

Supplement to
THE STRUCTURE OF THE PROJECTIVE INDECOMPOSABLE MODULES
OF THE SUZUKI GROUP $Sz(8)$ IN CHARACTERISTIC 2

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Table 1. *Tensor products of simple $Sz(8)$ -modules*

$4_a \otimes 16_a$	$4_b \otimes 16_b$	$4_c \otimes 16_c$
4_c	4_a	4_b
16_c	16_a	16_b
4_c	4_a	4_b
I	I	I
4_a	4_b	4_c
I	I	I
4_b	4_c	4_a
I	I	I
4_a	4_a	4_b
I	I	I
4_c	4_a	4_b
16_c	16_a	16_b
4_c	4_a	4_b

$4_a \otimes 16_b$	$4_b \otimes 16_c$	$4_c \otimes 16_a$
4_b	4_c	4_a
I	I	I
4_c	4_a	4_b
I	I	I
4_a	4_b	4_c
I	I	I
4_b 4_c	4_a 4_c	4_a 4_b
I 16_c	I 16_a	I 16_b
4_c	4_a	4_b
I I	I I	I I
4_a 4_b	4_b 4_c	4_a 4_c
I	I	I
4_c	4_a	4_b
I	I	I
4_b	4_c	4_a

$16_a \otimes 16_a$	$16_b \otimes 16_b$	$16_c \otimes 16_c$
I	I	I
4_b	4_c	4_a
16_b	16_c	16_a
4_b	4_c	4_a
I I	I I	I I
$4_c \ 4_c$	$4_a \ 4_a$	$4_b \ 4_b$
I I	I I	I I
$4_a \ 4_a \ 4_b$	$4_b \ 4_b \ 4_c$	$4_a \ 4_c \ 4_c$
I I 16 _b	I I 16 _c	I I 16 _a
$4_a \ 4_b \ 4_b \ 4_c$	$4_a \ 4_b \ 4_c \ 4_c$	$4_a \ 4_a \ 4_b \ 4_c$
I I 16 _a 16 _c	I I 16 _a 16 _b	I I 16 _b 16 _c
$4_a \ 4_c \ 4_c$	$4_b \ 4_a \ 4_b$	$4_c \ 4_b \ 4_c$
I I I I	I I I I	I I I I
$4_a \ 4_a \ 4_b \ 4_b$	$4_b \ 4_b \ 4_c \ 4_c$	$4_c \ 4_a \ 4_b \ 4_b$
I I I I 16 _b	I I I I 16 _c	I I I I 16 _a
$4_a \ 4_a \ 4_b \ 4_b \ 4_b$	$4_b \ 4_b \ 4_c \ 4_c \ 4_b$	$4_c \ 4_a \ 4_b \ 4_b \ 4_b$
I I I I 16 _a 16 _c	I I I I 16 _a 16 _b	I I I I 16 _b 16 _c
$4_a \ 4_a \ 4_b \ 4_b \ 4_c$	$4_b \ 4_b \ 4_c \ 4_c \ 4_c$	$4_c \ 4_a \ 4_b \ 4_b \ 4_c$
I I I I 16 _a 16 _b	I I I I 16 _a 16 _c	I I I I 16 _b 16 _a
$4_a \ 4_b \ 4_b \ 4_c$	$4_a \ 4_b \ 4_c \ 4_c$	$4_b \ 4_a \ 4_c$
I I 16 _b	I I 16 _c	I I 16 _a
$4_a \ 4_b \ 4_c$	$4_b \ 4_a \ 4_c$	I
I	I	I

Table 2. The socle series of the projective covers of the 16-dimensional simple modules of $S\ell_2(8)$

Π_{16_a}	Π_{16_b}	Π_{16_c}
I	I	I
4_a	4_c	4_a
16_b	16_c	16_a
4_b	4_c	4_a
I I	I I	I I
$4_c \ 4_c$	$4_a \ 4_a$	$4_b \ 4_b$
I I	I I	I I
$4_a \ 4_a \ 4_b$	$4_b \ 4_b \ 4_c$	$4_a \ 4_c \ 4_c$
I I 16 _b	I I 16 _c	I I 16 _a
$4_a \ 4_b \ 4_b \ 4_c$	$4_b \ 4_c \ 4_c \ 4_b$	$4_c \ 4_a \ 4_b \ 4_b$
I I I I 16 _b	I I I I 16 _c	I I I I 16 _a
$4_a \ 4_a \ 4_b \ 4_b \ 4_b$	$4_b \ 4_b \ 4_c \ 4_c \ 4_b$	$4_c \ 4_a \ 4_b \ 4_b \ 4_b$
I I I I 16 _a 16 _c	I I I I 16 _a 16 _b	I I I I 16 _b 16 _a
$4_a \ 4_a \ 4_b \ 4_b \ 4_c$	$4_b \ 4_b \ 4_c \ 4_c \ 4_c$	$4_c \ 4_a \ 4_b \ 4_b \ 4_c$
I I I I 16 _a 16 _b	I I I I 16 _a 16 _c	I I I I 16 _b 16 _a
$4_a \ 4_b \ 4_b \ 4_c$	$4_a \ 4_c \ 4_c \ 4_c$	$4_b \ 4_a \ 4_c$
I I 16 _b	I I 16 _c	I I 16 _a
$4_a \ 4_b \ 4_c$	$4_b \ 4_a \ 4_c$	I
I	I	I

Table 3. The socle series of the projective covers of the 4-dimensional simple modules of $Sz(8)$

Π_{4_a}	Π_{4_b}	Π_{4_c}
4_a		4_c
I 16 _a		I 16 _c
$4_c \ 4_a \ 4_b$	$4_a \ 4_b \ 4_c$	$4_b \ 4_c \ 4_a$
I I I 16 _c	I I I 16 _a	I I I 16 _b
$4_c \ 4_e \ 4_a \ 4_b \ 4_b$	$4_a \ 4_e \ 4_b \ 4_c \ 4_c$	$4_b \ 4_b \ 4_c \ 4_a \ 4_a$
I I I I I 16 _a 16 _b	I I I I I 16 _b 16 _c	I I I I I 16 _c 16 _a
$4_c \ 4_e \ 4_a \ 4_a \ 4_b \ 4_b$	$4_a \ 4_e \ 4_b \ 4_b \ 4_c \ 4_c$	$4_b \ 4_b \ 4_c \ 4_c \ 4_a \ 4_a$
I I I I I I 16 _c 16 _b	I I I I I I 16 _a 16 _c	I I I I I I 16 _b 16 _a
$4_c \ 4_e \ 4_e \ 4_a \ 4_b \ 4_b$	$4_a \ 4_e \ 4_a \ 4_b \ 4_c \ 4_c$	$4_b \ 4_b \ 4_b \ 4_c \ 4_a \ 4_a$
I I I I I I I 16 _c 16 _b	I I I I I I I 16 _a 16 _c	I I I I I I I 16 _b 16 _a
$4_c \ 4_e \ 4_e \ 4_a \ 4_a \ 4_b \ 4_b$	$4_a \ 4_e \ 4_b \ 4_b \ 4_b \ 4_c \ 4_c$	$4_b \ 4_b \ 4_c \ 4_c \ 4_c \ 4_a \ 4_a$
I I I I I I I I 16 _a 16 _b	I I I I I I I I 16 _b 16 _c	I I I I I I I I 16 _c 16 _a
$4_c \ 4_e \ 4_e \ 4_e \ 4_a \ 4_b \ 4_b$	$4_a \ 4_e \ 4_b \ 4_b \ 4_b \ 4_b \ 4_c$	$4_b \ 4_b \ 4_c \ 4_c \ 4_c \ 4_c \ 4_a$
I I I I I I I I I 16 _a 16 _b	I I I I I I I I I 16 _b 16 _c	I I I I I I I I I 16 _c 16 _a
$4_c \ 4_e \ 4_e \ 4_e \ 4_e \ 4_a \ 4_b$	$4_a \ 4_e \ 4_b \ 4_b \ 4_b \ 4_b \ 4_b \ 4_c$	$4_b \ 4_b \ 4_c \ 4_c \ 4_c \ 4_c \ 4_c \ 4_a$
I I I I I I I I I I 16 _a 16 _b	I I I I I I I I I I 16 _b 16 _c	I I I I I I I I I I 16 _c 16 _a
$4_c \ 4_e \ 4_e \ 4_e \ 4_e \ 4_e \ 4_a$	$4_a \ 4_e \ 4_b \ 4_b \ 4_b \ 4_b \ 4_b \ 4_b \ 4_c$	$4_b \ 4_b \ 4_c \ 4_c \ 4_c \ 4_c \ 4_c \ 4_c \ 4_a$
I I I I I I I I I I I 16 _a 16 _b	I I I I I I I I I I I 16 _b 16 _c	I I I I I I I I I I I 16 _c 16 _a
$4_c \ 4_e \ 4_e \ 4_e \ 4_e \ 4_e \ 4_e$		
I I I I I I I I I I I I 16 _a 16 _b		
4_e		4_e
I I I I I I I I I I I I 16 _a		I I I I I I I I I I I I 16 _a
$4_e \ 4_e$		$4_e \ 4_e$
I I I I I I I I I I I I 16 _b		I I I I I I I I I I I I 16 _b
$I \ 16_a$		$I \ 16_c$

Table 4. The socle series of the projective cover of the trivial modules of $Sz(8)$

APPENDIX.

The character table of $Sz(8)$ is										
Π_I		1A	2A	4A	4B	5A	7A	7B	7C	13A
I		1	1	1	1	1	1	1	1	1
$4_a 4_b 4_c$		14a	-2	2i	-1	0	0	0	1	1
		14b	-2	-2i	-1	0	0	0	1	1
		35a	3	-1	0	0	0	0	-a1	-a2
		35b	3	-1	0	0	0	0	-a3	-a2
		35c	3	-1	0	0	0	0	-a2	-a1
		64	0	0	-1	1	1	-1	-1	-1
		65a	1	1	0	β_1	β_2	β_3	0	0
		65b	1	1	1	0	β_3	β_2	0	0
		65c	1	1	1	0	β_2	β_3	0	0
		91	-5	-1	-1	1	0	0	0	0

where $a_1 = z + z^5 + z^{12}$, $a_2 = z^2 + z^3 + z^{11}$, $a_3 = z^4 + z^6 + z^7 + z^9$ with $z = e^{2\pi i / 13}$
and $\beta_1 = z + z^6$, $\beta_2 = z^2 + z^5$, $\beta_3 = z^3 + z^4$ with $z = e^{2\pi i / 7}$

The 2-modular character table of $Sz(8)$ is										
	1A	5A	7A	7B	7C	13A	13B	13C		
	1	1	1	1	1	1	1	1	1	1
	4a	-1	-1	- β_1	-1	- β_2	-1	- β_3	α_1	α_2
	4b	-1	-1	- β_3	-1	- β_1	-1	- β_2	α_3	α_1
	4c	-1	-1	- β_2	-1	- β_3	-1	- β_1	α_2	α_3
	4e	-1	-1	β_1	β_2	β_3	-1 + α_1	-1 + α_2	α_1	α_3
	16a	-1	β_3	β_1	β_2	β_3	-1 + α_3	-1 + α_2	-1 + α_3	-1 + α_1
	16b	-1	β_1	β_3	β_2	β_1	-1 + α_2	-1 + α_3	-1 + α_1	-1 + α_2
	16c	-1	β_2	β_2	β_3	β_1	-1 + α_2	-1 + α_3	-1 + α_1	-1
	64	-1	1	1	1	1	-1	-1	-1	-1

The decomposition matrix for $Sz(8)$ has the form

$$\begin{pmatrix} I & 4_a & 4_b & 4_c & 16_a & 16_b & 16_c \\ 1 & \left(\begin{matrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 14_a & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 14_b & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 35_n & 3 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 35_b & 3 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 35_c & 3 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 65_n & 5 & 2 & 3 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 65_b & 5 & 3 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 65_c & 5 & 3 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 91 & 7 & 3 & 3 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 64 & & & & & & & & & & & 1 \end{matrix} \right) \end{pmatrix}$$

The Cartan matrix for the principal block for $Sz(8)$ is given by

$$\begin{pmatrix} I & \left(\begin{matrix} 160 & 72 & 72 & 72 & 20 & 20 & 20 \\ 4_a & 72 & 34 & 32 & 32 & 10 & 8 \\ 4_b & 72 & 32 & 34 & 32 & 9 & 10 & 9 \\ 4_c & 72 & 32 & 32 & 34 & 8 & 9 & 10 \\ 16_a & 20 & 8 & 10 & 9 & 2 & 2 & 2 \\ 16_b & 20 & 9 & 8 & 10 & 2 & 2 & 4 \end{matrix} \right) \end{pmatrix}$$