 1 0 1 −1 1 −1 1 4
0 0 0 0 −1 1 −1 0 0 0 −1 0 0 −1 4
−1 0 0 −1 0 0 0 0 0 1 −1 1 0 0 −1 4
0 0 0 0 0 0 0 1 0 −1 1 −1 1 0 1 −1 4
−1 −1 1 0 0 0 1 0 1 0 0 0 1 1 0 0 0 4

We take the opportunity to correct a mistake in [6], where we stated on the last page that $\mathrm{Aut}(K_{10})$ is irreducible, but not absolutely irreducible as a subgroup of $GL_{10}(\mathbb{Z})$. The group given there has indeed this property, but is only a subgroup of index two of the full automorphism group of the lattice. We note, however, that meanwhile examples of maximal finite irreducible, but not absolutely irreducible, subgroups of $GL_n(\mathbb{Z})$ (even $GL_n(\mathbb{Q})$) have been found, cf. [4] for $n = 36$ and [5] for $n = 20$, thus answering this question of H. Zassenhaus.

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